

Rondo: Scalable and Reconfiguration-Friendly Randomness Beacon

Xuanji Meng

Tsinghua University

Email: mxj21@mails.tsinghua.edu.cn

Zhaoxin Yang

Tsinghua University

Email: zhaoxin_yang@tsinghua.edu.cn

Wenbo Xu

Blockchain Platform Division, Ant Group

Email: xuwenbo.xwb@antgroup.com

Ying Yan

Blockchain Platform Division, Ant Group

Email: fuying.yy@antgroup.com

Xiao Sui

Shandong University

Email: suixiao@mail.sdu.edu.cn

Kang Rong

Blockchain Platform Division, Ant Group

Email: rongkang.rong@antgroup.com

Shenglong Chen

Blockchain Platform Division, Ant Group

Email: shenglong.chensl@antgroup.com

Sisi Duan*

Tsinghua University

Email: duansisi@tsinghua.edu.cn

Abstract—We present Rondo, a scalable and reconfiguration-friendly distributed randomness beacon (DRB) protocol in the partially synchronous model. Rondo is the first DRB protocol that is built from batched asynchronous verifiable secret sharing (bAVSS) and meanwhile avoids the high $O(n^3)$ message cost, where n is the number of nodes. Our key contribution lies in the introduction of a new variant of bAVSS called batched asynchronous verifiable secret sharing with partial output (bAVSS-PO). bAVSS-PO is a weaker primitive than bAVSS but allows us to build a secure and more efficient DRB protocol. We propose a bAVSS-PO protocol Breeze. Breeze achieves the optimal $O(n)$ messages for the sharing stage and allows Rondo to offer better scalability than prior DRB protocols. Additionally, to support the reconfiguration, we introduce Rondo-BFT, a dynamic and partially synchronous Byzantine fault-tolerant protocol inspired by Dyno (S&P 2022). Unlike Dyno, Rondo-BFT provides a communication pattern that generates randomness beacon output periodically, making it well-suited for DRB applications.

We implement our protocols and evaluate the performance on Amazon EC2 using up to 91 instances. Our evaluation results show that Rondo achieves higher throughput than existing works and meanwhile offers better scalability, where the performance does not degrade as significantly as n grows.

I. INTRODUCTION

Randomness beacon, a notion formalized by Rabin [1], refers to a service that provides a continuous stream of shared randomness. Randomness beacon is useful in numerous applications such as Proof-of-Stake based blockchains [2, 3], blockchain sharding [4], electronic voting [5], asynchronous Byzantine fault-tolerant (BFT) consensus protocols [6]–[9], anonymous communication [10]–[13], smart contract based decentralized finance (DeFi) applications and non-fungible tokens (NFTs) [14, 15].

While there exists some trusted centralized randomness sources [16, 17], many recent efforts have been made to

distributed randomness beacons (DRB) that remove the need for a trusted third party [18]–[27]. DRB can typically be built from three different cryptographic primitives: verifiable delay functions (VDF) [22], threshold cryptosystems that require distributed key generation (DKG) [24]–[27], and verifiable secret sharing (VSS) [18, 19, 21, 23]. VDF-based approaches enable efficient DRB with unlimited and open participation. However, the fixed delay introduced by VDF only works in synchronous networks, where a known upper bound exists for message transmission and processing. Meanwhile, threshold cryptosystems make it possible to build DRB in partially synchronous networks (where the upper bound for message transmission and processing is unknown [28]) or fully asynchronous networks (there does not exist an upper bound). The DKG setup, however, especially during system reconfiguration (allowing the joining and leaving of nodes), can be costly. VSS-based DRB protocols thus become a perfect choice to avoid the need for DKG.

Existing VSS-based DRB protocols all employ a paradigm that combines VSS and Byzantine fault-tolerant state machine replication (BFT-SMR), assuming a system with n nodes and at most t of them are corrupted. A typical VSS-based protocol proceeds as follows. First, multiple VSS instances are used for each node to share its secret. Then, the BFT-SMR commits the secrets of $t + 1$ nodes. Finally, nodes reconstruct and aggregate the secrets, ensuring that the aggregated beacon value is unbiased and unpredictable. The paradigm works both as a standalone randomness beacon protocol and an *on-chain* randomness beacon provider. In the latter case, the BFT-SMR can be replaced with a blockchain, and the randomness values generated can be used on-chain, e.g., for smart contract-based DeFi applications.

Such a paradigm suffers from some limitations we seek to address. First, as parallel VSS instances are needed for every

round (by default, one beacon is expected to be generated in each round), the performance of the protocols usually degrades significantly as n grows. Second, most VSS-based protocols [19, 21, 23] still build DRB under the synchronous timing assumption and are not that robust in wide area networks. Third, although VSS-based protocols are reconfiguration-friendly, it is still unclear how to build reconfigurable randomness beacon in systems that do not make a synchronous assumption. Indeed, the only partially synchronous DRB, Spurt [18], does not support reconfiguration. Meanwhile, the only reconfiguration-friendly DRB solutions, RandPiper [19] and Optrand [29], assume a synchronous network.

Thus, an interesting research question to answer is:

Can we build a distributed randomness beacon protocol that is both scalable and reconfiguration-friendly in the partially synchronous network?

Our approach and our results. We propose Rondo, a scalable and the first reconfiguration-friendly randomness beacon protocol in the partially synchronous model. Our starting point is using batched asynchronous verifiable secret sharing (bAVSS) [37]–[39] to build DRB. Specifically, bAVSS allows the *dealer* to share a batch of B secrets at once, and the secret can be reconstructed independently. As we can share B secrets at once, the VSS phase only needs to be executed once every B rounds and the amortized latency of beacon outputs can be significantly reduced compared to previous works. A recent concurrent work of ours, Hashrand [31], also utilizes this idea.

Unfortunately, all known bAVSS protocols have $O(n^2)$ messages. As n parallel instances are required to build DRB, the DRB protocol has $O(n^3)$ messages and the bAVSS phase might be the bottleneck of the system. Additionally, just as mentioned above, it is still unclear how to support reconfiguration in a partially synchronous network.

Compared to prior VSS-based DRB protocols, Rondo has two crucial building blocks to make it scalable and reconfiguration-friendly: A new primitive called the *batched asynchronous verifiable secret sharing with partial output (bAVSS-PO)* and a new dynamic Byzantine fault-tolerant state machine replication protocol. A comparison of Rondo and existing works is summarized in Table I.

▷ *Batched asynchronous verifiable secret sharing with partial output (bAVSS-PO) and the Breeze protocol.* We propose a new bAVSS-PO primitive and a protocol called Breeze. bAVSS-PO has two nice properties we need for building DRB. First, it supports batching. Accordingly, the amortized latency of beacon output can be significantly reduced. Second, bAVSS-PO has a weaker *commitment* property than conventional bAVSS and is the *weakest* property we envision for DRB.

Our Breeze protocol has $O(n)$ messages for the sharing stage, which is optimal for VSS. Indeed, $O(n)$ messages are required even for a dealer to share the secret shares. The communication paradigm is inspired by that for consistent broadcast (CBC) [40]. By setting the batch size B as $O(\log n)$, Breeze achieves $O(\lambda n)$ communication and is the best result known so far. Indeed, existing state-of-the-art protocols achieve $O(\lambda n)$ amortized communication at the cost of $B = O(n)$ [39, 41]. Meanwhile, to support batching, we use Bulletproofs [42] for batch verification inspired by hbACSS [38]. Compared

to the approach used in hbACSS, our approach reduces the computational cost by a factor of B for batch verification.

▷ *Dynamic Byzantine fault-tolerant state machine replication (BFT-SMR).* Although VSS supports reconfiguration of the nodes relatively easily (e.g., via proactive secret sharing [43, 44] or asking the nodes to start fresh VSS instances), SMR cannot easily do so in the partially synchronous network. Inspired by a recent work on dynamic BFT called Dyno [45], we propose Rondo-BFT, a new partially synchronous and reconfiguration-friendly BFT protocol, which might be of independent interest. Different from Dyno that can be viewed as a reconfiguration-friendly version of PBFT [46], Rondo-BFT can be viewed as a reconfiguration-friendly version of HotStuff [47]. The HotStuff-style pipelining communication pattern is a better *fit* for randomness beacon, mainly because the beacons can be output periodically.

Our contributions. We make the following contributions:

- We provide a new DRB protocol Rondo. Rondo consists of a new bAVSS-PO primitive and dynamic BFT-SMR. The batching feature of bAVSS-PO and the dynamic feature of BFT-SMR allow us to achieve scalability and reconfiguration simultaneously. To our knowledge, Rondo is the first reconfiguration-friendly DRB in the partially synchronous model.
- We propose Breeze, a bAVSS-PO protocol. Breeze is both communication-efficient and computation-efficient. Breeze achieves $O(n)$ messages for the sharing stage, in contrast to $O(n^2)$ by prior bAVSS protocols. Meanwhile, the computational cost for batch verification is reduced by a factor of B compared to existing state-of-the-art.
- We propose Rondo-BFT, a dynamic BFT-SMR protocol in the partially synchronous model. Rondo-BFT can be viewed as a dynamic version of HotStuff that supports the joining and leaving of nodes, which is of independent interests.
- Our evaluation results on Amazon EC2 using up to 91 instances across four different regions show that Rondo achieves high throughput (beacon/min) and scalability. Unlike prior works, VSS does not become the bottleneck of the system anymore.

II. RELATED WORK

AVSS, ACSS, and HACSS. In the unconditional security setting (also known as information-theoretical model where the only assumption is authenticated channels), AVSS usually incurs high communication [48]–[50]. In this setting, there are two types: perfectly secure AVSS [51, 52] and statistically secure AVSS [53, 54]. In the computational setting, Cachin et al. [55] (assuming the DL assumption) proposed the first practical AVSS with optimal $O(n^2)$ messages assuming $n > 3t$. Many follow-up works optimize the communication complexity [56, 57]. If AVSS achieves the *completeness* property, it is sometimes called asynchronous complete secret sharing (ACSS). Additionally, if ACSS supports a high threshold (e.g., $2t + 1$ out of $3t + 1$ for the $n > 3t$ setting), it is also called high-threshold ACSS (HACSS) [56, 58, 59]. In this work, we propose a new bAVSS-PO primitive that has a weaker *commitment* property than ACSS and uses it to build a more efficient Breeze protocol. Our technique of supporting batching, however, is generic and can be integrated with any

| protocol | timing | fault tolerance | liveness | unpredictability | bias-resistance | re-configurable | communication | message | computation | crypto. primitives | crypto. assumptions* | setup |
|--------------------------|---------|-----------------|----------|------------------|-----------------|-----------------|--------------------------------------|----------|------------------------------------|-----------------------------|-------------------------------|-----------------|
| Cachin et al. [30] | async. | 1/3 | ✓ | ✓ | ✓ | ✗ | $O(\lambda n^2)$ | $O(n^2)$ | $O(n)$ | th-sig PVSS | CDH | DKG |
| RandHerd [26] | async. | 1/3 | ✓ | ✓ | ✓ | ✗ | $O(\lambda c^2 \log n)$ [♣] | $O(c^2)$ | $O(c^2 \log n)$ [♣] | +CoSi | DLog | DKG |
| Hashrand [31] | async. | 1/3 | ✓ | ✓ | ✓ | ✗ | $O(\lambda n^2 \log n)$ | $O(n^2)$ | $O(\lambda n \log n)$ [†] | AwVSS [¶] +Hash | RO | secure channels |
| BRandPiper [19] | sync. | 1/2 | ✓ | ✓ | ✓ | ✓ | $O(\lambda n^3)$ | $O(n^3)$ | $O(n^2)$ | VSS | q -SDH | SRS |
| GRandPiper [19] | sync. | 1/2 | ✓ | $t + 1$ | ✓ | ✓ | $O(\lambda n^2)$ | $O(n^2)$ | $O(n^2)$ | PVSS | q -SDH | SRS |
| GRandline [32] | sync. | 1/2 | ✓ | ✓ | ✓ | ✗ | $O(\lambda n^2)$ | $O(n^2)$ | $O(n)$ | +pairing +th-sig | AGM +Co-OMDL ^{††} | DKG |
| Optrand [29] | sync. | 1/2 | ✓ | ✓ | ✓ | ✓ | $O(\lambda n^2)$ | $O(n^2)$ | $O(n^2)$ [‡] | PVSS | q -SDH | SRS |
| Drand [33] | sync. | 1/2 | ✓ | ✓ | ✓ | ✗ | $O(\lambda n^2)$ | $O(n^2)$ | $O(n)$ | +Pairing th-sig | +SXDH Gap-CDH | +PKI DKG |
| Scrape [34] | sync. | 1/2 | ✓ | ✓ | ✓ | ✗ | $O(\lambda n^4)$ | $O(n^3)$ | $O(n^2)$ | PVSS +Pairing | DDH ^{**} | CRS +PKI |
| Spurt [18] | partial | 1/3 | ✓ | ✓ | ✓ | ✗ | $O(\lambda n^2)$ | $O(n^2)$ | $O(n^2)$ [‡] | PVSS +Pairing | DBDH | CRS +PKI |
| Dfinity [35] | partial | 1/3 | ✓ | ✓ | ✓ | ✗ | $O(\lambda n^3)$ [§] | $O(n^3)$ | $O(n)$ | th-sig | CDH | DKG |
| Rondo (this work) | partial | 1/3 | ✓ | ✓ | ✓ | ✓ | $O(\lambda n^2 \log n)$ | $O(n^2)$ | $O(n)$ | bAVSS-PO | DLog | CRS +PKI |

TABLE I: Comparison of existing randomness beacon protocols. *All protocols in the table assume random oracles. †All the protocols count the complexity of exponentiation, except Hashrand [31], which counts hash functions. ‡The computational cost is $O(n^2)$ for the dealer and $O(n)$ for other nodes. **This scheme can be instantiated both under the Decisional Diffie Hellman (DDH) assumption in the random oracle model and in the plain model under the Decisional Bilinear Square (DBS) assumption. §The complexity is $O(\lambda n^2)$ in the best case. ¶AwVSS stands for asynchronous weak verifiable secret sharing. The main difference between this scheme and ordinary VSS is that it allows nodes to reconstruct \perp in the reconstruction phase. ††Co-OMDL stands for co-one-more discrete logarithm [36], which is stronger than ordinary OMDL assumption. ♣ Randherd employs a committee-sampling approach that samples a committee of size c for the protocol to enhance the scalability. Such an optimization can be applied to almost all other protocols.

ACSS protocols. To demonstrate it, we integrate our approach with HAVEN [56] to build a batched HACSS protocol called BatchHAVEN. A concurrent work HAVEN++ [41] shares a lot similarity with BatchHAVEN. Additionally, our Breeze protocol shares some similarity with another concurrent work by Das et al. [59]. The protocol by Das et al. is a conventional ACSS protocol and it has $O(n^2)$ messages but Breeze is a bAVSS-PO protocol with $O(n)$ messages.

Batched VSS and packed VSS. Batched AVSS allows the dealer to share many secrets in parallel [37]–[39, 41]. hbACSS [38] proposes a batched ACSS protocol leveraging Bulletproofs [42]. Recently, Shoup and Smart (SS) proposed a lightweight batched AVSS using only lightweight cryptographic primitives, i.e., hash functions [37]. Compared to hbACSS, SS is less communication-efficient, which might be less interesting in wide-area networks. Our approach of supporting batching for Breeze is inspired by hbACSS. Compared to hbACSS, we reduce the computational cost by a factor of B for batch verification. Additionally, we found some subtle mistakes in hbACSS (in which case the verification might fail) and we fix them in our work.

Another relevant notion is packed secret sharing, which packs many secrets in one sharing [60]. Packed VSS does not suit our needs as all secrets need to be reconstructed at once.

Dynamic BFT and reconfiguration for SMR. Reconfiguration for crash fault-tolerant (CFT) protocols or synchronous BFT protocols are relatively easy. Many CFT protocols have

studied efficient reconfiguration that does not degrade the overall performance [61]–[63]. Prior synchronous randomness beacon protocols such as RandPiper [19] and OptRand [29] also support reconfiguration for BFT-SMR.

BFT-SMaRt is a partially synchronous BFT protocol that supports reconfiguration [64], where membership requests are treated as a special type of client requests and order all requests together. It was mentioned in Dyno [45] that by simplifying treating the *membership requests* (requests for joining or leaving) as standard transactions, a partially synchronous BFT protocol may have subtle liveness issues. In this work, we follow the definitions by Dyno and provide a new dynamic BFT protocol that can be viewed as dynamic HotStuff [47].

Randomness Beacon. Chainlink [65] is an industrial solution that has been deployed in a considerable number of applications. The randomness beacon is generated using verifiable random function (VRF) and thus requires trusted setup.

A concurrent work of ours, Hashrand [31], uses batched asynchronous weak verifiable secret sharing (BAwVSS) to construct a randomness beacon protocol. The BAwVSS primitive, being a weaker primitive than conventional AVSS, allows the randomness beacon protocol to have a lower amortized communication complexity. However, the *weak commitment* property of BAwVSS leads to a Monte-Carlo beacon, which terminates in a deterministic number of rounds, i.e., there is no guarantee on an output every round. In contrast, we provide a conventional DRB protocol in this work.

III. SYSTEM MODEL AND BUILDING BLOCKS

We consider a system consisting of a finite set of nodes $\{P_1, P_2, \dots\}$, where a fraction of them may fail arbitrarily (Byzantine failures) and they are controlled by an adversary \mathcal{A} . Nodes that are not Byzantine are called *honest* or *correct* nodes. The set of nodes that participate in the system may change dynamically. We assume a standard public-key infrastructure, i.e., each node P_i has a public/private key pair (pk_i, sk_i) and the public key pk_i is known by all nodes. We use a collision-resistant hash function $H(\cdot)$ mapping a message of arbitrary length to a fixed-length output.

We follow the notations in Dyno [45] for our definition of a reconfigurable system. In particular, we use *configuration* to denote the successive membership of the system. The configuration number c grows monotonically, beginning with 0. In each configuration c , there is a fixed set of nodes M_c and we call these nodes *members* of configuration c . Each node changes its configuration via configuration installation. Every correct node is aware of the initial configuration M_0 (i.e., the set of nodes when the system is started).

Let n_c denote the number of nodes in M_c (i.e., $|M_c|$) and t_c denote the number of c -faulty nodes in M_c . Dyno defines two assumptions regarding the relationship of n_c and t_c . We assume the *standard assumption* and additionally require a new *one-correct* assumption as summarized below.

- (a) **Standard assumption:** For any configuration M_c , $t_c \leq \lfloor \frac{|M_c|-1}{3} \rfloor$.
- (b) **One-correct assumption:** There exists at least one correct replica in M_0 that never leaves the system.

We may omit the subscript for n_c and t_c when no ambiguity arises. For instance, when we describe our bAVSS-PO protocol, we use n and t to denote the number of nodes and the number of faulty nodes in each instance.

We assume that all nodes have an agreement of a common reference string (CRS): $crs = (\mathbb{G}, \mathbf{g}, h)$, which contains a cyclic group \mathbb{G} , a vector of generators \mathbf{g} and another single generator h .

We consider the partially synchronous model [66], where there exists an unknown global stabilization time (GST) such that after GST, messages sent between two correct nodes arrive within a fixed delay.

Randomness beacon. We consider a system producing randomness beacon in this work. The protocol proceeds in *epochs* and each epoch has several *rounds*. By default, the protocol produces a beacon output in each round. In our work, a batch of secrets is shared once in each epoch and a randomness beacon output is revealed in each round. We borrow the notions from prior works [18, 20] and further refine the security goals of the randomness beacon protocol as follows.

- **Unpredictability.** Let r_1, r_2, \dots, r_i be the beacon outputs generated so far. The protocol is unpredictable if for every future beacon output r_j such that $j > i + 1$, for any PPT adversary \mathcal{A} as mentioned above and $i \geq 1$, there exists a negligible function $\text{negl}(\lambda)$ such that:

$$\Pr[(j, r'_j) \leftarrow \mathcal{A}(r_1, r_2, \dots, r_i) : r'_j = r_j] \leq \text{negl}(\lambda)$$

- **Bias-resistance.** Let $r_i(k)$ be the k^{th} bit of the beacon output r_i , $|r_i|$ be the number of bits of r_i , and $\mathcal{A}(r_1, r_2, \dots, r_{i-1})$ be the adversary having access to all the beacon outputs generated so far. The randomness beacon protocol is bias-resistant if for any PPT adversary \mathcal{A} where $i \geq 1$, and $k = 1, \dots, |r_i|$, there exists a negligible function $\text{negl}(\lambda)$ such that:

$$\left| \Pr \left[\begin{array}{l} (k, r'_i(k)) \leftarrow \mathcal{A}(r_1, r_2, \dots, r_{i-1}) : \\ r'_i(k) = r_i(k) \end{array} \right] - \frac{1}{2} \right| \leq \text{negl}(\lambda)$$

$$|\Pr[r_i(k) = 0] - \frac{1}{2}| \leq \text{negl}(\lambda)$$

- **Liveness.** A beacon output is generated in every round except with a negligible probability.
- **Public verifiability.** The randomness beacon protocol is public verifiable if any users that do not participate in the protocol can verify the beacon outputs.

We consider the public verifiability property an *optional* property for a randomness beacon protocol. Indeed, if the protocol serves as an oracle for some systems to query randomness beacons, public verifiability is necessary. In contrast, if the protocol only provides randomness beacons to participants of the protocol (e.g., providing on-chain randomness beacon), public verifiability is not necessary.

A. Building Blocks

Dynamic Byzantine fault-tolerant state machine replication (BFT-SMR or BFT for short) and types of requests. In BFT, a node *delivers or commits requests* submitted by clients. A node then sends a reply to the corresponding client. The client computes a final response based on the reply messages. There are two types of requests: regular request that consists of *normal* transactions; membership requests that consists of requests for membership changes. Membership request might be either a JOIN request (for adding a new node) or a LEAVE request (for removing a node). In our case, a valid JOIN request contains the public key of the sender as an unique identifier. Requests are also called *transactions* and we use them interchangeably in this work. We consider dynamic BFT in our work to build reconfigurable randomness beacon. The correctness of dynamic BFT is defined below.

- **Agreement:** If a correct node in epoch e delivers a request rq , then every correct node in the same epoch e eventually delivers rq .
- **Enhanced total order:** If a correct node in epoch e delivers a request with a sequence number k , and another correct node delivers a request rq' in epoch e' with the same sequence number, then $e = e'$ and $rq = rq'$.
- **Liveness:** If a correct client submits a request rq , then eventually a correct node in some configuration delivers rq .
- **Consistent delivery:** A correct client submitting rq will deliver a correct response which is consistent with the state in some configuration where rq is delivered.

Batched asynchronous verifiable secret sharing with partial output (bAVSS-PO). We introduce a new primitive bAVSS-PO and we first present the non-batched version AVSS-PO. Similar to conventional AVSS, an AVSS-PO protocol also consists of two stages: sharing and reconstruction. In the sharing stage, a node, also called a dealer, shares a secret s to all nodes.

In the reconstruction stage, nodes interact to reconstruct secret s . The sharing stage of AVSS-PO is *verifiable* and any node that completes the sharing stage receives some validation data that can be verified by any node. Additionally, every node produces some *partial* output during the sharing stage. Once a node receives the validation data proving that some node has completed the sharing state, the node can verify whether its partial output is *valid*. A secure AVSS-PO protocol satisfies the following properties.

- **Privacy.** If a correct dealer shared s using ID. d , then before any correct node starts reconstruction for ID. d , \mathcal{A} has no information about s .
- **Liveness.** 1) If the dealer P_d is correct throughout the sharing stage, then all correct nodes complete the sharing stage; 2) If at least $t + 1$ correct nodes validate their partial outputs and start reconstruction for ID. d , then every correct node P_i reconstructs some z_i for ID. d .
- **Correctness.** Once $t + 1$ correct nodes have completed the sharing for ID. d , there exists a fixed value z such that the following holds: 1) if the dealer shared s using ID. d and is correct throughout the sharing stage, then $z = s$ and 2) if a correct node P_i reconstructs z_i for ID. d , then $z_i = z$.
- **Commitment.** Once some node completes the sharing stage, the following holds: 1) at least $t + 1$ correct nodes that receive the validation data can validate their partial outputs; 2) there exists a secret s , such that in the reconstruction stage, if any honest node outputs s' , $s' = s$.

bAVSS-PO is the batched version of AVSS-PO. Namely, the dealer shares a batch of B shares $\{s_1, \dots, s_B\}$ in one instance and all the secrets are independent to each other.

Batched polynomial commitment. We extend Bulletproofs [42] to build polynomial commitments for batching and use them to build our protocol. Following Bulletproofs, our scheme requires a common reference string (CRS). Formally, a polynomial commitment P for a batch of secrets consists of three sub-algorithms:

- $\text{BatchCommit}(crs, \{S_i | i \in [1, B]\}, p) \rightarrow \hat{S}$ is given the common reference string crs , B polynomials $\{S_i | i \in [1, B]\}$, and the degree p . It outputs a commitment vector \hat{S} .
- $\text{BatchEval}(crs, \{S_i | i \in [1, B]\}, \hat{S}, (y_1, y_2, \dots, y_m), p) \rightarrow (V, \Phi)$ is given the common reference string crs , B polynomials $\{S_i | i \in [1, B]\}$, polynomial commitment \hat{S} , evaluation points arranged as vectors (y_1, y_2, \dots, y_m) , the degree of the polynomials p . It outputs the evaluations arranged as a matrix V , and the corresponding proof vector Φ .
- $\text{BatchVerifyEval}(crs, \hat{S}, \mathbf{v}, \mathbf{y}, \phi, p) \rightarrow \text{true} \setminus \text{false}$ takes as input the common reference string crs , a commitment \hat{S} , evaluation point \mathbf{y} , evaluation proof ϕ , and the degree of S_i , denoted by p . It outputs a Boolean.

Let \mathbb{G} be a cyclic group of a prime order q with a generator g . Given a polynomial $R(x)$ of degree p : $R(x) = r_0 + r_1x + \dots + r_px^p$, where the coefficients $r_i \in \mathbb{Z}_q$, Bulletproofs commitment for $R(x)$ is $\prod_{i=0}^p g_i^{r_i}$. A Bulletproofs commitment for $R(x)$ uniquely determines $R(x)$.

We also need the binding and hiding property of commitments [56]:

- **Binding.** Let $crs \leftarrow \text{Setup}(1^\kappa, \mathbb{F}, D)$. For any PPT adversary $\mathcal{A}(crs)$ that outputs a batching commitment \hat{S} , the degree p of the polynomials, and two evaluations $e = \langle \mathbf{v}, \mathbf{y}, \phi \rangle$ and $e' = \langle \mathbf{v}', \mathbf{y}', \phi' \rangle$, there exists a negligible function $\text{negl}(\lambda)$ such that:

$$\Pr \left[\begin{array}{l} (\hat{S}, e, e', p + 1) \leftarrow \mathcal{A}(crs) : \\ \mathbf{y} = \mathbf{y}' \wedge \mathbf{v} \neq \mathbf{v}' \\ \wedge \text{BatchVerifyEval}(crs, \hat{S}, \mathbf{v}, \mathbf{y}, \phi, p) \\ \wedge \text{BatchVerifyEval}(crs, \hat{S}, \mathbf{v}', \mathbf{y}', \phi', p) \end{array} \right] \leq \text{negl}(\lambda)$$

- **Hiding.** Let $crs \leftarrow \text{Setup}(1^\kappa, \mathbb{F}, D)$, p be an arbitrary integer less than D , and $I \subset \mathbb{F}$ be an arbitrary set of evaluation points with $|I| \leq p$. Randomly choose a batch of polynomials $\{S_i | i \in [1, B]\} \leftarrow \mathbb{F}[x]$ of degree p and construct its commitment $\hat{S} = \text{BatchCommit}(pp, \{S_i | i \in [1, B]\}, p)$. For all PPT adversaries \mathcal{A} , there exists a negligible function $\text{negl}(\lambda)$ such that:

$$\Pr \left[\begin{array}{l} E = \text{BatchEval}(crs, \{S_i | i \in [1, B]\}, \hat{S}, I, p, n, |I|) : \\ (x, \mathbf{v}) \leftarrow \mathcal{A}(crs, \hat{S}, E) \\ \mathbf{v}[k] = S_k(x), k \in [1, n] \wedge x \notin I \end{array} \right] \leq \text{negl}(\lambda)$$

In this paper, given a batch of polynomials $\{S_i | i \in [1, B]\}$, we use a matrix \mathbf{s} to denote the vectors of coefficients of them, and use $\mathbf{s}[i, :]$ to denote the i^{th} row of \mathbf{s} , which is the coefficient vector of S_i .

For completeness, we also present the definitions of other related primitives in Appendix A.

IV. OVERVIEW OF OUR APPROACH

Rondo follows the generic workflow of conventional randomness beacon protocols: in the commitment phase, each node first disseminates its secret shares via an AVSS protocol; in the agreement phase, a BFT-SMR protocol is executed to agree on a *common subset* of the secrets; in the reconstruction phase, nodes reconstruct the secrets corresponding to the common subset and then output the beacon values. Our contributions focus on both AVSS and the BFT-SMR, as summarized below.

Breeze. We propose Breeze, a bAVSS-PO protocol and bAVSS-PO is a new variant of AVSS as defined in Sec. III-A. As mentioned in the introduction, bAVSS-PO does not achieve the conventional *completeness* property of AVSS and instead achieve a weaker *commitment* property. Our insight is that the weaker property is sufficient for randomness beacon but allows us to build a more efficient protocol.

The communication paradigm of Breeze is inspired by consistent broadcast (CBC) [40]: nodes only needs to interact with the dealer so the sharing stage achieves $O(n)$ messages, which is optimal. The idea is to let each node validate its share and sends a digital signature to the dealer. After the dealer collects $2t + 1$ signatures, some secret s shared by the dealer is *fixed*. If correct nodes reconstruct the secret, they will output the same s , i.e., the commitment property.

Our Breeze protocol allows a dealer to share a batch of B secrets $\{s_1, \dots, s_B\}$ at once. A crucial property is that the reconstruction is flexible. Namely, each secret could be separately recovered by more than $t + 1$ honest nodes. In

this way, the commitment phase only needs to be executed once every epoch and the amortized latency of beacon outputs becomes very close to the latency of the BFT-SMR.

The technique of support batching is inspired by hbACSS [38] that uses Bulletproofs for verification of the evaluations of the sharing polynomials. The evaluations of the polynomials can be expressed by an inner product of the coefficients and the evaluation points. To evaluate a batch of polynomials of degree p for a group of evaluation points and generate the corresponding proof, Bulletproofs [42] inner-product argument can be obtained as follows,

$$[P_i \leftrightarrow P_j] : \hat{S}[m] = \prod_{k=0}^p g_k^{s_k^{[m,k]}} \wedge v = \langle \mathbf{s}_{m,:}, \mathbf{y}_{:,j} \rangle, m \in [1, B]$$

where $i, j \in [1, n]$ and $m \in [1, B]$. Namely, the prover P_i (i.e., the dealer) shares B polynomials $\{S_m | m \in [1, B]\}$ and each verifier (i.e., a node) P_j verifies all the polynomials at a given evaluation point y_j .

The advantage of using Bulletproofs, also as mentioned in hbACSS, is that its commitment size is constant and the proof size is logarithmic. The underlying idea of Bulletproofs is using a so called *folding* technique to recursively halve the degree of the polynomials to one.

Nevertheless, the drawback of the above approach is that each polynomial requires a halving computation so the computational cost is high. We therefore provide a more computation-efficient verification mechanism. The idea is to use a group of random numbers to sum the polynomials up to a combined one. As a result, our approach can eliminate a B factor of the computation complexity. Meanwhile, we found some subtle mistakes in the pseudocode of hbACSS and we provide a fixed version in our work.

Dynamic BFT. As mentioned previously, Rondo-BFT can be viewed as a *dynamic* HotStuff protocol. When we transform HotStuff into dynamic BFT, we face some unique safety and liveness challenges.

The safety challenge arises because the linear message complexity of HotStuff makes it extremely challenging to achieve the *agreement* property, a *required* property for building dynamic BFT as pointed out by previous work [45]. Indeed, the leader is the only node that collects the quorum certificates (a sufficiently large number of matching votes, also called QC or certificates) and the certificates are crucial for correct replicas to deliver client requests. For instance, when a membership request for adding a new node to the system is delivered, a faulty leader may choose to send the certificate to nodes in the system but not the new node. In this way, the new node will not join the system. To overcome this challenge, we carefully add some constraints for nodes that request to join or leave the system and nodes may need to forward their received messages to other nodes. Therefore, the agreement property is achieved by correct nodes nearly simultaneously for membership requests (i.e., all correct nodes deliver the membership requests with a bounded network delay). Meanwhile, agreement is eventually achieved for regular requests.

The liveness challenge mainly arises during view changes. This is because multiple nodes from different configurations may *compete* for becoming a new leader, again due to the

fact that agreement is challenging. To handle this problem, we additionally introduce some procedures in the normal-case operation. In particular, before any correct node installs a new configuration, it needs to collect certificates from more than two-third nodes in the current configuration. Accordingly, once a new configuration is installed, more than one-third correct nodes in the previous configuration are aware of the new configuration. Finally, only one leader will be selected once a view change occurs.

▷ **Initialization.**

01: - At the beginning of Rondo, set epoch $e \leftarrow 0$, round $r \leftarrow 1$.

02: - At the beginning of round r , **if** $r > B$ **then**

03: - Set $r \leftarrow r \bmod B$, $e \leftarrow e + 1$.

04: - Let CS_e denote common subset and $CS_e \leftarrow \emptyset$.

05: - Let $S_e \leftarrow [\perp]$ be the shares received from other nodes.

06: - Let M_e be the members of epoch e .

▷ **The commitment phase.**

07: - **if** $P_i \in M_e$ **and** $r \bmod B = 1$ **then**

08: - Select the random secrets $\mathbf{s}^i \leftarrow (s_1^{(i)}, \dots, s_B^{(i)})$.

09: - Query the Breeze $_i$.Share($in, share, \mathbf{s}^i$) for Breeze $_i$ and participate in other Breeze instances

10: - **Upon** completing the sharing stage of Breeze $_j$

11: - $S_e \leftarrow S_e \cup \{cer_{e,j}\}$, in which $cer_{e,j}$ is the validation data for Breeze $_j$.

▷ **The agreement phase.**

12: - **if** $r \bmod B = 1$ **then**

13: - Wait until $|S_e| \geq t+1$, RondoBFT.Propose $_i(e, r, (S_e, m))$ and let $S'_e \leftarrow S_e$ // m consists of additional input data

14: - Introduce the following predicate to the input S'_e of Rondo-BFT: S'_e is valid only if S_e consists of at least $t+1$ valid certificates.

15: - Wait until RondoBFT.Return $_i(e, r, (S'_e, M, *))$

16: - $CS_e \leftarrow S'_e$, $M_{e+1} \leftarrow M$.

17: - **else**

18: - RondoBFT.Propose $_i(e, r, (-, m))$

19: - Wait until RondoBFT.Return $_i(e, r, (-, *))$

▷ **The reconstruction phase.**

20: - **if** $r \bmod B = 1$ **then**

21: - For $j \in CS_e$, invoke ParVerify($cer_{e,j}, C, set_i^{e,j}$)

22: - For $k \in [1, B]$

23: - Aggregate the shares received from nodes in CS_e

24: - $r_k^{(i)} \leftarrow \sum_{j \in CS_e} s_k^{(j)}$.

25: - **Upon** a reconstruction request for round r :

26: - Output Breeze.Reconstruct($in, reconstruct, r, r_k^{(i)}$).

Fig. 1: The pseudocode of Rondo for node P_i , round r and epoch e . The ParVerify is functions provided by Breeze.

V. RONDO

We show the pseudocode of our randomness beacon protocol Rondo in Figure 1 and the workflow in Figure 2. The protocol consists of three phases: the *commitment* phase where nodes disseminate their secret shares via n parallel Breeze instances; the *agreement* phase where nodes agree on a common subset (denoted as CS_e) via BFT-SMR; the *reconstruction* phase where nodes reconstruct the secrets.

Rondo is epoch-based and there are B rounds in each epoch. Informally, a randomness beacon value is supposed to be produced in every round of the protocol and every epoch has B rounds, where B is the batch size for bAVSS-PO. Meanwhile, nodes may send requests for membership changes, i.e., a JOIN or LEAVE request.

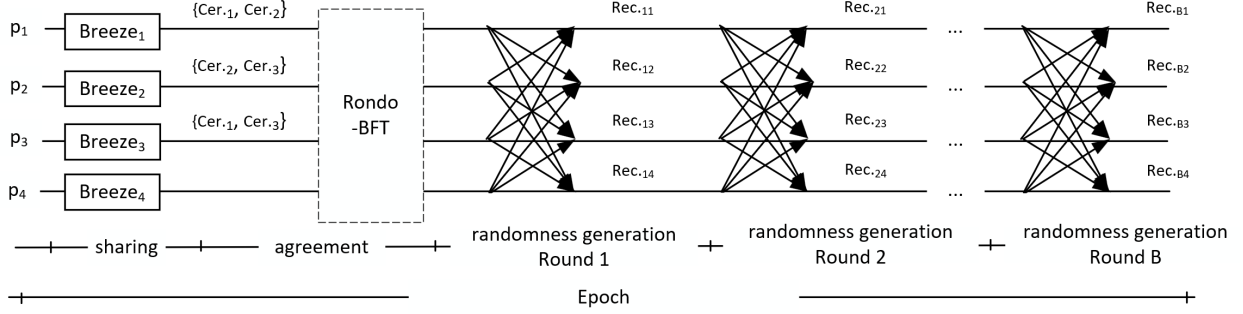


Fig. 2: The Rondo protocol.

Each phase proceeds as follows. First, every node executes the commitment phase once every B rounds (i.e., an epoch). In particular, node P_i starts an bAVSS-PO instance Breeze_i and shares B secrets s^i (lines 07-11). Second, the agreement phase is executed in every round r and there are two cases.

- If r is the first round of an epoch, every node waits until $n - t$ Breeze instances have completed and then proposes (S_e, m) to Rondo-BFT (lines 12-13). Here, S_e consists of the validation data for Breeze instances. Jumping ahead a little bit, we use cer_i to denote the validation data for Breeze_i . Additionally, m denotes the input data to Rondo-BFT. Every node then waits for the output of Rondo-BFT (line 15).
- If r is not the first round of an epoch, every node simply proposes $(-, m)$ to Rondo-BFT and then waits for the output of Rondo-BFT (lines 18-19).

There are two cases for m mentioned above. If we use Rondo as a dedicated randomness beacon protocol, the field m is either \perp or some membership requests. Alternatively, if Rondo is a system that generates on-chain randomness beacon, m may additionally include a batch of regular requests.

Finally, the reconstruction phase is by default triggered in every round. In each round r , nodes reconstruct the $(r \bmod e)^{\text{th}}$ secret among the B secrets, where e is the epoch number. As $|CS_e| \geq t + 1$, the reconstruction phase in fact reconstructs the secrets shared in $t + 1$ Breeze instances. We thus require each node to first verify its shares based on CS_e via the ParVerify function (line 21, to be described in Sec. VI) and then aggregate its shares in CS_e (lines 22-24). In this way, we only need one reconstruction instance to reconstruct all the secrets corresponding to CS_e (lines 25-26).

As our protocol supports reconfiguration of the nodes, we additionally use M_e to denote the members of epoch e . To simplify our protocol and do not waste the secrets shared by Breeze, we only support membership requests at the boundary of epochs. Namely, any membership requests are included in the m field as input to Rondo-BFT in the first round of an epoch (line 13). After Rondo-BFT outputs the results that involve membership requests, nodes update their M_{e+1} accordingly (line 16). Starting from epoch $e + 1$, nodes use M_{e+1} to identify the set of nodes in the system. We leave the discussion about the details in Sec. VII when we present Rondo-BFT. We show the proof for Rondo in Appendix B.

VI. BREEZE: COMPUTATION-EFFICIENT BAVSS-PO

In this section, we first present the workflow of the Breeze protocol using the batched polynomial commitment (defined in Sec. III) in a black-box manner and then discuss how the corresponding functions are instantiated.

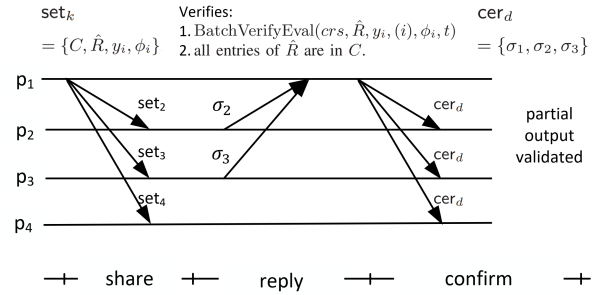


Fig. 3: The Breeze protocol.

A. The Main Workflow

The sharing stage (Figure 4). The sharing stage involves three communication phases: share, reply, and confirm, also as shown in Figure 3. In the share phase, the dealer P_d first generates B secrets $\{s_1, \dots, s_B\}$ and the shares for the nodes. Additionally, P_d generates correctness proofs for the shares using our new batching polynomial commitment BatchCommit and BatchEval. The dealer P_d also creates a vector commitment C for the shares by querying the vCom function (line 04). We instantiate the vector commitment using Merkle Tree.

In the reply phase, if each node P_i validates the share it receives from P_d , P_i sends a digital signature for C to P_d . At least $2t + 1$ matching signatures form a *certificate* cer_d . After collecting cer_d , P_d sends cer_d to the nodes.

The share each node receives from P_d is called a *partial output* for the batch of secrets. Additionally, cer_d is used as the *validation data* for the sharing stage. After receiving cer_d , each node can verify whether its partial output is valid. As the nodes may not receive cer_d in the sharing stage (i.e., when P_d is faulty), we also define a ParVerify function for each node to verify whether its partial output is valid. Namely, each node may trigger the ParVerify function even if the node does not complete the sharing stage.

Reconstruction stage (Figure 5). Each reconstruction procedure recovers one secret in the batch of B secrets and we use

idx as the index of the secret to be reconstructed. Specifically, each node P_i may hold the shares $\{R_k(i) | k \in [1, B]\}$ as the partial output of the sharing stage. Before participating in the reconstruction stage, each node needs to validate its partial output by invoking the ParVerify function (if it has not done so yet). If so, P_i sends its idx^{th} share $R_{idx}(i)$ to all nodes. After receiving $t + 1$ valid shares, P_i interpolates the corresponding points and recovers the secret $R_{idx}(0)$.

B. Instantiating Batched Polynomial Commitment

The batched polynomial commitment scheme consists of three functions: BatchCommit, BatchEval and BatchVerifyEval.

Batching commitments (Figure 6). The BatchCommit function outputs a vector \hat{S} as commitment for input polynomials. Informally, each entry of \hat{S} is a Bulletproofs commitment for corresponding input polynomial.

Batching evaluation (Figure 7). The BatchEval function evaluates input polynomials at specified points (y_1, y_2, \dots, y_n) . By slightly abusing notation, we use an evaluation point y_i to imply a vector $(1, y_i, y_i^2, \dots, y_i^p)$. Thus, given a coefficient vector \mathbf{s} , the evaluation at point y_i can be written as an inner product $\mathbf{s} \cdot y_i$ (line 03). The function consists of two parts: using inner product to compute the polynomial evaluations (lines 03-06) and invoking InnerProductProof to generate proof for the evaluations (lines 07-12).

Batching verification (Figure 8). The function verifies the correctness of the output of the BatchEval function. Briefly speaking, each node re-computes the inner product statement (lines 01-04) and uses the VerifyInnerProduct function to validates its correctness (line 05).

C. System Optimization in Rondo

When Breeze is used in Rondo, we further optimize the reconstruction stage to reduce the computational overhead. The idea is that the reconstruction stage reconstructs $t + 1$ secrets (that belong to the common subset CS_e) at a time and every node computes the Lagrange interpolation as follows:

$$R_{idx} = \sum_{i=1}^{t+1} S_{idx,i}(i) \prod_{j \neq i, j \in CS_e} \frac{-j}{i-j}$$

This costs $O(n^2)$ multiplication per round. Hence, in each epoch, each node needs to perform $O(n^2B)$ multiplications.

An interesting fact is that the product term is determined by the common subset CS_e and CS_e is fixed before the reconstruction stage (for any secret) begins. Therefore, we can pre-compute this product once in every epoch. Using this trick, each node only has to perform $O(n^2)$ multiplication during reconstruction, B times lower than trivial case.

D. Discussion

Breeze achieves $O(\lambda n(B + \log n))$ communication if instantiate the certificate using aggregate signatures. If we set B as $O(\log n)$, Breeze achieves $O(\lambda n)$ amortized communication. Our result is the most optimal result so far, as existing state-of-the-art protocols achieve $O(\lambda n)$ amortized

communication by setting B as $O(n)$ [39, 41]. We show the proof of Breeze in Appendix C.

The use of batched polynomial commitment for verification of the shares is inspired by hbACSS [38]. Our BatchEval function can be viewed as a computation-efficient function of that in hbACSS. In particular, BatchEval combines B polynomials into one polynomial (line 04 of Figure 7). Our function first generates a group of random numbers. These random numbers are used make a random linear combination of the input polynomials. The proof of the combined polynomial can also prove the correctness of the evaluations of the B polynomials. The computational complexity is thus B times lower than that in hbACSS.

Moreover, it is worth mentioning that our protocol does not have the subtle issues in hbACSS. First, the *hbPolyCommit-Core* algorithm [38, Fig.1] in hbACSS does not use the correct *crs* and the verifier will never be able to verify the shares. We fix this in line 09 of Figure 7. Second, in the sub-algorithm InnerProductProof of hbACSS, a Merkle Tree is used. When the size of the input vector is even, it is impossible to add the final column of coefficient matrix to the Merkle tree. We therefore introduce a parameter *flag* and use it when generating Merkle tree (lines 22-28) to distinguish the cases.

Our improvement for using batched polynomial commitment to support batched AVSS is generic and can be used for any bAVSS protocols. For instance, if we integrate our batched polynomial commitment with an ACSS protocol HAVEN [56], we obtain a batched ACSS protocol BatchHAVEN. Compared to hbACSS, BatchHAVEN only assumes discrete logarithm (DLog) in the random oracle model. Besides, BatchHAVEN does not require PKI. We leave the discussion about BatchHAVEN in Appendix D.

VII. RONDO-BFT: DYNAMIC BFT

We present Rondo-BFT, a dynamic HotStuff protocol [47]. HotStuff is a leader-based partially synchronous protocol. The agreement of a block (called normal-case operation) in HotStuff involves four phases: propose, prepare, pre-commit, and commit. Following the notion of Rondo, membership changes only occur at the boundary of the epochs. Each epoch has B rounds and up to B blocks can be delivered in an epoch. The protocol is view-based and there is only one leader in each view v and configuration. We use $\text{LEADER}(v, M)$ to denote the leader of view v given members of a configuration M . A *view change* mechanism might be started to elect a new leader. When we present the pseudocode in this section, we highlight the workflow different from HotStuff (which is designated to handle membership requests) in **green**.

A. Rondo-BFT-Specific Data Structures

Blocks and branches. A block is in the form $b = [pl, view, height, rq]$, where pl is the hash of the parent block of b , $view$ is the view number, $height$ is the number of blocks on the branch led by b , rq is a batch of client requests. The first block on a branch is set as an empty *genesis block* with height 0. For a block b , we use $b.x$ to denote the element x of b . If block b satisfies $b.height \in [eB + 1, eB + B]$, then b is a block in epoch e . If b is the first block in epoch e , i.e., $b.height \bmod B = 1$, then $b.rq$ may contain membership


```

▷ Initialization: Set  $flag_1 \leftarrow \text{false}$ . //for validation of the partial output
▷ The share phase.
01: - Upon receiving (ID.d, in, share,  $s_1, \dots, s_B$ ), as a dealer (i.e.,  $P_i = P_d$ ):
02: - For  $k \in [1, B]$ , randomly choose a recovery polynomial  $R_k \in \mathbb{F}[x]$  of degree  $t$  s.t.  $R_k(0) = s_k$ .
03: - Compute  $\hat{R} \leftarrow \text{BatchCommit}(crs, \{R_k | k \in [1, B]\}, t)$ .
04: - Compute  $C \leftarrow \text{vCom}(\hat{R}[1], \dots, \hat{R}[B])$ . //create a root commitment
05: - Compute  $(Y, \Phi) \leftarrow \text{BatchEval}(crs, \{R_k | k \in [1, B]\}, \hat{R}, (1, \dots, n), t)$ . //evaluates of the polynomials
06: - For  $k \in [1, n]$ ,  $y_k \leftarrow Y[:, k]$ ,  $\phi_k \leftarrow \Phi[k]$ .
07: - For  $k \in [1, n]$ , send (ID.d, send,  $set_k$ ) to node  $P_k$ , where  $set_k = \{C, \hat{R}, y_k, \phi_k\}$ . //executed by any node
▷ The reply phase.
08: - Upon receiving (ID.d, send,  $set_d$ ) from the dealer  $P_d$  for the first time:
09: - Verify that  $\text{BatchVerifyEval}(crs, \hat{R}, y_i, (i), \phi_i, t)$  returns true.
10: - Verify that all entries of  $\hat{R}$  are in  $C$ .
11: - Set  $flag_1 \leftarrow \text{true}$  and create a digital signature  $\sigma_i$  for  $C$ .
12: - Send (ID.d, reply,  $\sigma_i$ ) to dealer  $P_d$ .
▷ The confirm phase.
13: - if  $P_i$  is the dealer (i.e.  $P_i = P_d$ ) then
14: - Let  $cer_i$  be the set of signatures received from other nodes,  $cer_i \leftarrow [\perp]$ .
15: - Upon receiving (ID.d, reply,  $info_{j,i}$ ) from  $P_j$  for the first time:
16: - If  $\sigma_j$  is a valid signature for  $C$ , then  $cer_i \leftarrow cer_i \cup \{\sigma_j\}$ .
17: - if  $|cer_i| \geq 2t + 1$  then
18: - Send (ID.d, confirm,  $C, cer_i$ ) to all nodes.
19: - Output (ID.d, out, shared,  $set_i$ ).
20: - Upon receiving (ID.d, confirm,  $C, cer_d$ ) from  $P_d$  for the first time where  $cer_d$  is a valid certificate for  $C$  //executed by any node
21: - if  $flag_1 = \text{true}$  then //the partial output is validated
22: - Output (ID.d, out, shared,  $set_i$ )
▷ func ParVerify( $cer_d, C, set_i$ ):
23: - if  $cer_d$  is a valid certificate for  $C$  and  $C = set_i.C$  and  $\text{BatchVerifyEval}(crs, \hat{R}, y_k, (i), \phi_i, t)$  then
24: - Output true. //the partial output is validated
25: - Output false.

```

Fig. 4: The sharing stage of Breeze, for node P_i and tag ID.d. A certificate cer_d is a valid certificate for C if cer_d consists of $2t + 1$ digital signatures for C .

```

01: - Upon Breeze.Reconstruct (in, share,  $s, idx, R_{idx}(i)$ ): //reconstruct the  $idx^{th}$  secret.
02: - Send (reconstruct-share,  $C, \hat{R}[idx], y_i[idx], \phi_i[idx]$ ) to node  $P_j$ , for  $j \in [1, n]$ .
03: - Upon receiving (reconstruct-share,  $C, \hat{R}[idx], y_m[idx], \phi_m[idx]$ ) from node  $P_m$ :
04: - if  $\hat{R}[idx]$  is in  $C$  and  $\text{BatchVerifyEval}(crs, \hat{R}[idx], y_m[idx], (m), \phi_m[idx], t)$  returns true then
05: - if  $t + 1$  valid reconstruct-share messages with the same  $C$  are received then
06: - Interpolate  $R[idx]$  from the  $t + 1$  valid points.
07: - Output (out, reconstructed,  $R_{idx}(0)$ )

```

Fig. 5: The reconstruction stage of Breeze for the idx^{th} secret in the batch. Code is shown for node P_i and tag ID.d.

```

func BatchCommit( $crs, \{S_i | i \in [1, B]\}, p$ ):
01: - For  $j \in [1, B]$ , compute  $\hat{S}[j] = \prod_{i=1}^{p+1} g_i^{s[j,i]}$ , where  $s[j, :]$ 
are the coefficients vector of  $S_j$ .
02: - Output  $\hat{S}$ 

```

Fig. 6: The BatchCommit function.

requests. Otherwise, $b.rq$ only contains regular requests. In addition, $b.parent$ denotes the parent block of b .

Message format. A message m contains several fields: $m.type$, $m.view$, $m.block$, $m.justify$. $m.type \in \{\text{new-view}, \text{prepare}, \text{pre-commit}, \text{commit}\}$. $m.view$ is the view number of m . m is called a message for block b , if $m.block = b$. $m.justify$ includes quorum certificate(s) or digital signature(s). For a message m proposed by P_i , we use $m.sig = \sigma_i$ to denote a signature for the tuple $(m.type, m.view, \text{hash}(m.block))$.

Quorum certificates (QC) and view-change certificate. A QC consists of $2t + 1$ matching votes for a message m , which is instantiated by an aggregate signature in our work. There are

three types of QCs in the normal-case operation: *prepareQC*, *precommitQC*, *commitQC*. Given a quorum certificate qc for m , $\text{type}(qc)$ is $m.type$ and $\text{block}(qc)$ is $m.block$. We use $qc.x$ to denote the element x of $m.block$. Meanwhile, view-change certificates are generated during view changes. A view change certificate vc for view v consists of new-view messages in view v from more than two-third nodes in a configuration. Based on vc , each node P_i checks whether it is the correct new leader and then enters the new view. Any node can verify the identity of P_i via vc . Here, vc is also called a view-change certificate for P_i .

Rank of QCs and blocks. We use the notion of *rank* [67]–[69] in our work. Given a QC qc for a block, the $\text{rank}(qc)$ function does not explicitly return a value. Instead, we only care if the rank of a block is higher than that of another one. In our protocol, ranks equal heights by default. During view changes, a block with a higher view has a higher rank than other blocks. The rank of qc is the same with that of $\text{block}(qc)$.

Local state. Each node maintains the following state parameters: (i) the current view number $cview$; (ii) a quorum

```

func BatchEval( $crs, \{S_i | i \in [1, B]\}, \hat{S}, (y_1, y_2, \dots, y_n), p$ ):
01: - Parse  $crs$  as  $(\mathbb{G}, \mathbf{g}, h)$ .
02: - Randomly select a blind mask  $\mathbf{d}$ , then set  $\hat{D}$  as the
    commitment of  $\mathbf{d}$ .
03: -  $V \leftarrow \mathbf{s} \cdot \mathbf{y}, V_D \leftarrow \mathbf{d} \cdot \mathbf{y}$ .
04: - Compute  $\boldsymbol{\sigma} \leftarrow \sum_{i=1}^B \gamma_i \mathbf{s}[i, :]$ , in which  $\gamma_i$  is the random
    number generated by Fiat-Shamir heuristic.
05: - Compute  $\widehat{SD} \leftarrow \text{BatchCommit}(crs, \boldsymbol{\sigma} + \mathbf{d}, p)$  as a
    commitment of the polynomial covered with blind mask.
06: - Compute  $V' \leftarrow \sum_{i=1}^B \gamma_i V[i, :] + V_D$  as the corresponding
    evaluations.
07: - For  $i \in [1, n]$ , generate the inner product statement  $stmt_i$ 
    and the challenges  $z_i \leftarrow H(stmt_i)$ .
08: - Compute  $\widehat{SD}'[i] \leftarrow \widehat{SD} h^{z_i \cdot V'[i]}$ .
09: -  $\mathbf{h}^z \leftarrow (h^{z_1}, \dots, h^{z_n})$ .
10: -  $\Phi \leftarrow \text{InnerProductProof}((\mathbb{G}, \mathbf{g}, \mathbf{h}^z), \widehat{SD}', \mathbf{y}, p, \boldsymbol{\sigma} + \mathbf{d})$ .
11: - For  $i \in [1, n]$ ,  $\Phi[i] \leftarrow \Phi[i] || \hat{D} || V_D[i]$ .
12: - Output  $(V, \Phi)$ .

func InnerProductProof( $(\mathbb{G}, \mathbf{g}, \mathbf{h}), A, \mathbf{y}, p, \mathbf{a}$ ):
13: - Initialize the proof and a flag,  $\Phi \leftarrow \perp, flag \leftarrow 0$ 
14: - if  $p = 1$  then
15:   - Affix the proof  $\Phi$  with the only entry of the vector.
16:   - Output  $\Phi$ .
17: - if  $p$  is odd then
18:   - Cut off the last entry of  $\mathbf{a}$  and let  $\tilde{a}$  denote the negative
    value of this entry. Set this entry into the proof. Also, cut off the last
    entry of  $\mathbf{y}$  and  $\mathbf{g}$ .
19:   - Set  $flag \leftarrow 1, p \leftarrow p - 1, A[j, i] \leftarrow A[j, i] g_p^{\tilde{a}[j]} h^{\tilde{a}[j] \mathbf{y}[p, i]}$ .
20:   - Let  $p \leftarrow p/2$ , then halve  $\mathbf{a}, \mathbf{y}, \mathbf{g}$  into sub-matrices: let  $\mathbf{a}_L, \mathbf{y}_L, \mathbf{g}_L$ 
    be the left halves and  $\mathbf{a}_R, \mathbf{y}_R, \mathbf{g}_R$  be the right halves.
21:   - Set the following local parameters:  $\mathbf{c}_L \leftarrow \mathbf{a}_L \mathbf{y}_R,$ 
     $\mathbf{c}_R \leftarrow \mathbf{a}_R \mathbf{y}_L, L[i] \leftarrow \mathbf{g}_R^{\mathbf{a}_L} h^{z_i \mathbf{c}_L[i]}, R[i] \leftarrow \mathbf{g}_L^{\mathbf{a}_R} h^{z_i \mathbf{c}_R[i]}$ .
22:   -  $\Phi[i] \leftarrow \Phi[i] || (L[i], R[i])$ .
23:   - Build a Merkle Tree over all transcript as follows:
24:   - if  $flag = 1$  then
25:     -  $leaf_i \leftarrow H(\mathbf{g}, p, h^{z_i}, \mathbf{y}[:, i], \mathbf{A}[i], L[i], R[i], \tilde{a})$ 
26:     -  $z \leftarrow \text{roothash}, b_i \leftarrow \text{MerkleBranch}(i)$ .
27:   - else
28:     -  $leaf_i \leftarrow H(\mathbf{g}, p, h^{z_i}, \mathbf{y}[:, i], \mathbf{A}[i], L[i], R[i])$ .
29:     -  $z \leftarrow \text{roothash}, b_i \leftarrow \text{MerkleBranch}(i)$ .
30:   -  $\Phi[i] \leftarrow \Phi[i] || (z, b_i)$ .
31:   -  $A'[i] \leftarrow L^{z^2} [i] A[i] R^{z^{-2}} [i], \mathbf{g}' \leftarrow \mathbf{g}_L^{z^{-1}} \mathbf{g}_R^{z^1}$ .
32:   -  $\mathbf{y}' \leftarrow z^{-1} \mathbf{y}_L + z^1 \mathbf{y}_R, \mathbf{a}' \leftarrow z^{-1} \mathbf{a}_L + z^1 \mathbf{a}_R$ .
33:   -  $crs' = (\mathbb{G}, \mathbf{g}', h)$ .
34:   - Output  $\Phi = \Phi || \text{InnerProductProof}(crs', A', \mathbf{y}', p', \mathbf{a}')$ .

```

Fig. 7: The BatchEval function.

certificate *highQC*, i.e., the highest *prepareQC* for which the node has sent pre-commit message; (iii) a *lockedQC*, i.e., the highest QC for which the node has sent a commit message; (iv) *confirmQC*, i.e., the *commitQC* for the latest delivered block; (v) the last voted block *lv*; (vi) the current system configuration *M* and its epoch *e*; and (vii) the *temporary membership* of a configuration *TM*.

B. The Protocol

Normal-case operation (Figure 9). In the propose phase, the leader proposes a new block *b* with requests $(*, m)$ (also specified in the API in Figure 1). In particular, P_i extends *block(highQC)* by setting *block(highQC)* as the parent block of *b*, where *highQC* is the *prepareQC* with the highest rank

```

func BatchVerifyEval( $crs, \hat{S}, \mathbf{v}, \mathbf{y}, \phi, p$ ):
01: - Parse  $crs$  as  $(\mathbb{G}, \mathbf{g}, h)$ .
02: - Generate the inner product statement  $stmt = (\widehat{SD}, \mathbf{y}, \mathbf{v}')$ 
03: -  $z \leftarrow H(stmt)$ 
04: - Compute  $\widehat{SD}' \leftarrow \widehat{SD} h^{z \mathbf{v}'}$ .
05: - Output  $\text{VerifyInnerProduct}((\mathbb{G}, \mathbf{g}, h^z), \widehat{SD}', \mathbf{y}, p, \phi)$ .

func VerifyInnerProduct( $(\mathbb{G}, \mathbf{g}, h), A, \mathbf{y}, p, \phi$ ):
06: - if  $p = 1$  then
07:   - Parse  $\mathbf{a}$  from  $\phi$ .
08:   - Output  $(A = g_0^{\mathbf{a}} h^{\mathbf{a} \mathbf{y}})$ .
09: - if  $p$  is odd then
10:   - Parse  $\tilde{a}$  from  $\phi$  and update the parameters  $\phi, A, \mathbf{y}, \mathbf{g}, p$ .
11:   - Parse  $(L, R)$  from  $\phi$ .
12:   - Compute  $leaf \leftarrow H(\mathbf{g}, p, h^{z_i}, \mathbf{y}, A, L, R, \tilde{a})$ .
13: - else
14:   - Parse  $(L, R)$  from  $\phi$ .
15:   - Compute  $leaf \leftarrow H(\mathbf{g}, p, h^{z_i}, \mathbf{y}, A, L, R)$ .
16:   - Parse  $c \leftarrow \text{roothash}$  and  $b$  from  $\phi$ .
17:   - Verify whether the Merkle Tree commitment is  $(leaf, c, b)$ .
    Otherwise, output false.
18:   - Halve  $\mathbf{g}$  into two sub-matrix  $\mathbf{g}_L$  and  $\mathbf{g}_R$ .
19:   -  $A' \leftarrow L^{c^2} A R^{c^{-2}}, \mathbf{g}' \leftarrow \mathbf{g}_L^{c^{-1}} \mathbf{g}_R^{c^1}$ .
20:   -  $\mathbf{y}' \leftarrow c^{-1} \mathbf{y}_L + c^1 \mathbf{y}_R$ .
21:   -  $crs' = (\mathbb{G}, \mathbf{g}', h)$ 
22:   - Output  $\text{VerifyInnerProduct}(crs', A', \mathbf{y}', p', \phi)$ .

```

Fig. 8: The BatchVerifyEval function

(to be described shortly). Here $b.height = eB + r$ (lines 01-02). If $b.height \bmod B = 1$, membership requests might be included in *m*. We distinguish JOIN and LEAVE requests here. For LEAVE requests, the leader directly includes the requests in *m* (line 04). For each JOIN request (join, pk_s), the leader assigns an id *s* to the new node and then includes a message (add, s, pk_s) in *m*. Meanwhile, P_i adds P_s to its *temporary membership TM* (line 05). After the requests are packed, the leader sets *b.rq* as *m* and sends (prepare, *cview*, *b*, *highQC*) to *TM* (line 06). Upon receiving a (prepare, *cview*, *b*, *qc*) message from the leader such that *b* is an extension of *block(qc)* (line 08), P_i checks whether one of the conditions is satisfied for block *b* in lines 09-10. One noteworthy requirement we introduce is highlighted in line 09. This check requires that b' (the parent block of *b*) has already been delivered. Additionally, if b' was proposed in the previous view, P_i also checks whether $\text{block}(\text{lockedQC}) = b'$. Such a change is crucial for our protocol to achieve liveness. After the conditions are checked, P_i updates its *lv* as *b* and sends a (prepare, *cview*, *b*, σ_i) message to the leader $\text{LEADER}(\text{cview}, M)$, where σ_i is a digital signature for the tuple (prepare, *cview*, *hash(b)*). If $b.height \bmod B = 1$ and *b* consists of some (add, s, pk_s) request, P_i helps P_s complete the state transfer via a catchup message (lines 11-12). We ignore the details on state transfer as it largely follows previous work [45].

After collecting $n - t$ matching prepare messages, the signatures in the prepare messages form a *prepareQC*. P_i updates its *highQC* as *prepareQC* and enters the pre-commit phase. Such a procedure is repeated in the commit phase, similar to that in HotStuff. We highlight the changes we made here. First, in the pre-commit phase, the leader sends the pre-commit message to all nodes in $TM \cup M_{e-1}$, including the previous configuration, existing configuration, and the new nodes to be added (line 14). If P_i is a node in M_{e-1} but not

▷ **Initialization of local parameters:** $cview \leftarrow 1, highQC, lockedQC, confirmQC \leftarrow \perp, lv \leftarrow \perp, TM \leftarrow M_0$.
 ▷ **Initialization of global parameter:** epoch e , round r .
 ▷ **The prepare phase in epoch e , round r .**
 01: - **As a leader**, upon $RondoBFT.Propose_i(e, r, (*, m))$ $//P_i = LEADER(cview, M_e)$
 02: - Let b' denote $block(highQC)$. Generate a block b such that $b.rq \leftarrow \perp, b.height \leftarrow eB + r, b.parent = b'$.
 03: - **if $b.height \bmod B = 1$ then**
 04: - For any LEAVE request $rq \in m, m \leftarrow m$.
 05: - For any JOIN request $(join, pk_s) \in m$, assign the sender P_s with an ID s , set $m \leftarrow m \cup (Add, s, pk_s)$ and $TM \leftarrow M \cup P_s$.
 06: - $b.rq \leftarrow (*, m)$, broadcast (prepare, $cview, b, highQC$) to TM .
 07: - **As a node in M_e** , upon receiving a message (prepare, $cview, b, qc$) from $LEADER(cview, M)$:
 08: - Let b' denote $b.parent$, b_l denote $block(lockedQC)$. Verify that $b.height \in [eB+1, eB+B]$ and qc is a *prepareQC* for b' .
 09: - **if $(b'.view = cview$ and $rank(b') \geq rank(lv)$) or $(b'.view < cview$ and $lv.view < cview$ and $b' = b_l)$ and b' is delivered) then**
 10: - **if $b.height \bmod B = 1$ or b contains no membership requests then $lv \leftarrow b$, send (prepare, $cview, b, \sigma_i$) to $LEADER(cview, M)$**
 11: - **if $b.height \bmod B = 1$ then**
 12: - For any ADD request $(ADD, s, pk_s) \in V$, set $TM \leftarrow M \cup P_s$ and send local state to P_s in a catchup message.
 ▷ **The pre-commit phase in epoch e , round r .**
 13: - **As a leader**, Wait for (prepare, $cview, b, *$) messages from $\lceil \frac{2|M|+1}{3} \rceil$ nodes in M :
 14: - Generate a valid *prepareQC* qc_b for b , $highQC \leftarrow qc_b$, broadcast (pre-commit, $cview, b, qc_b$) to $TM \cup M_{e-1}$.
 15: - **As a node in M_{e-1}** , Upon receiving a message (pre-commit, $cview, b, qc$):
 16: - **if qc is a *prepareQC* in epoch e and $P_i \notin M_e$ and P_i has delivered all blocks in epoch $e-1$ then P_i leave the system**
 17: - **As a node in M_e** , wait for a message $m = (\text{pre-commit}, cview, b, qc)$ from $LEADER(cview, M)$:
 18: - **if $b.height \bmod B = 1$ then broadcast m to all node in $M_{e-1} \setminus M_e$**
 19: - Set $highQC \leftarrow qc$, send a signed message (pre-commit, $cview, b, \sigma_i$) to $LEADER(cview, M)$.
 ▷ **The commit phase in epoch e , round r .**
 20: - **As a leader**, wait for (pre-commit, $cview, b, *$) messages from $\lceil \frac{2|M|+1}{3} \rceil$ nodes in M :
 21: - Generate a valid *precommitQC* qc for b , $lockedQC \leftarrow qc_b$, broadcast (commit, $cview, b, qc$) to TM .
 22: - **As a node in M** , wait for a message (commit, $cview, b, qc$) from $LEADER(cview, M)$:
 23: - Set $lockedQC \leftarrow qc$ and send a signed message (commit, $cview, b, \sigma_i$) to $LEADER(cview, M)$.
 ▷ **The decide phase in epoch e , round r .**
 24: - **As a leader**, wait for (commit, $cview, b, *$) messages from $\lceil \frac{2|M|+1}{3} \rceil$ nodes in M :
 25: - Generate a valid *commitQC* qc_b for b , updates $commitQC \leftarrow qc_b$, broadcast (decide, $cview, b, qc_b$) to TM .
 26: - **As a node in M_e** , wait for a message (decide, $cview, b, qc$) from $LEADER(cview, M)$:
 27: - Update $confirmQC \leftarrow qc$. Deliver every block b' on the branch led by b in the following method:
 28: - Deliver every block on the branch led by b' . Let e' denote $\lfloor \frac{b.height-1}{B} \rfloor$, r' denote $b.height - e'B$
 29: - **if $b'.height \bmod B = 0$ then broadcast (commit, $cview, b, qc$) to $M_e \cup M_{e+1}$**
 30: - **if $b'.height \bmod B = 1$ then**
 31: - Parse $b'.rq$ as $(S_{e'}, m)$. Set $M_{e'+1} \leftarrow M$. For any LEAVE request $(leave, pk_j) \in rqs, M_{e'+1} \leftarrow M_{e'+1} \setminus P_j$.
 32: - For any JOIN request $(add, j, pk_j) \in m$, set $M_{e'+1} \leftarrow M_{e'+1} \cup P_j$.
 33: - Set $TM \leftarrow M \cup M_{e'+1}, round \leftarrow r' + 1, RondoBFT.Return_i(e', r', (S_{e'}, M_{e'+1}, m))$
 34: - **if $b'.height \bmod B \neq 0$ and $b'.height \bmod B \neq 1$ then $round \leftarrow r' + 1, RondoBFT.Return_i(e', r', b'.rq)$.**
 35: - **As a node in M_{e+1}** , upon receiving (decide, v, b, qc) messages from $\lceil \frac{2|M_e|+1}{3} \rceil$ nodes in M_e :
 36: - **if qc is a valid *commitQC* for b and $b.height = eB$ then**
 37: - Deliver b and all the ancestors of b
 38: - $cview \leftarrow v + 1$

Fig. 9: Normal-case operation of Rondo-BFT for P_i in epoch e and round r .

in M_e (i.e., P_i requests to leave the system), it waits until receiving the pre-commit message in epoch e and delivering all the blocks in epoch $e-1$ before leaving the system (lines 15-16). In addition, if P_i is a node in M_e , after receiving the pre-commit message for the first block in epoch e , P_i forwards this message to all nodes in M_{e-1} (line 18). As mentioned in Sec. IV, such changes ensure that the agreement property for the membership requests is achieved by all correct nodes nearly simultaneously. Second, in the decide phase, the leader sends the commit message to all nodes in TM (line 25). If P_i is a node in M_e , after receiving the decide message for block b , P_i executes additional procedures before delivering b (lines 28-33). In particular, if b consists of membership requests, P_i outputs the new configuration in addition to the client requests. If $b.height \bmod B = 0$, P_i broadcasts a (decide, $cview, b, qc$) message to $M_e \cup M_{e+1}$, including the current and the next configuration. The idea is again for correct nodes to achieve agreement nearly simultaneously so nodes can correctly install

a new configuration. If P_i is a node in the new configuration M_{e+1} , after receiving the decide message for the last block in epoch e from $\lceil \frac{2|M_e|+1}{3} \rceil$ nodes in M_e , P_i delivers the block (lines 35-38) and then installs the new configuration (which also match lines 2-3 in Figure 1).

The view change protocol (Figure 10). Each node P_i may trigger view change in view v (lines 02-03). When view change happens, P_i sets $cview \leftarrow cview + 1$ and broadcasts a (new-view, $cview, \perp, (highQC, confirmQC)$) message to TM (line 04). Upon receiving a new-view message m from node P_j , P_i forwards this message to other nodes following the Msgforwarding rule (lines 17-22): 1) if m contains a *prepareQC* with a height higher than that of $highQC$, P_i updates its $highQC$; 2) if m contains a *commitQC* qc with a height higher than that of $confirmQC$, P_i updates $confirmQC$ to qc and delivers the corresponding blocks; 3) if *prepareQC* has a lower height than that of $highQC$ and $P_j \in M$, P_i forwards m to $LEADER(cview, M)$. The

```

▷ The new-view phase.
01: - As a node, a view change is triggered when one of the following conditions is satisfied:
02:   - Timeout occurs during "wait for" in normal-case operation).
03:   - Receiving new-view messages for view  $v$  from a node set  $X$  such that  $X \in M$ ,  $|X| \geq \frac{|M|+1}{3}$ , and  $v \geq cview$ .
04:   - Upon view change: set  $cview \leftarrow cview + 1$  and broadcast  $(new-view, cview, \perp, (highQC, confirmQC))$  to  $TM$ .
05:   - Upon receiving a new-view message, forward it to other nodes following the Msgforwarding rule.
▷ The view-change phase.
06: - As a new leader candidate, upon receiving  $(new-view, cview, \perp, *)$  messages from nodes  $X$ , let  $b$  denote  $block(commitQC)$ :
07:   - if  $P_i = LEADER(cview, M_e)$  and  $b.height \in [(e-1)B, eB)$  and  $X \in M_e$  and  $|X| \geq \lceil \frac{2|M_e|+1}{3} \rceil$  then
08:     -  $vc \leftarrow$  received new-view messages,  $b_{high} \leftarrow$  the block with the highest rank for which a valid  $prepareQC$  is included in  $vc$ .
09:     - Broadcast  $vc$  and  $(pre-commit, v, b_{high}, highQC)$  and switch to the pre-commit phase for  $b_{high}$ .
10:     - After  $b_{high}$  is delivered, switch to the normal-case operation for an extension block of  $b_{high}$ .
11: - As a node, upon receiving  $(new-view, cview, \perp, *)$  messages from nodes  $X$ , let  $b$  denote  $block(commitQC)$ :
12:   - Let  $b_{high}$  denote the block with the highest rank for which a valid  $prepareQC$  is included in  $VC$ .
13:   - if  $P_j = LEADER(cview, M_e)$  and  $b.height \in ((e-1)k, ek]$  and  $X \in M_e$  and  $|X| \geq \lceil \frac{2|M_e|+1}{3} \rceil$  then
14:     - if  $rank(b_{high}) \geq rank(lockedQC)$  then
15:       - Wait for a pre-commit message for  $b_{high}$  and then switch to the pre-commit phase for  $b_{high}$ .
16:     - After  $b_{high}$  is delivered, switch to the normal-case operation for an extension block of  $b_{high}$ .
▷ The Msgforwarding rule.
17: - Upon receiving  $m = (new-view, v, \perp, (qc_1, qc_2))$  from  $P_j$  such that  $qc$  is a valid QC for  $b$ :
18:   - if  $qc_1$  is a  $prepareQC$  and  $rank(qc_1) > rank(highQC)$  then  $highQC \leftarrow qc_1$ 
19:   - if  $qc_2$  is a  $commitQC$  and  $rank(qc_2) > rank(confirmQC)$  then
20:     - Deliver  $b$  and all the ancestors of  $b$ 
21:     - if  $P_i \in M$  and  $P_i$  has broadcast new-view message  $m'$  for view  $v$  then forward  $m'$  to  $LEADER(cview, M)$ 
22:     - if  $rank(qc) < rank(highQC)$  and  $P_j \in M$  then forward  $m$  to  $LEADER(cview, M)$ 

```

Fig. 10: The view change protocol for Rondo-BFT.

Msgforwarding rule enforces all correct nodes to be aware of the view change and addresses the liveness challenge mentioned in Sec. IV.

After receiving the $(new-view, cview, \perp, *)$ messages from a set of nodes X , every node P_i checks whether $P_i = LEADER(cview, M_e)$, $X \in M_e$, and $|X| \geq \lceil \frac{2|M_e|+1}{3} \rceil$. If so and the height of $block(confirmQC)$ is between $[(e-1)B, eB)$, P_i is the leader in view $cview$ (lines 06-07). These new-view messages form a view-change certificate vc . Let b_{high} be the block with the highest rank for which a valid $prepareQC$ is included in vc . P_i broadcasts a $(pre-commit, v, b_{high}, highQC)$ message to M and proceeds to the pre-commit phase for b_{high} (lines 08-09). After b_{high} is delivered, P_i switches to the normal-case operation (line 10). If P_i is not the new leader, after receiving vc and a pre-commit message from node P_j for block b , P_i verifies whether P_j is the new leader based on vc and $rank(b) \geq rank(block(lockedQC))$. If so, P_i proceeds to the pre-commit phase for b (lines 10-14). After b is delivered, P_i switches to the normal-case operation (lines 15-16).

Discussion. Rondo-BFT achieves $O(n)$ message complexity in normal-case operation and has $O(n^2)$ messages only during configuration changes and view changes. Namely, nodes need to forward their messages to all nodes across configurations and views. This is "unavoidable" as we need the agreement property by correct nodes nearly simultaneously. As we use aggregate signatures to instantiate quorum certificates, Rondo-BFT achieves $O(\lambda n^2 \log n)$ communication. We show the proof of Rondo-BFT in Appendix B.

VIII. IMPLEMENTATION AND EVALUATION

We implement Rondo in Golang, including both Breeze and Rondo-BFT. We use the kyber crypto library¹ to implement our batched polynomial commitment scheme. In the same library, we also implement the batch verification functions of hbACSS [38] to compare the computational cost. We use gRPC as the communication library. We use HMAC to realize the authenticated channel and SHA256 as the hash function. Our implementation involves around 13,000 new LOC for the system and about 1,000 LOC for evaluation.

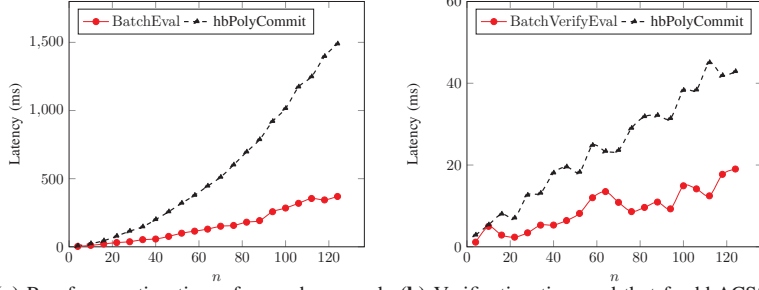
We evaluate the performance of our protocols on Amazon EC2 using up to 91 virtual machines (VMs). We use *c5.xlarge* instances for our evaluation. The *c5.xlarge* instance has 16 virtual CPUs and 32GB memory. We distribute the nodes evenly in four different regions: us-west-2 (Oregon, US), us-east-2 (Ohio, US), ap-southeast-1 (Singapore), and eu-west-1 (Ireland). We vary the network size and batch size B in our evaluation. We use n to denote the network size where $n = 3t + 1$ nodes are launched in each experiment.

For performance evaluation, we focus on the computational cost of our batched polynomial commitment scheme and the throughput of Rondo. To assess the computational cost, we compare our scheme with the hbPolyCommit function in hbACSS. For the DRB protocol, we compare Rondo with that using threshold Boneh-Lynn-Shacham (TBS) signature using the TBS implementation in kyber. Indeed, some industrial level randomness beacon protocols such as Drand² and Dfinity³ also use TBS. Such a scheme requires DKG setup, which might be expensive. In our evaluation, we ignore the latency for DKG and only assess the performance of generating randomness beacon using TBS.

¹The kyber crypto library: <https://github.com/dedis/kyber>

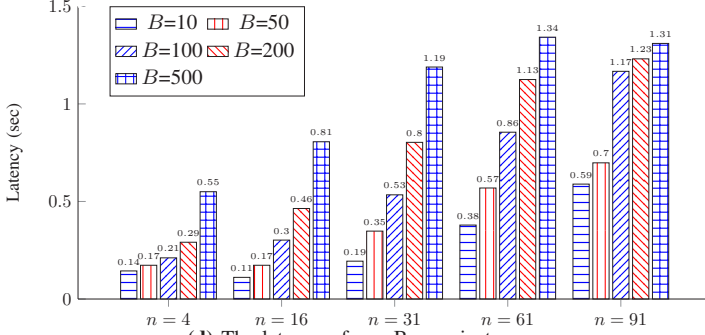
²Drand: <https://github.com/drand/drand>

³Dfinity: <https://github.com/dfinity-side-projects/random-beacon>

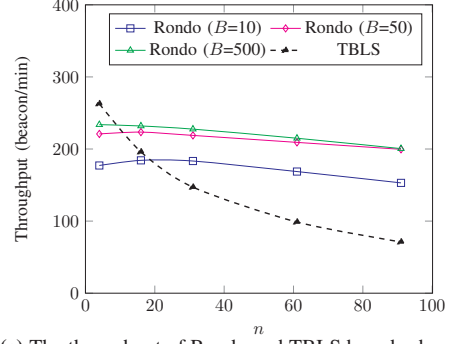


(a) Proof generation time of our scheme and that for hbACSS. (b) Verification time and that for hbACSS. (c) Latency of reconstructing B secrets where $B = 2n$.

| n | latency (ms) | | |
|-----|--------------|--------------|-------------|
| | default | optimization | improvement |
| 4 | 15.630 | 14.125 | 1.505 |
| 16 | 140.332 | 137.1935 | 3.138 |
| 32 | 449.246 | 437.856 | 11.390 |
| 64 | 1678.881 | 1633.421 | 45.460 |
| 128 | 6161.158 | 5920.885 | 240.273 |



(d) The latency of one Breeze instance.



(e) The throughput of Rondo and TBLS based scheme.

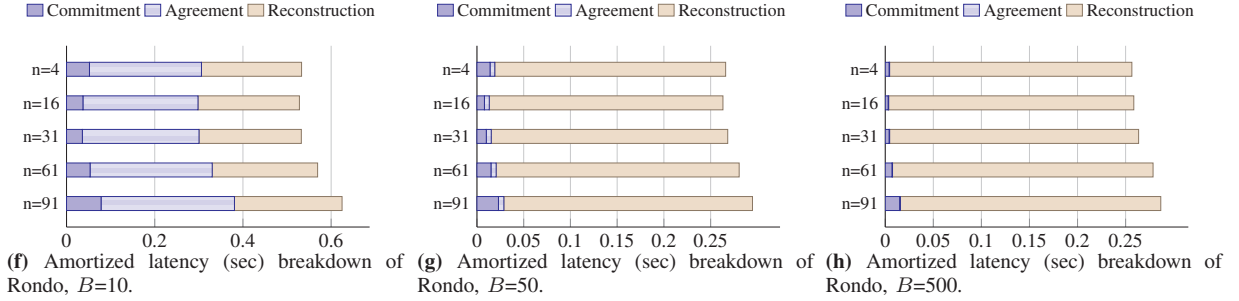


Fig. 11: Evaluation results.

Batched polynomial commitment. We first compare the computational cost of hbPolyCommit and our BatchEval function. As no network communication is needed, we evaluate the performance on one machine with Intel i7 CPU with 2.70GHz and 16GB memory. Figure 11a illustrates the proof generation time with different n , where B is set as $2n$. Our BatchEval function consistently outperforms hbPolyCommit. When n is greater than 100, BatchEval is 4x-5x faster than hbPolyCommit. In addition, the latency of BatchEval does not increase as significantly when n grows.

Meanwhile, we show the verification time of the BatchVerifyEval function and hbPolyCommit in Figure 11b. Our BatchVerifyEval function is 2x-2.5x faster than hbPolyCommit in all the experiments we conducted.

Reconstruction. We set $B = 2n$ and evaluate the latency of reconstructing B secrets. As shown in Figure 11c, our optimized reconstruction phase improves the performance of the default reconstruction approach. Besides, the improvement is more significant as n and B grow, which matches our

analysis in VI-C.

Performance of Breeze. We evaluate the performance of Breeze on EC2 by varying B and n . As shown in Figure 11d, given certain n , the latency grows as B grows. This is expected as a higher computation cost is involved when B grows. Meanwhile, given certain B , the latency grows as n grows. This is because higher communication is involved. All of our experiments were completed within 1.3 seconds. The highest latency happens when $n = 61$ and $B = 500$ (The latency is slightly higher than that for $n = 91$ and $B = 500$, and we believe this is caused by the variance of network delay), so the amortized latency for sharing each secret is only 2.68 ms.

Performance of Rondo. We compare the performance of Rondo and that using TBLS. For Rondo, we assess the latency of each epoch of Rondo (which involves running the commitment phase with n Breeze instances, Rondo-BFT for B rounds, and reconstruction of secrets in each round) and then report the throughput accordingly. In practice, the performance can be further optimized, e.g., the commitment phase can be

executed prior to the beginning of an epoch.

We report the throughput of Rondo with different B in Figure 11e. As B grows, the throughput of our Rondo also increases. In our case, the throughput is very close to the peak when $B=50$. For a network with more than 4 nodes, Rondo consistently outperforms that using TBLS and Rondo does not require DKG. Furthermore, unlike almost all previous works [18, 19, 29, 32, 33, 35] (and TBLS) where the throughput degrades significantly as n grows, the throughput of Rondo does not degrade much so our protocol offers better scalability.

We show the amortized latency breakdown (for generating each beacon) in Figure 11f-11h. Our results show that when B is small, the bottlenecks of the system are the agreement and reconstruction phases. While as B grows, the overhead is dominated by the reconstruction phase. The results show that our Breeze and Rondo-BFT protocols are efficient and are not the bottleneck of the system.

IX. CONCLUSION

We present Rondo, a scalable distributed randomness beacon protocol and the first reconfiguration-friendly protocol in the partially synchronous model. We propose a new primitive called batched asynchronous verifiable secret sharing with partial output (bAVSS-PO) and a bAVSS-PO protocol Breeze, which is both communication-efficient and computation-efficient. To support reconfiguration, we propose Rondo-BFT, a dynamic Byzantine fault-tolerant protocol that supports efficient reconfiguration. We show that Rondo achieve both high throughput and better scalability than prior works.

REFERENCES

- [1] M. O. Rabin, "Transaction protection by beacons," *Journal of Computer and System Sciences*, vol. 27, no. 2, pp. 256–267, 1983.
- [2] Y. Gilad, R. Hemo, S. Micali, G. Vlachos, and N. Zeldovich, "Algorand: Scaling Byzantine agreements for cryptocurrencies," in *SOSP*. ACM, 2017, pp. 51–68.
- [3] A. Kiayias, A. Russell, B. David, and R. Oliynykov, "Ouroboros: A provably secure proof-of-stake blockchain protocol," in *Annual international cryptography conference*. Springer, 2017, pp. 357–388.
- [4] E. Kokoris-Kogias, P. Jovanovic, L. Gasser, N. Gailly, and B. Ford, "Omniledger: A secure, scale-out, decentralized ledger," *IACR Cryptology ePrint Archive*, vol. 2017, p. 406, 2017.
- [5] B. Adida, "Helios: Web-based open-audit voting," in *USENIX security symposium*, vol. 17, 2008, pp. 335–348.
- [6] A. Mostefaoui, H. Moumen, and M. Raynal, "Signature-free asynchronous byzantine consensus with $t < n/3$ and $o(n^2)$ messages," in *Proceedings of the ACM symposium on Principles of distributed computing*. ACM, 2014, pp. 2–9.
- [7] A. Mostefaoui and M. Raynal, "Signature-free asynchronous byzantine systems: from multivalued to binary consensus with $t < n/3$, $o(n^2)$ messages, and constant time," *Acta Informatica*, vol. 54, no. 5, pp. 501–520, 2017.
- [8] S. Duan, X. Wang, and H. Zhang, "Practical signature-free asynchronous common subset in constant time," in *CCS*, 2023.
- [9] H. Zhang and S. Duan, "PACE: Fully parallelizable BFT from reproposable byzantine agreement," in *CCS*, 2022.
- [10] S. Goel, M. Robson, M. Polte, and E. Siro, "Herbivore: A scalable and efficient protocol for anonymous communication," Cornell University, Tech. Rep., 2003.
- [11] J. Van Den Hooff, D. Lazar, M. Zaharia, and N. Zeldovich, "Vuvuzela: Scalable private messaging resistant to traffic analysis," in *SOSP*, 2015, pp. 137–152.
- [12] D. I. Wolinsky, H. Corrigan-Gibbs, B. Ford, and A. Johnson, "Dissent in numbers: Making strong anonymity scale," in *OSDI*, 2012, pp. 179–182.
- [13] R. Dingledine, N. Mathewson, P. F. Syverson *et al.*, "Tor: The second-generation onion router," in *USENIX security symposium*, vol. 4, 2004, pp. 303–320.
- [14] J. Bonneau, J. Clark, and S. Goldfeder, "On bitcoin as a public randomness source," *Cryptology ePrint Archive*, 2015.
- [15] K. Chatterjee, A. K. Goharshady, and A. Pourdamghani, "Probabilistic smart contracts: Secure randomness on the blockchain," in *ICBC*. IEEE, 2019, pp. 403–412.
- [16] M. Haahr, "Random.org: True random number service," *School of Computer Science and Statistics, Trinity College, Dublin, Ireland. Website (http://www.random.org)*. Accessed, vol. 10, 2023.
- [17] J. Kelsey, L. T. Brandão, R. Peralta, and H. Booth, "A reference for randomness beacons: Format and protocol version 2," National Institute of Standards and Technology, Tech. Rep., 2019.
- [18] S. Das, V. Krishnan, I. M. Isaac, and L. Ren, "Spurt: Scalable distributed randomness beacon with transparent setup," in *IEEE Symposium on Security and Privacy (SP)*. IEEE, 2022, pp. 2502–2517.
- [19] A. Bhat, N. Shrestha, Z. Luo, A. Kate, and K. Nayak, "Randpipe-reconfiguration-friendly random beacons with quadratic communication," in *CCS*, 2021, pp. 3502–3524.
- [20] K. Choi, A. Manoj, and J. Bonneau, "Sok: Distributed randomness beacons," *IEEE Symposium on Security and Privacy (SP)*, 2023.
- [21] I. Cascudo and B. David, "Scrape: Scalable randomness attested by public entities," in *International Conference on Applied Cryptography and Network Security*. Springer, 2017, pp. 537–556.
- [22] P. Schindler, A. Judmayer, M. Hittmeir, N. Stifter, and E. R. Weippl, "Randrunner: Distributed randomness from trapdoor vdfs with strong uniqueness," in *NDSS*, 2021.
- [23] P. Schindler, A. Judmayer, N. Stifter, and E. Weippl, "Hydrand: Efficient continuous distributed randomness," in *IEEE Symposium on Security and Privacy (SP)*. IEEE, 2020, pp. 73–89.
- [24] J. Camenisch, M. Drijvers, T. Hanke, Y.-A. Pignolet, V. Shoup, and D. Williams, "Internet computer consensus," in *PODC*, 2022, pp. 81–91.
- [25] A. Cherniaeva, I. Shirobokov, and O. Shlomovits, "Homomorphic encryption random beacon," *Cryptology ePrint Archive*, 2019.
- [26] E. Syta, P. Jovanovic, E. K. Kogias, N. Gailly, L. Gasser, I. Khoffi, M. J. Fischer, and B. Ford, "Scalable bias-resistant distributed randomness," in *IEEE Symposium on Security and Privacy (SP)*. IEEE, 2017, pp. 444–460.
- [27] C. Cachin, K. Kursawe, and V. Shoup, "Random oracles in constant-time: Practical asynchronous byzantine agreement using cryptography," *Journal of Cryptology*, vol. 18, no. 3, pp. 219–246, 2005.
- [28] C. Dwork, N. Lynch, and L. Stockmeyer, "Consensus in the presence of partial synchrony," *Journal of the ACM (JACM)*, vol. 35, no. 2, pp. 288–323, 1988.
- [29] A. Bhat, N. Shrestha, A. Kate, and K. Nayak, "Optrand: Optimistically responsive reconfigurable distributed randomness," in *NDSS*, 2023.
- [30] C. Cachin, K. Kursawe, and V. Shoup, "Random oracles in constant-time: practical asynchronous byzantine agreement using cryptography," in *Proceedings of the nineteenth annual ACM symposium on Principles of distributed computing*, 2000, pp. 123–132.
- [31] A. Bandarupalli, A. Bhat, S. Bagchi, A. Kate, and M. Reiter, "Hashrand: Efficient asynchronous random beacon without threshold cryptographic setup," *Cryptology ePrint Archive*, 2023.
- [32] R. Bacho, C. Lenzen, J. Loss, S. Ochsreither, and D. Papachristoudis, "Grandline: First adaptively secure dkg and randomness beacon with (almost) quadratic communication complexity," *Cryptology ePrint Archive*, 2023.
- [33] "drand - a distributed randomness beacon daemon," 2020, <https://github.com/drang/drang>.
- [34] I. Cascudo and B. David, "Scrape: Scalable randomness attested by public entities," in *International Conference on Applied Cryptography and Network Security*. Springer, 2017, pp. 537–556.
- [35] J. Camenisch, M. Drijvers, T. Hanke, Y.-A. Pignolet, V. Shoup, and D. Williams, "Internet computer consensus," in *Proceedings of the 2022*

- ACM Symposium on Principles of Distributed Computing, 2022, pp. 81–91.
- [36] R. Bacho and J. Loss, “Adaptively secure (aggregatable) pvss and application to distributed randomness beacons,” in *Proceedings of the 2023 ACM SIGSAC Conference on Computer and Communications Security*, 2023, pp. 1791–1804.
- [37] V. Shoup and N. P. Smart, “Lightweight asynchronous verifiable secret sharing with optimal resilience,” *Cryptology ePrint Archive*, 2023.
- [38] T. Yurek, L. Luo, J. Fairuze, A. Kate, and A. Miller, “hbacs: How to robustly share many secrets,” *Cryptology ePrint Archive*, 2021.
- [39] I. Abraham, P. Jovanovic, M. Maller, S. Meiklejohn, and G. Stern, “Bingo: Adaptively secure packed asynchronous verifiable secret sharing and asynchronous distributed key generation,” in *CRYPTO*, 2023.
- [40] M. K. Reiter, “Secure agreement protocols: Reliable and atomic group multicast in rampart,” in *Proceedings of the 2nd ACM Conference on Computer and Communications Security*, 1994, pp. 68–80.
- [41] N. Alhaddad, M. Varia, and Z. Yang, “Haven++: Batched and packed dual-threshold asynchronous complete secret sharing with applications,” *Cryptology ePrint Archive*, 2024.
- [42] B. Bünz, J. Bootle, D. Boneh, A. Poelstra, P. Wuille, and G. Maxwell, “Bulletproofs: Short proofs for confidential transactions and more,” in *2018 IEEE symposium on security and privacy (SP)*. IEEE, 2018, pp. 315–334.
- [43] R. Vassantlal, E. Alchieri, B. Ferreira, and A. Bessani, “Cobra: Dynamic proactive secret sharing for confidential bft services,” in *2022 IEEE symposium on security and privacy (SP)*. IEEE, 2022, pp. 1335–1353.
- [44] S. K. D. Maram, F. Zhang, L. Wang, A. Low, Y. Zhang, A. Juels, and D. Song, “Churp: dynamic-committee proactive secret sharing,” in *Proceedings of the 2019 ACM SIGSAC Conference on Computer and Communications Security*, 2019, pp. 2369–2386.
- [45] S. Duan and H. Zhang, “Foundations of dynamic bft,” in *IEEE Symposium on Security and Privacy (SP)*, 2022.
- [46] M. Castro and B. Liskov, “Practical byzantine fault tolerance and proactive recovery,” *TOCS*, vol. 20, no. 4, pp. 398–461, 2002.
- [47] M. Yin, D. Malkhi, M. K. Reiter, G. G. Gueta, and I. Abraham, “Hotstuff: Bft consensus with linearity and responsiveness,” in *PODC*, 2019.
- [48] M. Ben-Or, R. Canetti, and O. Goldreich, “Asynchronous secure computation,” ser. STOC ’93, 1993.
- [49] R. Canetti and T. Rabin, “Fast asynchronous byzantine agreement with optimal resilience,” in *STOC*, vol. 93. Citeseer, 1993, pp. 42–51.
- [50] A. Choudhury and A. Patra, “An efficient framework for unconditionally secure multiparty computation,” *IEEE Transactions on Information Theory*, vol. 63, no. 1, pp. 428–468, 2016.
- [51] Z. Beerliová-Trubíniová and M. Hirt, “Simple and efficient perfectly-secure asynchronous mpc,” in *International Conference on the Theory and Application of Cryptology and Information Security*. Springer, 2007, pp. 376–392.
- [52] A. Patra, A. Choudhury, and C. Pandu Rangan, “Efficient asynchronous verifiable secret sharing and multiparty computation,” *Journal of Cryptology*, vol. 28, pp. 49–109, 2015.
- [53] M. Ben-Or, B. Kelmer, and T. Rabin, “Asynchronous secure computations with optimal resilience,” in *Proceedings of the 13th annual symposium on Principles of distributed computing*. ACM, 1994, pp. 183–192.
- [54] A. Patra, A. Choudhury, and C. P. Rangan, “Efficient statistical asynchronous verifiable secret sharing with optimal resilience,” in *International Conference on Information Theoretic Security*. Springer, 2009, pp. 74–92.
- [55] C. Cachin, K. Kursawe, A. Lysyanskaya, and R. Strohli, “Asynchronous verifiable secret sharing and proactive cryptosystems,” in *Proceedings of the 9th ACM Conference on Computer and Communications Security*, 2002, pp. 88–97.
- [56] N. Alhaddad, M. Varia, and H. Zhang, “High-threshold avss with optimal communication complexity,” in *Financial Cryptography and Data Security: 25th International Conference, FC 2021, Virtual Event, March 1–5, 2021, Revised Selected Papers, Part II 25*. Springer, 2021, pp. 479–498.
- [57] S. Das, Z. Xiang, and L. Ren, “Asynchronous data dissemination and its applications,” in *CCS*, 2021, pp. 2705–2721.
- [58] H. Zhang, S. Duan, C. Liu, B. Zhao, X. Meng, S. Liu, Y. Yu, F. Zhang, and L. Zhu, “Practical asynchronous distributed key generation: Improved efficiency, weaker assumption, and standard model,” in *2023 53rd Annual IEEE/IFIP International Conference on Dependable Systems and Networks (DSN)*. IEEE, 2023, pp. 568–581.
- [59] S. Das, Z. Xiang, A. Tomescu, A. Spiegelman, B. Pinkas, and L. Ren, “Verifiable secret sharing simplified,” *Cryptology ePrint Archive*, Paper 2023/1196, 2023, <https://eprint.iacr.org/2023/1196>. [Online]. Available: <https://eprint.iacr.org/2023/1196>
- [60] M. Franklin and M. Yung, “Communication complexity of secure computation,” in *Proceedings of the twenty-fourth annual ACM symposium on Theory of computing*, 1992, pp. 699–710.
- [61] L. Lamport, “The part-time parliament,” *TOCS*, vol. 16, no. 2, pp. 133–169, 1998.
- [62] J. R. Lorch, A. Adya, W. J. Bolosky, R. Chaiken, J. R. Douceur, and J. Howell, “The SMART way to migrate replicated stateful services,” in *EuroSys*, Y. Berbers and W. Zwaenepoel, Eds., 2006, pp. 103–115.
- [63] D. Ongaro and J. Ousterhout, “In search of an understandable consensus algorithm,” in *ATC*, 2014, pp. 305–319.
- [64] J. Sousa, E. Alchieri, and A. Bessani, “State machine replication for the masses with bft-smart,” in *DSN*, 2014, pp. 355–362.
- [65] L. Breidenbach, C. Cachin, B. Chan, A. Coventry, S. Ellis, A. Juels, F. Koushanfar, A. Miller, B. Magauran, D. Moroz *et al.*, “Chainlink 2.0: Next steps in the evolution of decentralized oracle networks,” *Chainlink Labs*, vol. 1, pp. 1–136, 2021.
- [66] C. Dwork, N. Lynch, and L. Stockmeyer, “Consensus in the presence of partial synchrony,” *Journal of ACM*, vol. 32, no. 2, pp. 288–323, 1988.
- [67] R. Gelashvili, L. Kokoris-Kogias, A. Sonnino, A. Spiegelman, and Z. Xiang, “Jolteon and ditto: Network-adaptive efficient consensus with asynchronous fallback,” *arXiv preprint arXiv:2106.10362*, 2021.
- [68] X. Sui, S. Duan, and H. Zhang, “Marlin: Two-phase BFT with linearity,” 2022.
- [69] S. Duan, H. Zhang, X. Sui, B. Huang, C. Mu, G. Di, and X. Wang, “Dashing and star: Byzantine fault tolerance using weak certificates,” in *EuroSys*, 2024.
- [70] S. Das, T. Yurek, Z. Xiang, A. Miller, L. Kokoris-Kogias, and L. Ren, “Practical asynchronous distributed key generation,” in *2022 IEEE Symposium on Security and Privacy (SP)*. IEEE, 2022, pp. 2518–2534.
- [71] S. Das, Z. Xiang, L. Kokoris-Kogias, and L. Ren, “Practical asynchronous high-threshold distributed key generation and distributed polynomial sampling,” in *USENIX Security Symposium*, 2023.
- [72] G. Asharov and Y. Lindell, “A full proof of the bgw protocol for perfectly secure multiparty computation,” *Journal of Cryptology*, vol. 30, no. 1, pp. 58–151, 2017.

APPENDIX A ADDITIONAL PRELIMINARIES

Asynchronous verifiable secret sharing (AVSS). AVSS is an interactive protocol between a set of n nodes where the adversary \mathcal{A} controls t nodes and $t < n/3$. The dealer (a particular node) shares a secret among all nodes. For each AVSS instance, the set of nodes do not change. An AVSS protocol has two stages:

- **Sharing stage.** In the sharing stage, the dealer P_d is activated on an input message of the form $(ID.d, in, share, s)$, where $ID.d$ is the instance identifier (also called a *tag* of the instance) and s is the dealer’s secret. P_d begins the protocol by sharing s to the nodes. A node P_i has completed the sharing stage when it generates a local output of the form $(ID.d, out, shared)$.
- **Reconstruction stage.** After node P_i has completed the sharing stage, it may *start reconstruction* for $ID.d$ when

activated on a message $(ID.d, in, reconstruct)$. Eventually, the node outputs $(ID.d, out, reconstructed, z_i)$, in which case we say that P_i reconstructs z_i for $ID.d$.

An AVSS protocol can be denoted as (n, p, t) -AVSS, where $p + 1$ nodes are required to reconstruct s . When $p = t$, the protocol is also called *low-threshold* AVSS (or AVSS for short). When $p < n - t$ (e.g., $p = 2t$), the AVSS protocol is also called *high-threshold* AVSS (HAVSS) or dual-threshold AVSS (DAVSS). Without loss of generality, in this work, we consider the more generic dual-threshold AVSS with high threshold (i.e., HAVSS) in this work. Indeed, low-threshold AVSS is a special case of our construction.

An (n, p, t) -AVSS satisfies the following security properties.

- **Privacy.** If a correct dealer shared s using $ID.d$ and at most $p - t$ correct nodes started reconstruction for $ID.d$, then \mathcal{A} has no information about s .
- **Liveness.** 1) If the dealer P_d is correct throughout the sharing stage, then with overwhelming probability all correct nodes complete the sharing stage. 2) If some correct node completes the sharing stage, then all correct nodes complete the sharing stage. 3) If all correct nodes start reconstruction for $ID.d$, then with overwhelming probability every correct node P_i reconstructs some s_i for $ID.d$.
- **Correctness.** Once $p + 1$ correct nodes have completed the sharing for $ID.d$, there exists a fixed value z such that the following holds with overwhelming probability: 1) if the dealer shared s using $ID.d$ and is correct throughout the sharing stage, then $z = s$ and 2) if a correct node P_i reconstructs z_i for $ID.d$, then $z_i = z$.

Asynchronous complete secret sharing (ACSS). An ACSS is an AVSS scheme that additionally achieves the *completeness* property:

- **Completeness.** If a correct node completes the sharing stage, then there exists a polynomial $R(\cdot)$ of degree p such that $R(0) = s'$ and each correct node will hold a secret share s'_i such that $s'_i = R(i)$. If the dealer is correct, then $s' = s$.

To achieve the completeness property, most constructions known so far use the commitment scheme. If the commitments are additively homomorphic, we say it satisfies the ACSS satisfies the *homomorphic partial commitment* property [70, 71].

Batched HACSS (bHACSS). We now define bHACSS. More definitions regarding AVSS and ACSS can be found in Appendix A. In bHACSS, the dealer shares a batch of B secrets $\{s_1, \dots, s_B\}$ in one HACSS instance, where all the secrets are independent to each other. Compared to HACSS, HACSS with batching enjoys the benefits of low amortized communication complexity and computational complexity [38].

While the definition of bHACSS is an extended version of HACSS, we provide the definition of bHACSS in detail to facilitate the exposition of our work. The sharing stage remains the same as that in AVSS, besides that the secret s is not a single secret but a batch of B secrets $\{s_1, \dots, s_B\}$. For the reconstruction stage, each node may start reconstruction for $\{idx, ID.d\}$ when activated on a message $(idx, ID.d, in, reconstruct)$. This allows nodes to recon-

struct the idx^{th} secret for $ID.d$. bHACSS should satisfy the following properties.

- **Privacy.** If a correct dealer shared $\{s_1, \dots, s_B\}$ using $ID.d$ and at most $p - t$ correct nodes start reconstruction for $ID.d$, then \mathcal{A} has no information about $\{s_1, \dots, s_B\}$. We define the indistinguishability with an interactive game as below.

Definition 1. A bHACSS protocol satisfies *indistinguishability* if for any PPT adversary \mathcal{A} corrupting at most t nodes, \mathcal{A} wins the following game \mathcal{G}_{bhacss} against a challenger \mathcal{C} with negligible dominance.

 - 1) The challenger \mathcal{C} generates public parameters and then sends them to the adversary \mathcal{A} .
 - 2) \mathcal{A} selects a set of at most t nodes and corrupts them, then sends the identities of these nodes to \mathcal{C} .
 - 3) \mathcal{C} uniformly selects a batch of random numbers as the dealer's input and simulate all the honest nodes' operations as per the protocol in the sharing stage.
 - 4) When $p - t$ correct nodes start the reconstruction stage, \mathcal{C} halts the protocol.
 - 5) \mathcal{A} sends \mathcal{C} a number k indicating that it wants to make a guess about the k^{th} random number s_k . Then, \mathcal{C} samples a single bit $b \in \{0, 1\}$. If $b = 0$, \mathcal{C} sends the k^{th} random number in the dealer's inputs to \mathcal{A} . Otherwise, \mathcal{C} uniformly selects a random number to \mathcal{A} .
 - 6) \mathcal{A} makes a guess b' .
 - 7) \mathcal{A} wins the game if and only if $b = b'$.

Formally, we have

$$|Pr[b \leftarrow \mathcal{A} : b = b'] - \frac{1}{2}| \leq \text{negl}(\lambda)$$

- **Liveness.** 1) If the dealer P_d is correct throughout the sharing stage, then with overwhelming probability all correct nodes complete the sharing. 2) If some correct node completes the sharing for $ID.d$, then all correct nodes complete the sharing for $ID.d$. 3) If all correct nodes start reconstruction for $\{idx, ID.d\}$, then with overwhelming probability every correct node P_i reconstructs some $s_{idx}^{(i)}$ for $ID.d$.
- **Correctness.** Once $p + 1$ correct nodes have completed the sharing for $ID.d$, there exists a batch of fixed values $\{z_1, \dots, z_B\}$ such that the following holds with overwhelming probability: 1) if the dealer shared $\{s_1, \dots, s_B\}$ using $ID.d$ and is correct throughout the sharing stage, then $z_i = s_i$ for $i \in [1, B]$ and 2) if a correct node P_i reconstructs z'_{idx} for $ID.d$, then $z'_{idx} = z_{idx}$.
- **Completeness.** If a correct node completes the sharing stage, then there exists a batch of degree p polynomial $\{R_k(\cdot) | k \in [1, B]\}$ such that $R_k(0) = s'_k$ and each correct node will hold a batch of secret shares $\{s'_{ki} = R_k(i) | k \in [1, B]\}$. If the dealer is correct, then $s'_k = s_k$ for all $k \in [1, B]$.

APPENDIX B PROOF OF RONDO

Following previous work [18], we capture the unpredictability and bias-resistance properties based on the indistinguishability game defined below. Informally, the game specifies that the randomness beacon output is indistinguishable from a random number.

Definition 2. Rondo satisfies *indistinguishability* in an epoch if for any PPT adversary \mathcal{A} corrupting at most t nodes, \mathcal{A}

wins the following game \mathcal{G}_{beacon} against a challenger \mathcal{C} with negligible probability.

- 1) The challenger \mathcal{C} generates public parameters, including the previous randomness output, the round number as well as other public information in this epoch and then sends them to the adversary \mathcal{A} .
- 2) \mathcal{A} selects a set of at most t nodes and corrupts them, then sends the identities of these nodes to \mathcal{C} .
- 3) \mathcal{C} and \mathcal{A} interact to execute Rondo as follows:
 - \mathcal{C} sends inputs to the honest nodes and simulate their operations as per the protocol in the sharing phase, while \mathcal{A} has the ability to misbehave.
 - \mathcal{A} aggregates the messages it receives and run the BFT-SMR protocol with \mathcal{C} . In the protocol, \mathcal{A} is able to observe messages sent between every pair of nodes and reorder them.
 - After at least one honest node outputs a common subset CS , \mathcal{C} halt the execution.
- 4) \mathcal{A} sends an index k to \mathcal{C} indicating that it will make a guess of the beacon in the k^{th} round.
- 5) Then, \mathcal{C} samples a bit $b \in \{0, 1\}$. Subsequently, if $b = 0$, \mathcal{C} sends the corresponding beacon output of CS for the k^{th} round to \mathcal{A} . Otherwise, \mathcal{C} sends a real random number to \mathcal{A} .
- 6) \mathcal{A} makes a guess b' .
- 7) \mathcal{A} wins the game if and only if $b = b'$.

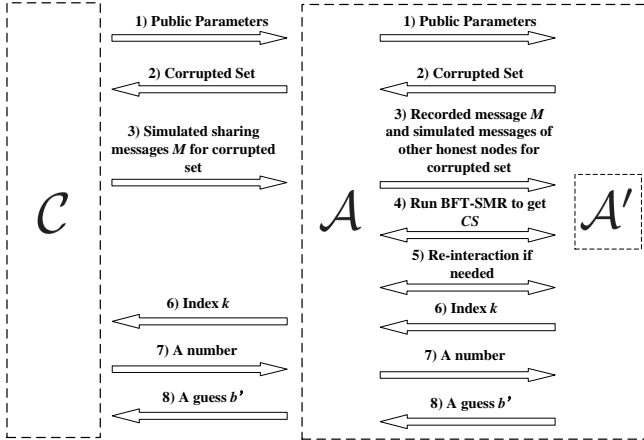


Fig. 12: Depiction of the reduction proof for beacon indistinguishability

Theorem 1. Assuming the $DLog$ assumption, Rondo satisfies indistinguishability.

Proof: According to the analysis in Sec. D, assuming $DLog$, Breeze achieves privacy. Now we show that the indistinguishability of Rondo can be reduced to the privacy of Breeze. Concretely, if there exists a PPT adversary \mathcal{A}' who can win the indistinguishability game \mathcal{G}_{beacon} of Rondo with overwhelming possibility, we can construct a PPT adversary \mathcal{A} to break the privacy of Breeze. This is achieved by acting as an adversary in the game $\mathcal{G}_{bavss-po}$ of $bAVSS-PO$ and a challenger in the game of randomness beacon, as shown below.

- 1) \mathcal{A} interacts with the challenger \mathcal{C} in the game \mathcal{G}_{bhacss} of Breeze to get the public parameters. Then \mathcal{A} sends the

parameters to \mathcal{A}' .

- 2) \mathcal{A}' selects a set of at most t nodes and corrupts them. Then \mathcal{A}' sends the identities of these nodes to \mathcal{A} , who then provide the identities to \mathcal{C} .
- 3) \mathcal{C} uniformly selects a batch of random numbers as the dealer's input and simulate all the honest nodes' operations as per the protocol in the sharing phase. Then \mathcal{A} records the messages received by the corrupted parties, denoted by M . Furthermore, it acts like the challenger in the game \mathcal{G}_{beacon} of the randomness beacon. Notably, \mathcal{A} sends the inputs to all honest nodes except for only one specified node, denoted as \mathbf{a} . For node \mathbf{a} , \mathcal{A} does not send to \mathbf{a} a real input but simulates what it has to broadcast in the sharing stage by directly sending M , the message \mathcal{A} has recorded.
- 4) \mathcal{A}' aggregate the messages it receives from \mathcal{A} and run the BFT-SMR protocol with \mathcal{A} . \mathcal{A}' is able to observe messages sent between every pair of nodes during this protocol and reorder them.
- 5) When at least one honest node outputs a CS , \mathcal{A} halts the execution. If $\mathbf{a} \notin CS$, \mathcal{A} generates a new randomness beacon instance and re-interacts with \mathcal{A}' as before until $\mathbf{a} \in CS$.
- 6) \mathcal{A}' sends an index k to \mathcal{A} , who transfers this to \mathcal{C} .
- 7) Then, \mathcal{A} receives a number from \mathcal{C} and transfers it to \mathcal{A}' . After that, \mathcal{A} waits for \mathcal{A}' to respond.
- 8) \mathcal{A} responds to \mathcal{C} whatever \mathcal{A}' sends to it.

We point out that in step 3), those recorded information only consists of the messages that the corrupted nodes received. However, these are enough for the simulation since \mathcal{A} does not need to simulate the view of the honest nodes. Furthermore, in step 5), the iteration cannot be done infinite times but at most polynomial times. This is adequate for \mathcal{A} , who can directly output 1 to \mathcal{C} without doing steps 6)-8). The probability of this situation is at most $\frac{1}{(n-t)^{poly(\ell)}}$ which is a negligible function.

Thus, if Rondo fails to achieve indistinguishability, \mathcal{A} can break the privacy of Breeze, a violation of our assumption. ■

Figure 12 depicts this procedure.

Theorem 2. Assuming the $DLog$ assumption, Rondo achieves unpredictability.

Proof: From Theorem 1, Rondo achieves indistinguishability. Assume \exists a PPT adversary \mathcal{A}' , corrupting at most t nodes, $i \geq 1, j > i + 1$, such that for any negligible function $\text{negl}(\lambda)$,

$$\Pr[(j, r'_j) \leftarrow \mathcal{A}(r_1, r_2, \dots, r_i) : r'_j = r_j] > \text{negl}(\lambda).$$

Then we can construct a PPT adversary \mathcal{A} utilizes \mathcal{A}' as a sub-algorithm to win the game \mathcal{G}_{beacon} in Definition 2 as follows:

Firstly, \mathcal{A} interacts with \mathcal{C} as per the game \mathcal{G}_{beacon} in step 1), then it invokes \mathcal{A}' with current information it receives from \mathcal{C} , including previous beacon output as well as public parameters. Moreover, \mathcal{A} corrupts the nodes that \mathcal{A}' chooses to corrupt and sends the identities of them to \mathcal{C} . After that, \mathcal{A} interact with \mathcal{C} as per the step 3), then sends the information it received from \mathcal{C} to \mathcal{A}' , who will return a 2-tuple (j, r'_j) . Subsequently, \mathcal{A} sends j to \mathcal{C} in step 4). In step 6), \mathcal{A} needs to make a guess. Here, if the number \mathcal{A} receives from \mathcal{C} is

equal to r'_j , let $b' = 0$. Otherwise, let $b' = 1$. Here, it is obvious that \mathcal{A} is PPT. Additionally, we have

$$|\Pr[b = b'] - \frac{1}{2}| > \text{negl}(\lambda).$$

As there does not exist such \mathcal{A} , Rondo is unpredictable. ■

Theorem 3. *Assuming the DLog assumption, Rondo achieves bias-resistance.*

Proof: From Theorem 1, Rondo achieves indistinguishability. Assume a PPT adversary \mathcal{A}' , $i \geq 1$, $k \in [1, |r_i|]$ for any negligible function $\text{negl}(\lambda)$,

$$\left| \Pr \left[\begin{array}{l} (k, r'_i(k)) \leftarrow \mathcal{A}(r_1, r_2, \dots, r_{i-1}) : \\ r'_i(k) = r_i(k) \end{array} \right] - \frac{1}{2} \right| \leq \text{negl}(\lambda)$$

or

$$|\Pr[r_i(k) = 0] - \frac{1}{2}| > \text{negl}(\lambda).$$

Then we can construct a PPT adversary \mathcal{A} utilizes \mathcal{A}' as a sub-algorithm to win the game $\mathcal{G}_{\text{beacon}}$ in Definition 2 as follows:

Firstly, \mathcal{A} interacts with \mathcal{C} as per the game $\mathcal{G}_{\text{beacon}}$ in step 1), then it invokes \mathcal{A}' with current information it receives from \mathcal{C} , including previous beacon output as well as public parameters. Moreover, \mathcal{A} corrupts the nodes that \mathcal{A}' chooses to corrupt and sends the identities of them to \mathcal{C} . After that, \mathcal{A} interact with \mathcal{C} as per the step 3), then sends the information it received from \mathcal{C} to \mathcal{A}' , who will return a 2-tuple $(k, r'_{i+1}(k))$. Subsequently, \mathcal{A} sends $i + 1$ to \mathcal{C} in step 4). Furthermore, in step 6), \mathcal{A} needs to make a guess. If the k^{th} bit of the number it receives from \mathcal{C} is equal to $r'_{i+1}(k)$, let $b = 0$. Otherwise, let $b = 1$. It is obvious that \mathcal{A} is PPT. Additionally, we have

$$|\Pr[b = b'] - \frac{1}{2}| > \text{negl}(\lambda).$$

As there does not exist such \mathcal{A} , Rondo is bias-resistant. ■

Theorem 4. *Rondo achieves liveness.*

Proof: In each epoch e there are at least $t + 1$ correct nodes acting as dealers of Breeze instances. Without loss of generality, we assume $\{\text{Breeze}_i, i \in [1, t+1]\}$ are $t+1$ correct instances. Subsequently, according to the liveness property of bAVSS-PO, each cer_i of Breeze_i is eventually received by all correct nodes and then be proposed in Rondo-BFT. Then, due to the liveness and agreement properties of our Rondo-BFT, every correct node will have the same common subset CS_e . Additionally, at least one cer_k of a correct instance Breeze_k is contained in CS_e . Finally, due to the liveness and commitment of bAVSS-PO, the batch of secrets shared by Breeze_k can be reconstructed by all the correct nodes. Thus, in each round of epoch e , there is a beacon output. ■

Theorem 5. *Rondo achieves public verifiability.*

Proof: As our Rondo-BFT protocol is signature-based, a set of $2t + 1$ matching signatures in the commit phase in the protocol can be used to prove the identities of CS_e in each epoch. ■

APPENDIX C PROOF OF BREEZE

We show the proof for our Breeze protocol.

Lemma 1. (Schwartz-Zippel) *Let $f(x) \in \mathbb{F}[x]$ be a nonzero polynomial of total degree at most p and \mathbb{F} is a finite set. Then if a is randomly chosen from \mathbb{F} , then we have:*

$$\Pr[f(a) = 0] \leq \frac{p}{|\mathbb{F}|}$$

Proof: The proof of this lemma directly comes from fundamental theorem of algebra. ■

Lemma 2. *Let $\mathcal{P}^{s,t}$ denotes the set of all degree- t univariate polynomials over \mathbb{F} whose constant term is s , where $|\mathcal{P}^{s,t}| = |\mathbb{F}|$. For any set of distinct non-zero elements $\alpha_1, \dots, \alpha_n \in \mathbb{F}$, any pair of values s, s' , any subset $\mathcal{C} \subset \{P_1, \dots, P_n\}$ where $|\mathcal{C}| = \ell \leq t$ and every $\vec{y} \in \mathbb{F}^\ell$, it holds that:*

$$\begin{aligned} & \Pr_{f(x) \in_R \mathcal{P}^{s,t}} [\vec{y} = (\{f(\alpha_i)\}_{P_i \in \mathcal{C}})] \\ &= \Pr_{g(x) \in_R \mathcal{P}^{s',t}} [\vec{y} = (\{g(\alpha_i)\}_{P_i \in \mathcal{C}})] \\ &= \frac{1}{|\mathbb{F}|^\ell} \end{aligned}$$

where $f(x)$ and $g(x)$ are chosen uniformly and independently from the set of polynomials $\mathcal{P}^{s,t}$ and $\mathcal{P}^{s',t}$, respectively.

Proof: Correctness of the lemma follows [72, Claim 3.2]. ■

Informally, Lemma 2 shows that for each ACSS instance, the distribution of the shares seen by the adversary is independent of the underlying secret.

Corollary 1. *For our Breeze protocol, the distributions of the shares of different Breeze instance seen by the adversary are independent to each other.*

Lemma 3. *Assuming the DLog assumption and a collision-resistant hash function, our polynomial commitment is binding and hiding for polynomials.*

Proof: Firstly, when taking a batch of polynomials $\{S_i, i \in [1, B]\}$ with coefficients $\{s_i = (s_{i,0}, \dots, s_{i, \text{deg}(S)})\}$, $i \in [1, B]$ as the input, the output of BatchCommit is a vector, in which every entry is a normal Bulletproofs of S_i with the form $\prod_{j=0}^{\text{deg}(S)} g_j^{s_{i,j}}$. Thus BatchCommit holds the same properties as Bulletproofs. Secondly, in the BatchEval algorithm, it evaluates the underlying polynomials. For each S , we use \mathbf{d} as a blind mask to cover S . Due to the randomness of \mathbf{d} , it achieves hiding property. Moreover, we generate a group of random numbers γ to sum the polynomials up. Then according to Lemma 1, if the verifier validates that $\sigma \cdot \mathbf{y} = \sum_{i=1}^B \gamma_i \mathbf{v}[i] + V_D$ by validating the proof, then with overwhelming probability, it holds that $s_i \cdot y = \mathbf{v}[i]$, $i \in [1, B]$. Hence, our polynomial commitment scheme achieves the hiding property. ■

Theorem 6. *Assuming the DLog assumption, our Breeze protocol is a secure bAVSS-PO protocol.*

Proof: We show that Breeze achieves liveness, correctness, commitment, and privacy.

Liveness-1. If the dealer P_d is correct, all nodes will receive the same root commitment C and polynomial commitments from P_d . Additionally, each node receives B points for B recovery polynomial. It is then not difficult to see that every correct node will eventually send a reply message to the dealer. Consequently, the dealer P_d will receive at least $2t + 1$ valid reply message and be able to send cer_d to all nodes. Therefore, in the confirm phase, each honest node will receive a cer_d for C and complete the sharing.

Liveness-2. If at least $t + 1$ correct nodes validate their partial outputs and start reconstruction for $\text{ID}.d$, the correct nodes must have received a valid cer_d for C . The cer_d of C contains at least $2t + 1$ valid signatures, so at least $t + 1$ correct nodes have received valid shares in the sharing stage. Thus, every correct node will receive at least $t + 1$ valid secret shares, C and $\text{id}x$ from other correct nodes, and thereby reconstruct the corresponding $z_i = R[\text{id}x](0)$.

Correctness-1. Assume that a correct dealer shared a batch of secrets $\{s_1, \dots, s_B\}$. The share polynomial evaluations at all $\{R_k(i)\}_{k \in [1, B]}$ lie on a batch of degree t polynomials. If at least $t + 1$ correct nodes complete the sharing, correct nodes will have a common root commitment C . Due to the binding property of BatchCommit, it must hold $z_i = s_i$, for $i \in [1, B]$.

Correctness-2. If a correct node P_i reconstructs z_i , it must have received at least $t + 1$ valid shares with the same vector commitment C . Any set of $t + 1$ points uniquely determines a polynomial of degree t . As nodes have agreed on the commitment C by validating the partial outputs, the commitments for different subsets of points must be the same due to the binding property of vector commitment. As BatchCommit is also binding, the reconstruction is therefore unique.

Commitment-1. Our cer_d is used as the validation data. If cer_d is formed, at least $t + 1$ correct nodes have received the partial outputs in the sharing stage. Thus, after receiving the validation data, these $t + 1$ correct nodes can validate their partial outputs by invoking ParVerify.

Commitment-2. First, it is clear that in the reconstruction stage, correct nodes will output if and only if they receive at least $t + 1$ valid shares. Therefore, at least $t + 1$ correct nodes have validated their partial outputs with the same C . Thereby, due to the binding property of BatchCommit, at least $t + 1$ correct nodes jointly hold $t + 1$ valid shares of the same sharing polynomial $R_{\text{id}x}$. Accordingly, the sharing polynomials are determined and every correct node output $R_{\text{id}x}[0]$.

Privacy. From Lemma 3, the commitments that the adversary \mathcal{A} received in the sharing phase, leak no information about the shares. Subsequently, the information that the adversary \mathcal{A} could learn is only the shares of the corrupted nodes, which are independent with the underlying secrets due to Lemma 2 and Corollary 1. Hence, the adversary \mathcal{A} has no ascendance in distinguishing the underlying secrets with a batch of random numbers when the dealer is honest. ■

Theorem 7. *Breeze achieves $O(n)$ message complexity and $O(\lambda n)$ amortized communication complexity.*

Proof: The protocol achieves a message complexity of $O(n)$, as nodes in the system only need to communicate

with the dealer. In the share phase, the dealer sends n messages, each of size $O(\lambda(B + \log n))$ so the communication is $O(\lambda n(B + \log n))$. In the reply phase, each node sends a signature to dealer so the communication is $O(\lambda n)$. In the confirm phase, the dealer sends a certificate to the nodes. This costs $O(\lambda n^2)$ communication. If we instantiate the certificate, the length of the certificate is $O(\lambda n \log n)$. Breeze then achieves $O(\lambda n(B + \log n))$. We can set B as $O(\log n)$ to achieve an optimal $O(\lambda n)$ amortized communication, as the amortized communication of Breeze is $O(\lambda(n + \frac{1}{B} \log n + \frac{1}{B} n^2))$.

Alternatively, if we instantiate the certificate using digital signatures, the amortized communication is $O(\lambda(n + \frac{1}{B} \log n + \frac{1}{B} n \log n))$ and we need to set $B = O(n)$ to get $O(\lambda n)$ amortized communication.

For the computation complexity, we calculate three kinds of computation: scalar multiplication, hash operation, exponentiation. In BatchEval, the most significant part is in line 03, which costs $O(nBp)$ scalar multiplication to obtain the evaluation V . For the hash operation, it is to compute the Fiat-Shamir heuristic and the Merkle Tree, which consumes $O(B + n)$ and $O(n \log p)$ respectively. Hence we have $O(B + n \log p)$ hash operation in total. Furthermore, the exponentiation required are BatchCommit, computation of L and R in line 20 and to compute A' in line 30. It is clear that BatchCommit for B polynomials with degree t costs $O(Bt)$. Besides, the sizes of L , R and A' are $O(p)$, and for each entry we need $O(1)$ exponentiation. We have $O(\log p)$ iteration, thus in total, it takes $O(p \log p)$ exponentiation. Therefore in total it costs $O(Bt + p \log p)$ exponentiation. ■

APPENDIX D

BATCHHAVEN: COMPUTATION-EFFICIENT BATCHED ACSS

In this section, we present the details of BatchHAVEN. Our protocol only based on the assumptions of discrete logarithm (DLog) and ROM without using any public key cryptography.

A. The Main Workflow

The workflow of BatchHAVEN largely follows the HAVEN framework [56]. The HAVEN protocol cannot be directly transformed into a batched version, since its polynomial commitment scheme is not suitable for batching. Thus we use our new batching polynomial commitment scheme as that in Breeze for batch verification (defined in Sec. III-A).

The sharing stage (Figure 13). The sharing stage involves three communication phases: a share phase where the dealer P_d disseminates the secret shares via a diagonal method; an echo phase where nodes exchange evaluations about the share polynomials so as to achieve completeness; a ready phase where nodes further exchange information and complete the sharing stage.

The main difference between BatchHAVEN to HAVEN is that we utilize our new polynomial commitment BatchCommit, BatchEval to generate commitments and correctness proof of evaluations for sharing polynomials. In the echo and ready phase, other nodes use BatchVerifyEval to validate the proof.

Moreover, for the vector commitment scheme vCom used in line 07, we also instantiate it with Merkle Tree and assume each polynomial commitment contains the witness to its own inclusion in C .

Reconstruction stage (Figure 14). Being different from HAVEN, the reconstruction procedure needs to be called B times in order to independently recover every secret in the batch. Similar to that for Breeze, we use the parameter idx as the index of the secret to be reconstructed.

Specifically, if P_i completes the sharing stage, it holds the shares $\{S_{ki}(i) | k \in [1, B]\}$. After receiving $p + 1$ valid messages, P_i interpolates the corresponding points and recovers the secret $R_{idx}(0)$.

Our technique works for the case of the low-threshold ACSS by setting p as t . To do this, one just needs to choose a recovery polynomial of degree t , while the rest of algorithms remain the same as those in BatchHAVEN.

B. Proof of BatchHAVEN

We show the proof for our BatchHAVEN protocol. As mentioned in Sec. III, the proof for our low-threshold ACSS is a special case of the proof.

Theorem 8. *Assuming the DLog assumption, our BatchHAVEN protocol is a secure bHACSS protocol.*

Proof: We show that BatchHAVEN achieves liveness, correctness, and completeness.

Liveness-1. If the dealer P_d is correct, all nodes will receive the same root commitment C and polynomial commitments from P_d . Additionally, each node receives one point on each share polynomial. It is then not difficult to see that every correct node will eventually send an echo message and receive $2t + 1$ valid echo messages. Similarly, each node will be able to send ready messages, receive $2t + 1$ valid ready messages, interpolate their own share polynomial, and complete the sharing.

Liveness-2. We show that if a correct node P_i has completed the sharing for ID. d , any other correct node P_j will also complete the sharing. We first show that P_j will eventually receive $2t + 1$ ready messages, and then show that P_j will output some value.

First, as P_i has completed the sharing, it must have received $2t + 1$ ready messages with the same root commitment C , among which at least $t + 1$ are sent from correct nodes. These $t + 1$ nodes will also send a ready message to all nodes if they have not done so yet. Hence, P_j will eventually receive $2t + 1$ ready messages with the root commitment C .

Second, as P_i completed the sharing, P_i must have sent a ready message with root commitment C . Here, P_i sends a ready message either because it has received $2t + 1$ echo messages with the same C or $t + 1$ ready messages. In both cases, it is not too difficult to see that at least one correct node has received $2t + 1$ echo messages with the same root commitment C , among which at least $t + 1$ are sent by correct nodes. Additionally, according to the binding property of BatchCommit, these correct nodes will also send consistent shares in their echo messages. Therefore, P_j can complete the interpolation in the delivery step and complete the sharing.

Liveness-3. All correct nodes start reconstruction for $\{idx, ID.d\}$. First, there are at least $n - t \geq p + 1$ correct nodes. As these nodes completed the sharing, they each have an evaluation at $S_{idx,i}(i)$ that will be accepted by others. Once $p + 1$ such points are received, each correct node can verify the correctness of points and recover a secret.

Privacy. From Lemma 3, the commitments that the adversary \mathcal{A} received in the sharing phase, leak no information about the shares. Subsequently, the information that the adversary \mathcal{A} could learn is only the shares of the corrupted parties, which are independent with the underlying secrets due to Lemma 2 and Corollary 1. Hence, the adversary \mathcal{A} has no ascendance in distinguishing the underlying secrets with a batch of random numbers when the dealer is honest.

Correctness-1. Assume that a correct dealer shared a batch of secrets $\{s_1, \dots, s_B\}$. Then, the share polynomial evaluations at all $\{S_{ki}(i)\}_{k \in [1, B], i \in [1..n]}$ lie on a batch of degree p polynomial that will recover $\{s_1, \dots, s_B\}$. If a correct node completes the sharing, correct nodes will have a common root commitment C . Due to the binding property of BatchCommit, it must hold $z_i = s_i$, for $i \in [1, B]$.

Correctness-2. If a correct node P_i reconstructs z_i , it must have received at least $p + 1$ valid shares with the same vector commitment C . Any set of $p + 1$ points uniquely determine a polynomial of degree p . As we have agreed on the commitment C , the commitments for different subsets of points must be the same due to the binding property of vector commitment. As BatchCommit is also binding, the reconstruction is therefore unique.

Completeness. This is implied by the correctness and the liveness-2. Indeed, once a correct node completes the sharing, each correct node p_i will receive at least $t + 1$ valid echo messages allowing each correct node to obtain its share polynomial S_i and therefore obtain $R(i) = S_i(i)$. ■

Theorem 9. *BatchHAVEN achieves $O(n^2)$ amortized message complexity and $O(\lambda n^3)$ amortized communication complexity.*

Proof: The protocol achieves a message complexity of $O(n^2)$, as only all-to-all communication is involved. In the send stage, the dealer sends n messages, each of size $O(\lambda n B)$. In the other two stages, each node broadcasts n messages, each of size $O(\lambda n B)$. Therefore, BatchHAVEN has $O(\lambda n^3)$ amortized communication.

For the computation complexity, we calculate three kinds of computation: scalar multiplication, hash operation, exponentiation. In BatchEval, the most significant part is in line 03, which costs $O(nBp)$ scalar multiplication to obtain the evaluation V . For the hash operation, it is to compute the Fiat-Shamir heuristic and the Merkle Tree, which consumes $O(B + n)$ and $O(n \log p)$ respectively. Hence we have $O(B + n \log p)$ hash operation in total. Furthermore, the exponentiation required are BatchCommit, computation of L and R in line 20 and to compute A' in line 30. It is clear that BatchCommit for B polynomials with degree t costs $O(Bt)$. Besides, the sizes of L , R and A' are $O(p)$, and for each entry we need $O(1)$ exponentiation. We have $O(\log p)$ iteration, thus in total, it takes $O(p \log p)$ exponentiation. Therefore in total it costs $O(Bt + p \log p)$ exponentiation. ■

▷ **The share phase.**

01: - Upon receiving (ID.d, in, share, s_1, \dots, s_B), as a dealer (i.e., $P_i = P_d$):

02: - For $k \in [1, B]$, randomly choose a recovery polynomial $R_k \in \mathbb{F}[x]$ of degree p s.t. $R_k(0) = s_k$.

03: - For $k \in [1, B], j \in [1, n]$, randomly choose a share polynomial $S_{kj} \in \mathbb{F}[x]$ of degree t s.t. $S_{kj}(j) = R_k(j)$.

04: - For $k \in [1, B], j \in [1, n]$, let $T_{kj} \leftarrow (R_k - S_{kj})$.

05: - Compute $\hat{R} \leftarrow \text{BatchCommit}(crs, \{R_k | k \in [1, B]\}, p)$, $\hat{S} \leftarrow \text{BatchCommit}(crs, \{S_{kj} | k \in [1, B], j \in [1, n]\}, t)$.

06: - For $j \in [1, n]$, compute $\hat{T}_j \leftarrow \text{BatchCommit}(crs, \{T_{kj} | k \in [1, B]\}, p)$.

07: - Compute $C \leftarrow \text{vCom}(\hat{R}, \hat{S}[1], \dots, \hat{S}[nB])$. //create a root commitment

08: - Compute $(Y^S, \Phi^S) \leftarrow \text{BatchEval}(crs, \{S_{kj} | k \in [1, B], j \in [1, n]\}, \hat{S}, (1, \dots, n), t)$. //evaluates of the polynomials

09: - For $k \in [1, n]$, $\hat{y}_k^S \leftarrow Y^S[:, k]$, $\phi_k^S \leftarrow \Phi^S[k]$.

10: - For $j \in [1, n]$, $(\hat{y}_j^T, \phi_j^T) \leftarrow \text{BatchEval}(crs, \{T_{kj} | k \in [1, B]\}, \hat{T}_j, (j), p)$.

11: - For $k \in [1, n]$, send (ID.d, send, set $_k$) to node P_k , where set $_k = \{C, \hat{R}, \hat{S}, \hat{y}_k^S, \phi_k^S, \hat{y}_k^T, \phi_k^T\}$.

▷ **The echo phase.**

12: - Upon receiving (ID.d, send, set $_i$) from the dealer P_d for the first time:

13: - Verify that $\text{BatchVerifyEval}(crs, \hat{S}, \hat{y}_i^S, (i), \phi_i^S, t)$ and $\text{BatchVerifyEval}(crs, \hat{T}_i, \hat{y}_i^T, (i), \phi_i^T, p)$ return true.

14: - Verify \hat{R} and all entries of \hat{S} are in C .

15: - For $k \in [1, B], j \in [1, n]$, compute $\hat{S}_{new}, y_{new}, \phi_{new}, \hat{S}_{new}[k] \leftarrow \hat{S}[k \times j], y_{new}[k, j] \leftarrow \hat{y}_i^S[k \times j], \phi_{new}[k, j] \leftarrow \phi_i^S[k \times j]$.

16: - For $j \in [1, n]$, send (ID.d, echo, info $_{i,j}$) to P_j , where info $_{i,j} = \{C, \hat{R}, \hat{S}_{new}, y_{new}[:, j], \phi_{new}[:, j]\}$.

▷ **The ready phase.**

17: - Upon receiving (ID.d, echo, info $_{j,i}$) from P_j for the first time:

18: - Verify \hat{R} and all entries of \hat{S}_{new} are in C .

19: - Verify $\text{BatchVerifyEval}(crs, \hat{S}_{new}, y_{new}[:, i], (i), \phi_{new}[:, i], t)$ is true.

20: - **If** $2t + 1$ echo messages with the same C are received and ready is not sent **then**

21: - Send (ID.d, ready, C) to all nodes.

22: - Upon receiving (ID.d, ready, C) from P_j for the first time:

23: - **If** $t + 1$ ready messages with the same C are received and ready is not sent **then**

24: - Send (ID.d, ready, C) to all nodes.

25: - **If** $2t + 1$ ready messages with the same C are received **then** //delivery

26: - Wait until P_i receives $t + 1$ valid echo messages with C .

27: - Then interpolate $\{S_{ji}, j \in [1, B]\}$ from these $B(t + 1)$ valid echo messages.

28: - Compute $y_{ji}^* \leftarrow \text{Eval}(crs, \{S_{ji}, j \in [1, B]\}, i)$.

29: - Output (ID.d, out, shared)

Fig. 13: The sharing stage of BatchHAVEN, for node P_i and tag ID.d.

01: - Upon receiving (in, share, $s, idx, S_{idx,i}(i)$): //reconstruct the idx^{th} secret.

02: - Send (ID.d, reconstruct-share, $C, \hat{S}_{idx,i}, S_{idx,i}(i), \phi_i^S[i]$) to node P_j , for $j \in [1, n]$.

03: - Upon receiving (ID.d, reconstruct-share, $C, \hat{S}_{idx,m}, S_{idx,m}(m), \phi_i^S[i]$) from node P_m :

04: - **If** $\hat{S}_{idx,m} = \hat{S}[idx \times m]$ and $\text{BatchVerifyEval}(crs, \hat{S}_{idx,m}, S_{idx,m}(m), (m), \phi_i^S[i], t)$ returns true **then**

05: - **If** $p + 1$ valid reconstruct-share messages with the same C are received **then**

06: - Interpolate R_{idx} from the $p + 1$ valid points

07: - Output (ID.d, out, reconstructed, $R_{idx}(0)$)

Fig. 14: The reconstruction stage of BatchHAVEN, for node P_i and tag ID.d.

APPENDIX E PROOF OF RONDO-BFT

We prove the correctness of the protocol based on the definitions of correctness specified in Sec. III. Based on the commonly used syntax in BFT, we use node and replica interchangeably.

Lemma 4. *Suppose that for any epoch $e_i \in [1, 2, \dots, E]$, any correct replica that sets configuration M_e to a non-empty value sets M_e to the same set. Let b and b' be blocks such that $b.height, b'.height \in [1, EB]$. If there exist valid $prepareQC$ s for both b and b' and $b.view = b'.view = v$, then b and b' are blocks in the same epoch.*

Proof: Assume, towards a contradiction, a $prepareQC$ qc for b and a $prepareQC$ qc' for b' are formed in view v , $b.height \in ((e - 1)k, ek]$, $b'.height \in ((e' - 1)B, e'B]$, and $e \neq e'$. W.o.l.g., we assume that $e \leq e'$. Based on the standard assumption, at least $\lceil \frac{2|M_e|+1}{3} \rceil$ replicas in M_e are correct and

at least $\lceil \frac{2|M_{e'}|+1}{3} \rceil$ replicas in $M_{e'}$ are correct. According to Figure 9, correct replicas in $M_{e'}$ begin to send messages for consensus in view v only after receiving $commitQC$ s from $\lceil \frac{2|M_{e'-1}|+1}{3} \rceil$ replicas included in $M_{e'-1}$, where the $commitQC$ s are formed for a block with a height of $(e' - 1)B$ in view $v' < v$. Therefore, at least $\lceil \frac{M_{e'-1}+1}{3} \rceil$ correct replicas in $M_{e'-1}$ have received a $commitQC$ for b_0 , set their system configuration as $M_{e'}$ before view v . These replicas won't send prepare messages for any blocks in an epoch lower than e' in any view $v' \geq v$. According to Figure 9 and Figure 10, a correct replica only sends commit (or pre-commit) message for b_0 after receiving a $precommitQC$ (or $prepareQC$) for b_0 . Then a valid $prepareQC$ for b_0 formed in epoch $e' - 1$ exists. Similar to the discussion for epoch e' and $e' - 1$, we can obtain through induction that at least $\lceil \frac{M_e|+1}{3} \rceil$ correct replicas in M_e have set their system configuration as M_{e+1} before view v . These replicas won't send prepare messageS for any blocks in an epoch lower than $e + 1$ in any view $v' \geq v$. Due to the quorum intersection, qc can't be formed, a contradiction. ■

Lemma 5. *Suppose that for any epoch $e_i \in [1, 2, \dots, E]$, any correct replica that sets configuration M_e to a non-empty value sets M_e to the same set. Let b and b' be blocks such that $b.height, b'.height \in [1, EB]$ and there exist valid *prepareQCs* for both b and b' . If b and b' are both blocks proposed in view v , then b and b' are blocks on the same branch.*

Proof: Assume, towards a contradiction, that b and b' are conflicting blocks. By Lemma 4, b and b' are blocks in the same epoch. Let e denote $b.height \% B$, we have $e \in [1, 2, \dots, E]$. Based on the standard assumption, for each epoch $e \in [1, 2, \dots, E]$, at least $\lceil \frac{2|M_e|+1}{3} \rceil$ replicas in M_e are correct. Then we consider two cases:

1) $b.height = b'.height$. At least one correct replica in M_e must have voted for *prepareQCs* for both blocks with the same rank. This is impossible because the pseudocode allows voting only once for each height in each view.

2) $b.height \neq b'.height$. We assume, w.o.l.g., $b.height \geq b'.height$. Let b_1 denote the block with the lowest height on the branch led by b such that $b_1.view = v$ and $b_1.height \geq b'.height$. According to Figure 9, within view v , a correct replica votes for a block only after receiving a valid *prepareQC* for the parent block of the block. As a *prepareQC* for b are formed in view v , a *prepareQC* for b_1 have been formed. Due to the quorum intersection, at least one correct replica p_j in M_e voted for both b_1 and b . If $b_1.height = b'.height$, similar to the discussion in case 1), that's a contradiction. If $b_1.height > b'.height$, we have that the view of $b_1.parent$ is lower than v by the definition of b_1 . According to Figure 9, b_1 is the first block voted by p_j in view v . After voting for b_1 , p_j should update its lv to b_1 and stop voting for blocks with lower heights than b_1 in view v . However, p_j has also voted for b' , a contradiction. ■

Lemma 6. *Suppose that for any epoch $e_i \in [1, 2, \dots, E]$, any correct replica that sets configuration M_e to a non-empty value sets M_e to the same set. Let b and b' be blocks proposed in the same epoch e . If there exists a valid *commitQC* qc for b and a valid *prepareQC* or *commitQC* qc' for b' such that $qc.view \leq qc'.view$, then b and b' are blocks on the same branch.*

Proof: Let v denote $qc.view$ and let v' denote $qc'.view$. Then $v \leq v'$. Based on the standard assumption, at least $\lceil \frac{2|M_e|+1}{3} \rceil$ replicas in M_e are correct. Note that a correct replica only sends commit (or pre-commit) message for b after receiving a *precommitQC* (or *prepareQC*) for b . As there exists *commitQC* qc formed for b in view v , at least $\lceil \frac{|M_e|+1}{3} \rceil$ correct replicas have locked at a *precommitQC* qc_b for b and sent commit messages for b in view v . As b and b' are blocks in the same epoch and qc' exists, at least one replica p_i of these correct replicas has voted for b' in view v' . Then we prove the lemma by induction over the view v' , starting from view v .

Base case: If $v' = v$, we consider four cases:

1) If $b.view = b'.view = v$, according to Figure 9, no matter qc' is a *prepareQC* or a *commitQC*, there exists *prepareQCs* for both b and b' formed in view v . By Lemma 5, b' cannot conflict with b .

2) If $b.view < v$ and $b'.view < v$, then p_i voted for for b and b' during view change in view v . According to Figure 10, p_i voted for only one block during view change. Then we have that $b = b'$.

3) If $b.view < v$ and $b'.view = v$, then correct replicas sent commit message for b during view change and voted for b' in normal cases in view v . No matter if qc' is a *prepareQC* or a *commitQC*, a valid *prepareQC* for b' exists. Note that within view v , a correct replica votes for a block only after receiving a valid *prepareQC* for the parent block of the block. Let b'_0 denote the block with the lowest height on the branch led by b' such that $b'_0.view = v$. Then we have that there exists a *prepareQC* for b'_0 formed in view v and the view of the parent block of b'_0 is proposed before view v . Due to the quorum intersection, at least one correct replica has sent commit message for b and prepare message for b'_0 . According to Figure 9, b'_0 and b' must extensions of b .

4) If $b.view = v$ and $b'.view < v$, then p_i voted for b' during view change in view v and qc' is a *commitQC*. Then similar to the discussion in case 3), b must be an extension of b' .

In all the cases, b' and b cannot conflict with each other.

Inductive case: Assume this lemma holds for view v' from v to $v + k - 1$ for some $k \geq 1$. We prove that it holds for $v' = v + k$. We consider two cases:

1) If $b'.view < v'$, then p_i voted for b' during view change in view v' . Let qc_l denote the *precommitQC* locked by p_i when p_i decided to vote for b' . According to Figure 9 and Figure 10, a correct replica only updates its *lockedQC* with a *precommitQC* with a higher rank than that of its *lockedQC*. Thus, $rank(qc_l) \geq rank(qc_b)$. Meanwhile, p_i has received a *prepareQC* $qc_{b'}$ for b' such that $rank(qc_l) \leq rank(qc_{b'})$ before sent commit message for b' . Therefore, $v \leq qc_{b'}.view < v'$. According to the inductive hypothesis, $b = b'$ or b and b' are blocks on the same branch.

2) If $b'.view = v'$, then p_i voted for b' in normal cases in view v' . Within view v' , if a block extends a block proposed before view v' , correct replicas vote for the block only after receiving a valid *commitQC* formed in view v' for the parent block of the block. Let b'_0 denote the block with the largest height on the branch led by b' such that $b'_0.view < v'$. No matter if qc' is a *prepareQC* or a *commitQC*, there exists a *precommitQC* for b'_0 formed in v' . Due to the quorum intersection, at least one correct replica p_j has sent commit message for b in view v and pre-commit message for b'_0 in view v' . Let qc_l denote the *precommitQC* locked by p_j when p_j decided to vote for b' . According to Figure 10, $rank(qc'_0) \geq rank(qc_l) \geq rank(qc_b)$. As $qc'_0.view = b'_0.view$, $b'_0.view > b.view$ or ($b'_0.view = b.view$ and $b'_0.height \geq b.height$). According to the inductive hypothesis, b'_0 is an extension b or $b_0 = b$. Therefore, b' is an extension b or $b = b'$.

Either way, b' and b cannot conflict with each other. This completes the proof. ■

Lemma 7. *Suppose that for any epoch $e_i \in [1, 2, \dots, E]$, any correct replica that sets configuration M_e to a non-empty value set M_e to the same set. Let b denote a block in epoch e and b'*

be a block in epoch e' , where $1 \leq e \leq e' \leq E$. If there exist a valid *commitQC* qc for b and a valid *QC* qc' for b' , where qc' is a *precommitQC* or a *commitQC*, then b and b' are blocks on the same branch.

Proof: Let v denote the view in which a *commitQC* for b is formed and let v' denote the view in which a *commitQC* for b' is formed. Based on the standard assumption, at least $\lceil \frac{2|M_e|+1}{3} \rceil$ replicas in M_e and at least $\lceil \frac{2|M_{e'}|+1}{3} \rceil$ replicas in $M_{e'}$ are correct. Then we prove the lemma by induction over the epoch e' , starting from view e .

Base case: If $e' = e$, b and b' are blocks in the same epoch. Note that a correct replica only sends commit (or pre-commit) message for b' after receiving a *precommitQC* (or *prepareQC*) for b' . By Lemma 6, no matter $v \leq v'$ or $v' \leq v$, b and b' must be blocks on the same branch.

Inductive case: Assume this lemma holds for view e' from e to $e+k-1$ for some $k \geq 1$. We prove that it holds for $e' = e+k$. Let b_0 denote the block on the branch led by b' such that $b_0.height = (e'-1)B+1$. Note that a correct replica only sends commit (or pre-commit) message for b' after receiving a *precommitQC* (or *prepareQC*) for b' . As a *commitQC* is formed for b' , at least $\lceil \frac{|M_{e'}|+1}{3} \rceil$ correct replicas in $M_{e'}$ have received *prepareQCs* for $b'.parent$ and sent prepare message to form a *prepareQC* for b' . Similarly, we have that for any block on the branch led by b' , a *prepareQC* for the block exists. Then at least $\lceil \frac{|M_{e'}|+1}{3} \rceil$ correct replicas in $M_{e'}$ have sent prepare message for b_0 to form a *prepareQC* for b_0 . Let $m = (\text{prepare}, v, b_0, qc)$ denote the prepare message in which b_0 is proposed. According to Figure 9, a correct replica p_i votes for b_0 only if one of the following two conditions are satisfied: (1) $b'.view = cview$ and $rank(b', lv) \geq 1$; (2) $b'.view < cview$ and $lv.view < cview$ and $b' = \text{block}(\text{lockedQC})$ and b' is delivered. Let b'_0 denote $b_0.parent$, then $b'_0.height = (e-1)B$. By Figure 9 and Lemma 6, any replica in $M_{e'}$ receives *commitQCs* for b'_0 , set their $cview$ as $b'_0.view + 1$, set their lv as b'_0 , and set their *lockedQC* as a *precommitQC* for b'_0 before voting for b_0 . Therefore, condition (1) can't be satisfied and condition (2) is satisfied at p_i . Let qc'_0 denote the *lockedQC* of p_i when p_i votes for b'_0 . According to the inductive hypothesis, the block of qc'_0 must be b'_0 and b'_0 must be an extension of b . Then b_0 and b' are extensions of b . This completes the proof. ■

Lemma 8. For any epoch $e > 1$, if a correct replica p_i sets the configuration as M_e , and another correct replica p_j sets the configuration as M'_e , then $M_e = M'_e$.

Proof: Assume, towards a contradiction, that $M_e \neq M'_e$. Note a correct replica installs system configurations in ascending order. Let e_0 denote the lowest epoch such that p_i and p_j have installed different configurations, i.e., for any epoch e' such that $1 \leq e' < e_0$, p_i and p_j installed the same configuration for epoch e' . As all correct replica has the same initial configuration, $e_0 > 1$. Let M_{e_0} denote the configuration installed by p_i and Let M'_{e_0} denote the configuration installed by p_j , then $M_{e_0} \neq M'_{e_0}$. As p_i sets its configuration as M_{e_0} in epoch e_0 , according to Figure 9 and Figure 10, p_i has installed configuration M_{e_0-1} for epoch $e_0 - 1$. Meanwhile, p_i has received a *commitQC* qc for a block b such that the branch led by b includes a block b_0 such

that $b_0.height = (e_0 - 1)B + 1$. M_{e_0} was computed based on M_{e_0-1} and b_0 using this method: for any JOIN request $(\text{add}, j, pk_j) \in b_0.rq$, $M_{e_0} \leftarrow M_{e_0-1} \cup p_j$; for any LEAVE request $(\text{leave}, pk_j) \in b_0.rq$, $M_{e_0} \leftarrow M_{e_0-1} \setminus p_j$. Similarly, p_j has received a *commitQC* qc' for a block b' such that the branch led by b' includes a block b'_0 such that $b'_0.height = (e_0 - 1)B + 1$. M'_{e_0} was computed based on M_{e_0-1} and b'_0 : for any JOIN request $(\text{add}, j, pk_j) \in b'_0.rq$, $M_{e_0} \leftarrow M_{e_0-1} \cup p_j$ and for any LEAVE request $(\text{leave}, pk_j) \in b'_0.rq$, $M_{e_0} \leftarrow M_{e_0-1} \setminus p_j$. According to Lemma 6 and Lemma 7, no matter whether b and b' are blocks in the same epoch, they are blocks on the same branch. Therefore, $b_0 = b'_0$ and $M_{e_0} = M'_{e_0}$, contradicting to the assumption. ■

Theorem 10. (Enhanced total order) If a correct replica p_i delivers a request with a sequence number k , and another correct replica delivers a request rq' with the same sequence number, then $rq = rq'$.

Proof: Let b_0 denote the block such that $rq \in b_0.rq$ and p_i delivers b_0 and let b'_0 denote another block such that $rq' \in b'_0.rq$ and p_j delivers b'_0 . Since rq and rq' are delivered with the same sequence number, b_0 and b'_0 are blocks in the same epoch e and $b_0.height = b'_0.height$. As p_i delivers b_0 , according to Figure 9 and Figure 10, p_i has received a valid *commitQC* qc for a block b such that the branch led by b includes b_0 . Similarly, p_j has received a valid *commitQC* for a block b' such that the branch led by b' includes b'_0 . By Lemma 8, for any epoch e_i , any correct replica that sets configuration M_{e_i} to a non-empty value set M_{e_i} to the same set. Then b and b' should be two blocks on the same branch from Lemma 6. Therefore, $rq = rq'$. ■

Lemma 9. Let M_e denote the latest configuration installed by at least one correct replica p_i before view v . After GST, if the designated leader $\text{LEADER}(v, M_e)$ is correct and timers are properly set up, a new leader can be successfully elected.

Proof: Let p_v denote $\text{LEADER}(v, M_e)$. The proof is divided into two parts. Firstly, we prove that in view v , no replica rather than $\text{LEADER}(v, M_e)$ can obtain a valid view-change certificate for itself. Subsequently, we prove that $\text{LEADER}(v, M_e)$ can collect a valid view-change certificate for itself and propose a block such that all correct replicas in M_{e+1} will vote for the block.

By Lemma 8, for any epoch $e' \in [1, e]$, each correct replica in $M_{e'}$ either sets the system configuration as empty or sets it to the same set $M_{e'}$. Then we prove, towards contradiction, that any replica rather than $\text{LEADER}(v, M_e)$ can't obtain a valid view-change certificate for itself. Assume that $p_j = \text{LEADER}(v, M_{e-1})$ and p_j obtains a view-change certificate from M_{e-1} for p_j in view v . As M_e has been installed by p_i , then p_i has received $(\text{commit}, v, b, qc)$ messages from $\lceil \frac{2|M_{e-1}|+1}{3} \rceil$ replicas in M_{e-1} and qc is a valid *commitQC* for b , where $b.height = eB$. Then there exists a set S consisting of at least $\lceil \frac{|M_{e-1}|+1}{3} \rceil$ correct replicas in M_{e-1} such that each replica in S has set its *confirmQC* as a *commitQC* for b before view v . In the view-change message for view v , each replica in S should broadcast its latest *commitQC*. Note a valid view-change certificate VC from M_{e-1} contains $\lceil \frac{2|M_{e-1}|+1}{3} \rceil$ messages. Therefore, the height of the highest *commitQC* in VC is no less than eB . According to Figure 10,

VC could not be a VC for p_j . We can prove the same applies to any epoch $e' \in [1, e]$ using the recursive method.

Then we prove that second part of the proof. As there exists a correct replica p_j that stays in the consensus system permanently, view-change messages from any correct replica in M_e can be received by p_j . p_j will forward their messages to p_v . Therefore, p_v can obtain a view-change certificate for itself. Then p_v can broadcast a pre-commit message for the block of the latest $prepareQC$ (denoted as b) and VC . After receiving VC , each correct replica in M_e will switch to the pre-commit phase for b . This completes the proof. ■

Theorem 11. (Liveness) *If a correct client submits a request rq , then a correct replica in some configuration c eventually delivers rq .*

Proof: Since each correct leader will propose blocks consisting of rq until rq is delivered, we only need to prove that after GST, a correct leader can be successfully elected and propose a new block such that more than two-thirds of the replicas vote for the block and reach an agreement. By Lemma 9, if the designated leader $LEADER(v, M_e)$ for view v is correct and timers are properly set up, a new leader can be successfully elected. As the $LEADER()$ function returns replicas in a rotation method, the probability of the designated leader for view v being correct exceeds $2/3$. Therefore, a correct replica in some configuration c eventually delivers rq . ■

Theorem 12. (Consistent delivery) *A correct client submitting m will deliver a correct response that is consistent with the state in com configuration where m is delivered.*

Proof: A correct client completes a request if it has received $t_e + 1$ matching replies in a configuration M_e . Before this, the client verifies the received configuration history to ensure that M_e has been committed. According to the total order and agreement properties, any correct replica will execute and deliver m following the same order. Therefore, all correct replicas will generate matching responses to the client. ■

Theorem 13. (Agreement) *If a correct node in epoch e delivers a request rq , then every correct node in the same epoch e eventually delivers rq .*

Proof: Let p_i denote the correct replica that delivers rq in epoch e . By Lemma 8, each correct replica in M_e either sets the system configuration as empty or sets it to the same set M_e . Let p_k denote another correct replica in M_e . Based on the protocol, p_k will stay in the consensus system until that for any height in epoch e , p_k has delivered a block and p_k has received a $prepareQC$ formed in epoch e' such that $e' > e$ and $p_k \notin M_{e'}$. By Theorem 10, p_k won't leave the system before delivering rq .

As there exists a correct replica p_j stays in the consensus system permanently and correct replicas broadcast their $prepareQCs$ and $commitQCs$ during view change or epoch change phase, $prepareQCs$ formed in higher epoch than e will be forwarded to p_j . According to Theorem 11 p_j will finally forward $prepareQCs$ and $commitQCs$ from p_i to p_k . This completes the proof. ■