Efficient Privacy-Preserving General Edit Distance and Beyond

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***Abstract*—Edit distance is an important non-linear metric that has many applications ranging from matching patient genomes to text-based intrusion detection. Depends on the application, related string-comparison metrics, such as weighted edit distance, Needleman-Wunsch distance, longest common subsequences, and heaviest common subsequences, can usually fit better than the basic edit distance. When these metrics need to be calculated on sensitive input strings supplied by mutually distrustful parties, it is more desirable but also more challenging to compute them in privacy-preserving ways. In this paper, we propose efficient secure computation protocols for private edit distance as well as several generalized applications including weighted edit distance (with potentially content-dependent weights), longest common subsequence, and heaviest common subsequence. Our protocols run 20+ times faster and use an order-of-magnitude less bandwidth than their best previous counterparts. Alongside, we propose a garbling scheme that allows free arithmetic addition, free multiplication with constants, and low-cost comparison/minimum for inputs of restricted relative-differences. Moreover, the encodings (i.e. wire-labels) in our garbling scheme can be converted from and to encodings used by traditional binary circuit garbling schemes with light to moderate costs. Therefore, while being extremely efficient on certain kinds of computations, the new garbling scheme remains composable and capable of handling generic computational tasks.**

I. Introduction

Edit Distance quantifies the *dissimilarity* of two strings by the minimal number of editing operations (insert, delete, and substitute) to transform one string to the other. It finds many interesting applications ranging from diagnosis and treatment of genetic diseases [1–3] to computer immunology [4] and intrusion detection [5], [6]. The basic edit distance can be generalized to settings where different costs are associated to different kinds of editing operations (known as *weighted edit distance*), and the costs of the edits can even depend on the values of the operands (e.g., *Needleman-Wunsch* [7] distance). To maximize utility, real world applications often favor variants of edit distance with weights empirically adjusted based on the likelihoods of various mutations [2], [6], [8].

To securely compute the basic edit distance, researchers have tried generic approaches using binary garbled circuit [9], [10]. However, due to the significant constant-factor blowups in translating the computation into binary circuits, the cost of these protocols are prohibitive for practical uses. Moreover, even leveraging all the recent technical breakthroughs [10–14] in generic secure computation, performance of the resulting protocols remains less than satisfactory to enable many real world applications. As a background-study part of this work, we have implemented a class of private-edit-distance-like secure computation protocols such as weighted edit distance, Needleman-Wunsch, longest common subsequence (LCS), and heaviest common subsequence (HCS), using all existing applicable optimizations including fixed-key hardware AES [12], [15], Half-Gate garbling [13], free-XOR technique [11]. We report the performance of these protocols in the "Best Prior" row of Table I, as well as in the performance charts of Figure 4, 5 in Section V-A and use them as baselines to evaluate our new approach. Note that our baseline performance numbers are already much better than any generic protocols we can find in the literature, simply because we have, for the first time, applied the most recent optimizations (such as Half-Gates, efficient AESNI-based garbling, and highly customized circuits) to solve this particular set of problems.

To circumvent the deficiency of the generic approach, researchers have proposed some interesting heuristic methods that exploit a public reference string to compute the basic edit distance over low-entropy genome strings [16], [17]. Wang et al. proposed to approximate edit distances by converting it to set-difference-size problem then used sketch algorithms to approximate the set-difference-size [16]. Asharov et al. proposed to divide strings into short segments and approximate the overall edit distance by accumulating the scores on the individual segments [17]. As a result, these methods achieved very high efficiency and scalability. These heuristic methods, however, have several notable limitations in common. First, they only work with low-entropy strings. When the entropy of the input strings increases, it is unclear how to find good reference strings as those papers did not discuss how to identify good reference strings, especially in a privacy-preserving manner. Second, they do not support more generalized string metrics such as weighted edit distance, Needleman-Wunsch, longest common subsequence (LCS), heaviest common subsequence (HCS) [18] which are more widely used in the fields than the basic edit distance [2], [6], [8]. Finally, they assume a weaker threat model that leaks more than what is allowed by the standard security definition of private edit distance [19]. See Section VI for more detailed discussion on the comparisons.

Thus, our work is motivated by the following two questions:

- *Can private edit distance be done more efficiently over arbitrary inputs according to the standard definition of security, without sacrificing accuracy?*
- *Can the result be extended to compute other more general edit-distance-like string metrics used in broader scenarios?*

Our approach to answering these questions is based on two

insights. First, most part of these computations can be more efficiently realized with *arithmetic* circuits than binary circuits. Second, we observe that, inherent to these applications, there is special *public* correlation of intermediate values used by many component computations, e.g., the inputs to the minimum circuit differ by some publicly inferable values. To leverage these insights, we propose a special garbling scheme and demonstrate with experiments that a range of edit-distance-like string-metrics can be securely computed an order-of-magnitude more efficiently than the best prior work.

Threat Model. In this paper, we focus on the semi-honest model but leave it as an interesting future work to prevent active attacks.

Contribution. We propose a new garbling scheme that exploits interesting characteristics of edit-distance-like computations. When applied to this particular class of computations, the proposed garbling scheme features (almost) free addition, low cost comparison, minimum, and public table lookup (with secret indices) operations. Its construction is conceptually simple and we formally prove its security. Finally, the scheme can also be extended to handle arbitrary functions through tethering with traditional garbling of binary circuits. We have implemented our scheme through exploiting the high-performance fixed-key AES cipher accelerated by Intel AESNI instructions.

We experimentally evaluate our approach by applying the proposed garbling scheme to securely compute edit distance, weighted edit distance, Needleman-Wunsch distance, LCS, HCS. Unlike the approximation approach, our protocols will always calculate *precise* results. Our experiments show that these new protocols run 20+ times faster and are an order-of-magnitude more bandwidth-efficient than the state-of-the-art protocols (see Table I for some highlights of the performance comparisons). Moreover, we stress that, unlike secure approximation protocols [16], [17], our approach keeps all the secret intermediate states of the computation, which can be obliviously used in a subsequent computation if needed.

II. Background

Notations. We denote the computational parameter by κ. We use "$a := b$" to denote assigning the value of b to a; use "$x \leftarrow S$" to denote uniformly sampling an element of the set S and assigning it to x.

A. Secure Garbling

First proposed by Yao [20], garbled circuits were later formalized by Bellare et al. [15] as a cryptographic primitive of its own interest. Following the notations of Bellare, Hoang, and Rogaway, a *garbling scheme* $\mathcal{G}$ is a 5-tuple $(\mathsf{Gb}, \mathsf{En}, \mathsf{Ev}, \mathsf{De}, f)$ of algorithms, where Gb is an efficient randomized *garbler* that, on input $(1^k, f)$, outputs (F, e, d); En is an *encoder* that, on input (e, x), outputs X; Ev is an *evaluator* that, on input (F, X), outputs Y; De is a *decoder* that, on input (d, Y), outputs y. The *correctness* of $\mathcal{G}$ requires that for every $(F, e, d) \leftarrow \mathsf{Gb}(1^k, f)$ and every x,

$$\mathsf{De}(d, \mathsf{Ev}(F, \mathsf{En}(e, x))) = f(x).$$

Bellare et al. have proposed three security notions for garbling: *privacy*, *obliviousness*, and *authenticity*, which we summarize as below.

- **Privacy:** there exists an efficient simulator $\mathcal{S}$ such that for every x,

$$\left\{(F, X, d) : \begin{array}{l}(F, e, d) \leftarrow \mathsf{Gb}(1^k, f),\\ X \leftarrow \mathsf{En}(e, x).\end{array}\right\} \approx \left\{\mathcal{S}(1^k, f, f(x)\right\}.$$

  where "$\approx$" symbolizes computational indistinguishability.
- **Obliviousness:** there exists an efficient simulator $\mathcal{S}$ such that for every x,

$$\left\{(F, X) : \begin{array}{l}(F, e, d) \leftarrow \mathsf{Gb}(1^k, f),\\ X \leftarrow \mathsf{En}(e, x).\end{array}\right\} \approx \left\{\mathcal{S}(1^k, f)\right\}.$$

- **ϵ-Authenticity:** for every efficient adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$,

$$\Pr\left(\begin{array}{c} Y \neq \mathsf{Ev}(F, X)\\ \textbf{and}\\ \mathsf{De}(d, Y) \neq \perp \end{array} : \begin{array}{l}(f, x) \leftarrow \mathcal{A}_1(1^k),\\ (F, e, d) \leftarrow \mathsf{Gb}(1^k, f),\\ X \leftarrow \mathsf{En}(e, x),\\ Y \leftarrow \mathcal{A}_2(1^k, F, X).\end{array}\right) \leq \epsilon.$$

Many optimizations have been proposed to improve circuit garbling in various aspects such as bandwidth [13], [21], [22], evaluator's computation [21], memory consumption [10], and using dedicated hardware [12], [13], [23]. State-of-the-art implementations of garbling schemes using AESNI can typically produce a garbled row of the garbled truth table in roughly every 25ns [12], [14], [23].

B. Edit Distance and its Variants and Generalizations

The edit distance (also known as *Levenshtein distance*) between any two strings $\mathtt{s}$ and $\mathtt{t}$ is the minimum number of edits needed to transform $\mathtt{s}$ into $\mathtt{t}$, where an *edit* is typically one of three basic operations: insert, delete, and substitute. Algorithm 1 is a standard dynamic programming approach to compute the edit distance between two strings. The invariant is that $\mathtt{D}_{i,j}$ always represents the edit distance between $\mathtt{s}[1..i]$ and $\mathtt{t}[1..j]$. Lines 1–2 initialize the first row of the matrix $\mathtt{D}$ while lines 3–4 initialize the first column. Within the main nested loops (lines 5–7), $\mathtt{D}_{i,j}$ is set at line 7 to the smallest of $\mathtt{D}_{i-1,j} + c_{ins}$, $\mathtt{D}_{i,j-1} + c_{del}$, and $\mathtt{D}_{i-1,j-1} + c_{sub}$, where c_{ins}, c_{del}, and c_{sub} correspond to the cost of *insert*, *delete*, and *substitute* a single character (at any position). For basic edit distance, $c_{ins} := 1$, $c_{del} := 1$, and $c_{sub} := (\mathtt{s}[i] = \mathtt{t}[j])\,?\,0 : 1$, i.e., each single-character insert, delete, and substitute incurs one unit cost while matching characters costs zero. Once the minimal edit distance is computed, it is easy to *backtrack* (from $\mathtt{D}_{i,j}$) a sequence of edits that transform $\mathtt{s}[1..i]$ to $\mathtt{t}[1..j]$, e.g., for the purpose of deriving an optimal alignment.

Weighted Edit Distance. More generally, c_{ins}, c_{del}, and c_{sub} can be adjusted to fit the goals of specific applications. For example, in diagnosing certain genetic diseases [1], [3], it is customary to set c_{ins} and c_{del} to integers between 5–10 while setting the substitution cost to 1. The rationale behind the cost gaps is that insertions and deletions (called *indel*s) occur much more rarely than substitution in DNA replication so we

TABLE I: Performance highlights (for κ-bit computational security)

κ		Edit Distance		Weighted Edit Distance		Needleman-Wunsch		LCS	
		Time (s)	B/W (GB)	Time (s)	B/W (GB)	Time (s)	B/W (GB)	Time (s)	B/W (GB)
87	Best Prior	125.33	24.62	156.77	31.34	422.24	96.99	90.23	16.92
	This Work	**5.35**	**2.04**	**10.01**	**7.17**	**35.11**	**12.29**	**4.83**	**1.53**
127	Best Prior	125.33	39.4	156.77	50.14	422.24	155.21	90.23	27.07
	This Work	**7.08**	**4.09**	**10.01**	**14.34**	**43.51**	**25.58**	**6.19**	**3.07**

All distances/scores are computed over two 4000-nucleotide genomes. The weight tables used in Weighted Edit Distance and Needleman-Wunsch are those described in Figure 1. "Best Prior" results are measured on efficient implementations combining the optimizations of Huang et al. [10] with emp-toolkit [14], the most efficient realization of Half-Gates [13] garbling to date. Note that the time costs of the baseline do not change as κ increases from 87 to 127.

Algorithm 1 EditDistance(s, t)

1: **for** $i \coloneqq 0$ to length(s) **do**
2: $\quad \mathtt{D}_{i,0} \coloneqq i$;
3: **for** $j \coloneqq 0$ to length(t) **do**
4: $\quad \mathtt{D}_{0,j} \coloneqq j$;
5: **for** $i \coloneqq 1$ to length(s) **do**
6: $\quad$ **for** $j \coloneqq 1$ to length(t) **do**
7: $\quad\quad \mathtt{D}_{i,j} \coloneqq \mathtt{min}(\ \mathtt{D}_{i-1,j} + c_{ins},\ \mathtt{D}_{i,j-1} + c_{del},$
$\quad\quad\quad \mathtt{D}_{i-1,j-1} + c_{sub}\ )$;

Algorithm 2 Longest common subsequence(s, t)

1: **for** $i \coloneqq 0$ to length(s) **do**
2: $\quad \mathtt{D}_{i,0} \coloneqq 0$;
3: **for** $j \coloneqq 0$ to length(t) **do**
4: $\quad \mathtt{D}_{0,j} \coloneqq 0$;
5: **for** $i \coloneqq 1$ to length(s) **do**
6: $\quad$ **for** $j \coloneqq 1$ to length(t) **do**
7: $\quad\quad \mathtt{D}_{i,j} \coloneqq \mathtt{max}(\ \mathtt{D}_{i-1,j},\ \mathtt{D}_{i,j-1},\ \mathtt{D}_{i-1,j-1} + \mathtt{w}_{i,j})$;

would expect the alignment contains proportionally less *indel*s to reflect this natural fact.

Needleman-Wunsch. As the statistical models of various operations were refined with respect to the symbols involved in the mutations, researchers [8], [24–26] have found many good reasons that justify varying the costs $c_{ins}, c_{del}, c_{sub}$ with the specific characters that are inserted, deleted, or substituted. In this case, c_{ins}, c_{del} and c_{sub} can be viewed as functions over the alphabet of all possible characters. For example, for genomes, they can be encoded as one- and two-dimensional tables (Fig. 1). Note that although the weight tables are publicly known, lookups over the arrays have to be obliviously computed because the indices used to lookup are secret.

A	G	C	T
5	5	6	6

(a) Example c_{ins} or c_{del}

	A	G	C	T
A	0	1	2	1
G	1	0	1	2
C	2	1	0	1
T	1	2	1	0

(b) Example c_{sub}

Fig. 1: Example weight tables of genomic Needleman-Wunsch

Longest common subsequence. Unlike edit distance, the length of longest common subsequence measures the *similarity* of two strings. Given strings s and t, the length of the longest common subsequence between them can be computed using dynamic programming similar to that for edit distance (Algorithm 2). Comparing to Algorithm 1, the only changes are the initialization values in line 2 and 3; and the logic to derive $D_{i,j}$ (line 7). The invariant now is that $\mathtt{D}_{i,j}$ always represents the length of LCS(s$[1..i]$, t$[1..j]$).

With basic LCS, the matching reward, $\mathtt{w}_{i,j}$, is designed as

$$\mathtt{w}_{i,j} = \begin{cases} 1, & \text{if } \mathtt{s}[i] = \mathtt{t}[j] \\ 0, & \text{otherwise} \end{cases}.$$

Heaviest Common Subsequence. As a generalization of LCS, researchers [18] have introduced the concept of *heaviest common subsequence* (HCS), just like Needleman-Wunsch generalizes edit distance. The idea is to let different characters reward differently when they match. Therefore, $\mathtt{w}_{i,j}$ can be viewed as a matrix (to be indexed by $\mathtt{s}[i]$ and $\mathtt{t}[j]$) where only the diagonal entries will be positive while the rest of the matrix are filled by 0s.

III. Our Approach

In this section, we describe an efficient garbling scheme for private edit-distance-like problems.

A. Insights and Design Decisions

First, we illustrate two important observations we made to motivate the design of our new garbling scheme.

Dominating Cost Factors. The dominating cost of solving the general edit distance problem lies in the oblivious computation of *addition*, *equality*, *minimum*, and an optional oblivious *table-lookup*, whose operands are in fact general number field

values. This is evident from the dynamic programming pseudocode of Algorithm 1. Therefore, it is our foremost priority to maximize the efficiency of these oblivious computations when designing new garbling schemes.

Bounded Difference Values. The edit distance computation makes a number of calls to the three-minimum function, which can be instantiated as two nested calls to the two-minimum function, i.e., $\min(a,b,c) = \min\big(\min(a,b), c\big)$. Our key observation is that edit distances can be calculated in a way that all invocations of two-minimum are made with a pair of inputs (a,b) where $a-b$ is bounded by a constant independent of the absolute values of a and b. This observation opens up an opportunity to speed up private edit distance computation by designing a two-minimum gadget that only works for two inputs that are not much apart from each other but is much more efficient than a generic minimum that handles all possible inputs. (Section III-B describes an actual implementation of such a secure two-minimum gadget.)

For example, we can show that, in computing the *basic* edit distance, every call to $\min(a,b)$ can be arranged so that $a-b \in \{-1,0,1,2\}$. A simple proof of this fact can be derived as below. First, because

$$\begin{aligned}&\min(D_{i-1,j}+1, D_{i,j-1}+1, D_{i-1,j-1}+c_{sub})\\ =&\min\big(\min(D_{i-1,j}+1, D_{i-1,j-1}+c_{sub}), D_{i,j-1}+1\big),\end{aligned}$$

let $m_{i,j} = \min(D_{i-1,j}+1, D_{i-1,j-1}+c_{sub})$, our goal is then to show

$$\begin{aligned}(D_{i-1,j}+1) - (D_{i-1,j-1}+c_{sub}) &\in \{-1,0,1,2\},\\ (D_{i,j-1}+1) - m_{i,j} &\in \{-1,0,1,2\}.\end{aligned}$$

Since all the quantities involved are integers, it suffices to show

$$-1 \le (D_{i-1,j}+1) - (D_{i-1,j-1}+c_{sub}) \le 2, \text{ and} \quad (1)$$
$$-1 \le (D_{i,j-1}+1) - m_{i,j} \le 2. \quad (2)$$

The triangle inequality of basic edit distance ensures,

$$|D_{i-1,j} - D_{i-1,j-1}| \le 1, \quad (3)$$
$$|D_{i,j-1} - D_{i-1,j-1}| \le 1. \quad (4)$$

Thus,

$$\begin{aligned}&|D_{i-1,j} - D_{i,j-1}|\\ =&|D_{i-1,j} - D_{i-1,j-1} - (D_{i,j-1} - D_{i-1,j-1})|\\ \le&|D_{i-1,j} - D_{i-1,j-1}| + |D_{i,j-1} - D_{i-1,j-1}| \le 2.\end{aligned}$$

Also because (3), (4), and $0 \le c_{sub} \le 1$, we know

$$\begin{aligned}-1 \le (D_{i-1,j}+1) - (D_{i-1,j-1}+c_{sub}) \le 2, \text{ and}\\ -1 \le (D_{i,j-1}+1) - (D_{i-1,j-1}+c_{sub}) \le 2.\end{aligned}$$

Since

$$\begin{aligned}(D_{i,j-1}+1) - (D_{i-1,j}+1) \le |D_{i,j-1} - D_{i-1,j}| \le 2,\\ (D_{i,j-1}+1) - (D_{i-1,j-1}+c_{sub}) \le 2,\end{aligned}$$

thus,

$$\begin{aligned}&(D_{i,j-1}+1) - m_{i,j}\\ =&(D_{i,j-1}+1) - \min\big((D_{i-1,j}+1) - (D_{i-1,j-1}+c_{sub})\big) \le 2.\end{aligned}$$

Finally, we have

$$\begin{aligned}&(D_{i,j-1}+1) - m_{i,j}\\ =&(D_{i,j-1}+1) - \min\big((D_{i-1,j}+1) - (D_{i-1,j-1}+c_{sub})\big)\\ \ge&(D_{i,j-1}+1) - (D_{i-1,j}+1) \ge -1.\end{aligned}$$

Therefore, we have shown both constraints (1) and (2) hold.

In general, similar observations about the constrained use of two-minimum gadgets can be deduced over extended applications including edit distance with (possibly content-specific) varying weights, and longest common sequence with (possibly contents-specific) varying weights. Here we state our general observation but formally prove it in appendix A. We stress that, unlike the illustrating example above, our proof for the general case does not need the triangle inequality.

Let $\mathtt{s}, \mathtt{t}, D_{i,j}, c_{ins}, c_{del}, c_{sub}$ be defined as in Section II-B, where c_{ins}, c_{del} are generalized to one-dimensional tables and c_{sub} is generalized to a two-dimensional table. Let

$$\begin{aligned}m_{i,j} &= \min\Big(D_{i,j-1} + c_{del}\big[\mathtt{t}[j]\big],\ D_{i-1,j-1} + c_{sub}\big[\mathtt{s}[i], \mathtt{t}[j]\big]\Big)\\ u_{i,j} &= \Big(D_{i,j-1} + c_{del}\big[\mathtt{t}[j]\big]\Big) - \Big(D_{i-1,j-1} + c_{sub}\big[\mathtt{s}[i], \mathtt{t}[j]\big]\Big)\\ v_{i,j} &= \Big(D_{i-1,j} + c_{ins}\big[\mathtt{s}[i]\big]\Big) - m_{i,j}\end{aligned}$$

Then, there exist $D_{i,j}$-independent public constants C_1, C_2, C_3, C_4 such that for all valid indices i, j.

$$C_1 \le u_{i,j} \le C_2, \qquad C_3 \le v_{i,j} \le C_4.$$

B. The Garbling Scheme

Basic Idea. As the circuit for edit distance computation mostly deal with integers, our intuition is to generalize the idea of garbling binary signals to work directly on arithmetic signals. Recall in Half-Gates [13], the garbler picks, for every wire in the circuit, a secret binary string $w_0 \leftarrow \{0,1\}^{128}$ to encode 0 and uses $w_1 = w_0 \oplus \Delta$ to encode 1, where Δ is a global secret value sampled from a large space (e.g., $\Delta \leftarrow \{0,1\}^{128}$). To generalize this idea, we will replace "$\oplus$", the adder of the binary field, with "$+_p$" which works on the public prime-p field (p is sufficiently large, e.g., $p > 2^{87}$). In our scheme, the garbler will first pick a global secret Δ uniform-randomly from $\mathbb{Z}_p$. Then, for every wire in the arithmetic circuit, the garbler picks k_0 uniform-randomly from $\mathbb{Z}_p$ to denote 0; and encode every integer signal $a \in \mathbb{Z}_p$ as $k_a = k_0 +_p a \times_p \Delta$ where "$+_p$" and "$\times_p$" denote mod-p addition and multiplication, respectively. For authentication purposes, we prefix a 40-bit all-zero tag to every wire key k_a as defined above. Since we only consider semi-honest adversaries who don't deviate from protocol specification, the security provided by the 40-zero authentication tags is actually *statistical*.

Notation for Wire-labels. In the rest of the paper, we always use upper-case letters (e.g., A) to name wires. If w_a^A denotes a wire-label, the superscript (in this example, A) signifies the name of the wire to which this wire-label is associated and the subscript (in this example, a) signifies the plaintext signal that the wire-label encodes. When the wire name is irrelevant in the context of the discussion, the superscript can be omitted. In our terminology, generating (or sampling) a fresh wire-label, say w_a^A, to encode a plaintext value a means first picking $k_0^A \leftarrow \mathbb{Z}_p$ (unless k_0^A for wire A has already been known) and then set $k_a^A := k_0^A +_p a \times_p \Delta$ and $w_a^A := 0^{40} \| k_a^A$. We require $w_a^A \in \{0,1\}^{128}$, so if $k_a^A < p$, leading zeros are padded in front to ensure w_a^A has exactly 128 bits.

Next, we show how every gadget needed in the private edit distance computation can be efficiently instantiated with this generalized definition of wire-labels.

Addition. To securely add two plaintext signals $a, b \in \mathbb{Z}_p$ on two wires A and B, which are represented by wire-labels

$$w_a^A = 0^{40} \| (k_0^A +_p a \times_p \Delta) \text{ and}$$
$$w_b^B = 0^{40} \| (k_0^B +_p b \times_p \Delta), \text{ respectively,}$$

it suffices for the garbler to set

$$w_0^C = w_0^A +_p w_0^B$$

while the evaluator locally computes

$$w_c^C := w_a^A +_p w_b^B.$$

Assuming there is no overflow[1], it is easy to verify that $w_c^C = (w_0^C +_p (a +_p b) \times_p \Delta)$, which is indeed the expected encoding of $a +_p b$ on wire C. Moreover, recall that if $a + b < p$, then $a + b = a +_p b$. Therefore, this essentially realizes addition over $\mathbb{Z}$ whenever $a + b < p$.

As a natural extension of secure addition, multiplying a secret value a of a wire A, encoded by wire-label

$$w_a^A = 0^{40} \| (k_0^A +_p a \times_p \Delta)$$

with a public constant c can simply be realized as:

1) the garbler sets $w_0^C = c \times_p w_0^A$; and
2) the evaluator locally derives the wire-label $w_z^Z = c \times_p w_a^A$.

Again, note that if $c \times a < p$ then $c \times a = c \times_p a$. Hence, it realizes constant multiplication over $\mathbb{Z}$ if $c \times a < p$.

Obviously, addition (or known-constant multiplication) is almost free—no expensive cryptographic computation nor network traffic is required—but only runs a mod-p addition (or mod-p multiplication, respectively) on each side of the protocol.

Equality. When computing c_{sub}, an equality test is needed to decide whether two input characters are identical. Let a, b be two integers whose values are publicly known to fall in $\{0, 1, \ldots, \zeta\}$. Let w_a and w_b be the wire-labels that encode signals a and b on wire A and B, respectively. To securely compute if a equals b, it suffices to first securely compute $d = a - b$ (which is almost free) so that the garbler knows k_0^D and Δ while the evaluator learns $w_d^D = 0^{40} \| (k_0^D +_p d \times_p \Delta)$, which encodes d; then noting $d \in \{-\zeta, \ldots, \zeta\}$,

1) the garbler samples a fresh pair of wire-labels w_0^Z and w_1^Z to encode signal 0 and 1 on the output-wire Z; and sends the following $2\zeta + 1$ ciphertexts

$$\mathbf{Enc}_{w_0^D}(w_1^Z, id); \text{ and}$$
$$\mathbf{Enc}_{w_i^D}(w_0^Z, id), \forall i \neq 0, i \in \{-\zeta, \ldots, \zeta\}$$

in a randomly permuted order. Note that id is the integer identifier of a gadget.

2) the evaluator tries to decrypt the above $2\zeta+1$ ciphertexts using w_d^D as the key. Note that only the ciphertext encrypted with key w_d^D will be successfully decrypted to reveal a valid wire-label w_z^Z that encodes $(a = b)?1 : 0$ to the evaluator. We stress that only a successful decryption will return a wire-label with a valid zero-tag.

Namely, the evaluator will learn w_1^Z if and only if $a = b$; and otherwise, will learn w_0^Z.

The cost of our secure equality is linear in the range of $a - b$. Recall that the cost of traditional binary garbled circuit based integer comparison is linear in the number of bits to represent the input numbers. Therefore, our approach will be more efficient when the input numbers are large while (for application-specific reasons) the difference between them can take only a few possible values.

Minimum. First, we observe that given two integers a, b, $\min(a, b) = a - \langle a - b \rangle$, where "$\langle \cdot \rangle$" is a function defined as follows,

$$\langle x \rangle = \begin{cases} x, & \text{if } x \geq 0; \\ 0, & \text{otherwise.} \end{cases}$$

In essence, "$\langle \cdot \rangle$" is a generalized comparison, which can be accomplished in our garbling scheme with a similar idea that enables secure comparison described above. That is, let X, Z be the input- and output-wire, respectively, and assume $x \in \{-\zeta, \ldots, \zeta\}$, the garbler simply sends the following $2\zeta+1$ ciphertexts in a randomly permuted order:

$$\mathbf{Enc}_{w_i^X}(w_0^Z, id), \forall i \in \{-\zeta, \ldots, -1\}; \text{ and}$$
$$\mathbf{Enc}_{w_i^X}(w_i^Z, id), \forall i \in \{0, \ldots, \zeta\}$$

where for $0 \leq i \leq \zeta$, w_i^Z is a freshly generated wire-label representing plaintext value i on the output-wire Z.

Based on a very similar analysis for the cost of secure comparison, our secure minimum gadget will outperform the traditional binary garbled circuit approach when the input numbers are large but the difference between these numbers can take very limited number of values. We have shown earlier in Section III-A that, thanks to the "bounded increments" property, this is indeed the case in edit-distance type of applications.

[1]For every specific computational task, this assumption can be guaranteed to hold by setting p to be a sufficiently large prime so that no intermediate values in the computation could overflow. For example, fixing p to the largest 88-bit prime suffices for edit-distance-based human genome comparisons. We also note that, without incurring significant overhead, it is possible to use a 128-bit prime p with the extension technique discussed in Section IV.

Table-lookup. A one-dimensional table of n entries can be viewed as an association-list

$$\{(0, v_0), (1, v_1), \ldots, (n-1, v_{n-1})\},$$

where v_is are bounded integer values. A table-lookup gadget can be treated as an unary gate with input-wire I and output-wire V. Given a wire-label w_i^I that encodes plaintext index i, a secure table look-up will output a wire-label $w_{v_i}^V$ that actually encodes v_i—the value in the table corresponding to the index i. In our scheme, this can be straightforwardly realized as follows:

1) the garbler generates fresh wire-labels $w_{v_0}^V, \ldots, w_{v_{n-1}}^V$ to encode $v_0, \ldots, v_{n-1}$ on the output-wire V; and sends the following n ciphertexts in a randomly permuted order:

$$\mathbf{Enc}_{w_i^I}(w_{v_i}^V, id),\ \forall i \in \{0, \ldots, n-1\}$$

   where w_i^I encodes i on the input index wire I.
2) the evaluator uses w_i^I as key to decrypt the above n ciphertexts. Due to the way the ciphertexts are constructed, precisely one of them will be successfully decrypted, revealing the wire-label $w_{v_i}^V$ that encodes v_i.

Moreover, looking up a multi-dimensional table with our scheme is readily reducible into a one-dimensional table lookup problem. Take the two-dimensional m-by-n-table lookup as an example. A two-dimensional table can always be mapped to a one-dimensional table by concatenating the rows, i.e., an index (i, j) (where $0 \le i < m, 0 \le j < n$) over the 2D-table can be translated into an index $k = i * m + j$ over a 1D-table of size mn. Since m is public, the affine mapping of wire-labels w_i^I and w_j^J (that encode the row and column indices) to the wire-label w_k^K (that encode the translated index) is almost free with our scheme. Once the translation is done, the secure 2D-table lookup reduces to sending and trial-decrypting mn ciphertexts—the same as the treatment to securely look up a 1D-table of mn entries.

Recall that with traditional binary circuit garbling schemes, a generic multiplexer-based secure table lookup is significantly more expensive because: 1) each index and each content integer in the table need to be encoded by multiple wire-labels; 2) n multiplexers would be needed to scan the table while the cost of each multiplexer depends on the bit length of the table content values as well as the length of the index. Alternatively, if the table is small, a secure table lookup can be realized as a giant garbled truth table like Huang et al. suggested [10]. However, it is unclear how this can be accomplished efficiently with AESNI support because $\log n$ keys (one key per bit of the index) are involved in producing every garbled row. A more straightforward solution would use SHA hashing, which, however, is orders-of-magnitude slower than AESNI instructions. In contrast, secure table lookup with our garbling scheme is significantly cheaper.

Handling the Initial Inputs. We assume the initial circuit inputs to our (arithmetic) circuit are in *bits* and the processing of these binary input values resembles that in binary garbled circuit protocols, i.e., the circuit generator's private input bits are encoded by wire-labels that are directly sent to the evaluator while the circuit evaluator's private input bits are translated to their corresponding wire-labels through oblivious transfer. Though we stress that the format of the wire-labels that encode the initial input bits conforms to the mod-p field notion of wire-labels required by our garbling scheme. Therefore, an (almost-free) set of addition and public-constant multiplication gadgets will translate the bits representation of input values into their arithmetic representations.

Efficient Implementation. Today's high-performance garbling schemes rely heavily on the premise that they can be built from an ideal block cipher instantiated by *fixed-key* AES. Fortunately, our scheme can be implemented using fixed-key AES accelerated by AESNI instructions. In constructing all the gadgets above, our garbling scheme requires only one cryptographic primitive, $\mathbf{Enc}_{w_{in}}(id, w_{out})$, where w_{in} and w_{out} are 128-bit wire-labels with valid zero-tags and $i < 2^{128}$ is an integer serving as a gadget counter. Inspired by the idea of Zahur et al. [13], we implement $\mathbf{Enc}_{w_{in}}(id, w_{out})$ as

$$\mathbf{Enc}_{w_{in}}(i, w_{out}) = \pi(K) \oplus K \oplus w_{out}$$

where $K = 2w_{in} \oplus i$ (note that $2w_{in}$ refers to doubling w_{in} in the finite field GF(2^{128})) and π is a random permutation that can be instantiated as a fixed-key AES. Thus, $\mathbf{Dec}_{w_{in}}(i, c)$ can be naturally defined as

$$\mathbf{Dec}_{w_{in}}(i, c) = \begin{cases} m := \pi(K) \oplus K \oplus c, & m \text{ has an all-zero tag;} \\ \perp, & \text{otherwise.} \end{cases}$$

where K is as was defined above.

Remarks on Security. Recall that we consider only semi-honest attackers who always follow the protocol. So the security provided by this 40-bit zero tag is statistical — it fails when two or more rows in the same gate happen to decrypt to wire-labels with valid zero tags so the evaluator would be confused. Thus, one may worry that the failure probability of the whole circuit with a total of n garbled rows in non-free gates would be $2^{-40} \cdot n$. We stress, however, this is not the case. This is because for every internal gate, the evaluator can always try all apparently-valid wire-labels in a subsequent gate to eliminate the confusion as this confusion will propagate with at most $2^{-40}n_1 \cdot 2^{-40}n_2$ probability where n_1, n_2 are the number of garbled rows in the two connected garbled gates, respectively. Since the number of rows in every garbled gate is always very limited in all applications discussed in this paper, we have $2^{-40}n_1 \cdot 2^{-40}n_2 \ll 2^{-40}$. Thus, the overall failure rate will only depend on the final layer of an application circuits, that is, $2^{-40} \cdot n_{out}$ where n_{out} is the number of garbled rows in non-free gates of the final layer. Note that n_{out} are highly limited (e.g., $4 \le n_{out} \le 20$) in all the applications considered in this paper as only a distance/score will be output. In Section IV-B, we give a technique to extend both the computational and statistical security to 128-bit, which makes the concern even more remote.

The Main Theorem. We formalize our garbling scheme in Figure 2. Note that equality, minimum and table-lookup

gadgets are all essentially realized in terms of a primitive operation we call *projection*. Secure projection can obliviously *project* an input signal a_i to its corresponding output signal b_i based on a publicly known map $\{(a_1, b_1), \ldots, (a_n, b_n)\}$. Thus, to prove the garbling scheme to be secure, it suffices to just consider addition and projection.

Next, we state and prove our main theorem.

Theorem 1: If π is an ideal block cipher that is used to realize $\mathbf{Enc}$ and $\mathbf{Dec}$ as described above. the scheme in Figure 2 satisfies the privacy, obliviousness, and authenticity definitions outlined in Section II-A.

Proof ***Privacy:*** Figure 3 describes a simulator $\mathsf{Sim}_{\mathsf{prv}}$ that can be used to show our garbling scheme is private. The construction of $\mathsf{Sim}_{\mathsf{prv}}$ is similar to Gb except for three changes that we highlighted in red: (1) $\mathsf{Sim}_{\mathsf{prv}}$ has a third input $f(x)$; (2) it uses $f(x)_i$ to replace t when producing the decoding information d^{O_i}; and (3) it calls En with an arbitrary legitimate input x_0 to produce X in the end.

For any x, consider (F, X, d) generated by

$$(F, e, d) \leftarrow \mathsf{Gb}(1^k, f)$$
$$X \coloneqq \mathsf{En}(e, x)$$

and the tuple (F', X', d') produced by $\mathsf{Sim}_{\mathsf{prv}}(1^k, f, f(x))$. Should $\mathsf{Sim}_{\mathsf{prv}}$ know x, then it would not replace t with $f(x)_i$ in producing $d_t^{O_i}$ but simply call $\mathsf{En}(\hat{e}, x)$ in the end to generate X'. Let the output distribution generated by this $\mathsf{Sim}^x_{\mathsf{prv}}$ be (F'', X'', d''). It is easy to see that (F, X, d) and (F'', X'', d'') are identically distributed. Now, to see $(F'', X'', d'') \approx (F', X', d')$, we note that

(1) the distinguisher cannot tell the two distributions apart by examining any garbled gates because x_0 is a legitimate input and $\mathsf{Sim}_{\mathsf{prv}}$ followed exactly the same procedure to handle all garbled gates as Gb does.

(2) For every output-wire O_i, for every $w_t^{O_i}$ the distinguisher does not learn, $d_t^{O_i}$ is no different from a random string (because π is an ideal cipher); from the $w_t^{O_i}$ learned by the distinguisher, the distinguisher can only get $f(x)_i$ from decrypting $d_t^{O_i}$, which is no different from what it would learn from examining (F, X, d).

Obliviousness: We simply observe that in $\mathsf{Sim}_{\mathsf{prv}}$, $f(x)$ is used only to compute d, which is dropped in the security definition of obliviousness. Thus, the simulator $\mathsf{Sim}_{\mathsf{obl}}$ can be derived from $\mathsf{Sim}^x_{\mathsf{prv}}$ by simply drop $f(x)$ in the input and the third component d in the output.

Authenticity: We note that due to the construction of $\mathbf{Enc}$, if the adversary $\mathcal{A}$ can provide any Y' such that $Y' \neq \mathsf{De}(d, \mathsf{Ev}(F, X)))$ and $\mathsf{De}(d, Y') = k \neq \perp$ (where $(F, e, d) \leftarrow \mathsf{Gb}(1^k, f), X \coloneqq \mathsf{En}(e, x)$), then $\mathcal{A}$ must know the $w_k = w_0 +_p k \times_p \Delta$ which is the output wire-label corresponding to k. However, without knowing w_0 and Δ, for any particular k, $\mathcal{A}$ can only guess $w_k = w_0 +_p k \times_p \Delta$ correctly with probability at most $1/p$. Hence, if we know from the public circuit that an output-wire can have at most n possible different k values, the adversary can only succeed in guessing a valid output wire-label with probability at most n/p. Thus, our scheme guarantees n/p-authenticity. ■

IV. Extensions

In this section, we discuss two extensions of our approach: 1) to support garbling of arbitrary computations; and 2) to accommodate the need for higher computational security guarantee.

A. Garbling Arbitrary Computations

Our garbling scheme as is described so far seems not efficient for generic computations because we haven't discussed how to *multiply* two secret values efficiently. Below we discuss how to extend our garbling scheme to realize generic secure computation. The basic idea is to tether our garbling scheme with the traditional binary circuit garbling, which is known to be generic and can leverage Half-Gate technique [13].

Arithemtic Wire-labels to Binary Wire-labels. Suppose the circuit garbler knows $w_0 = 0^{40} \| k_0$ and Δ, whereas the evaluator knows $w_a = 0^{40} \| (k_0 +_p a \times_p \Delta)$. Let the binary form of the integer a be $a_1 a_2, \ldots, a_n$. After conversion, we hope the the garbler learns wire-labels $w_{1,0}, \ldots, w_{n,0}$ and δ while the evaluator learns $w_{1,a_1}, \ldots, w_{n,a_n}$ such that $w_{i,a_i} = w_{i,0} \oplus a_i \delta$. We describe two methods to accomplish this goal that exhibit complementary tradeoffs between performance and generality.

1) **Via secret shares.** If the range of a is publicly known to be restricted to $\{0, \ldots, \zeta\}$. The basic idea is to let the garbler send a random permutation of

$$\mathbf{Enc}_{w_i}(i \oplus m), \ \forall i \in \{0, \ldots, \zeta\}$$

where m is a $\lceil \log \zeta \rceil$-bit secret mask picked by P_1. Thus, the evaluator who has w_a is able to recover $a \oplus m$. Then, the two parties can use traditional garbled circuit protocols [13] to run any followup computation over a by starting from their respective shares m and $a \oplus m$.

Per converting an arithmetic wire, it will cost $\zeta + 1$ encryptions to send the encrypted masked-shares, 176 encryptions to translate the garbler's input bits and 88 oblivious transfers (for the evaluator's 88-bit input) in the second stage of the secure computation. This approach would be preferred when ζ is known to be relatively small. As ζ grows too big, it becomes infeasible to transmit $O(\zeta)$ encryptions, in which case we can opt to the following alternative conversion method suitable for large ζs.

2) **Via generic secure modulo-arithmetic.** The basic idea is to construct a binary garbled circuit to securely compute $(k_a - k_0)/\Delta$ where "$-$" and "$/$" are mod-p subtraction and division, respectively. By requiring the garbler to locally compute $(\Delta^{-1} \bmod p)$, we can reduce the above computation into a secure mod-p subtraction followed by a secure mod-p multiplication, both realized by a traditional binary circuit garbling scheme.

Because $k_0, k_a, p \in \{0,1\}^{88}$, the cost of this approach is that of a traditional garbled circuit secure computation

$\mathsf{Gb}(1^k, f)$

$\Delta \leftarrow \{0,1\}^k$
for $I \in f.\mathsf{input\text{-}wires}$ **do**
  $w_0^I \leftarrow \mathbb{Z}_p$
$\hat{e} \coloneqq (w_0^1, \ldots, w_0^{|f.\mathsf{input\text{-}wires}|}, \Delta)$
for $g \in f.\mathsf{gates}$ **do**
  if g is Add-gate **then**
    $\{I_1, I_2\} \coloneqq g.\mathsf{input\text{-}wires}$
    $O \coloneqq g.\mathsf{output}$
    $w_0^O \coloneqq w_0^{I_1} +_p w_0^{I_2}$
  else if g is Proj_ϕ-gate **then**
    $I \coloneqq g.\mathsf{input\text{-}wire}$
    $O \coloneqq g.\mathsf{output\text{-}wire}$
    $\mathbb{Z}_\zeta \coloneqq g.\mathsf{domain}$
    for $t \in \mathbb{Z}_\zeta$ **do**
      $w_t^I \coloneqq w_0^I +_p t \times_p \Delta$
      $w_t^O \coloneqq w_0^O +_p \phi(t) \times_p \Delta$
      $c_t^g \leftarrow \mathbf{Enc}_{w_t^I}(g, w_t^O)$
    $\boldsymbol{c}^g \coloneqq \{c_0^g, \ldots, c_{\zeta-1}^g\}$
$\hat{F} \coloneqq (\boldsymbol{c}^1, \ldots, \boldsymbol{c}^{|f.\mathsf{Proj}|})$
for $O_i \in f.\mathsf{output\text{-}wires}$ **do**
  $\mathbb{Z}_\zeta \leftarrow i.\mathsf{domain}$
  for $t \in \mathbb{Z}_\zeta$ **do**
    $w_t^{O_i} \coloneqq w_0^{O_i} +_p t \times_p \Delta$
    $d_t^{O_i} \leftarrow \mathbf{Enc}_{w_t^{O_i}}(\mathsf{out} \| t)$
  $\boldsymbol{d}^i \coloneqq \{d_0^O, \ldots, d_{\zeta-1}^O\}$
$\hat{d} \coloneqq (\boldsymbol{d}^1, \ldots, \boldsymbol{d}^{|f.\mathsf{output\text{-}wires}|})$
return $(\hat{F}, \hat{e}, \hat{d})$

$\mathsf{En}(\hat{e}, \hat{x})$

$(w_0^1, \ldots, w_0^{|f.\mathsf{input\text{-}wires}|}, \Delta) \coloneqq \hat{e}$
for $x_i \in \hat{x}$ **do**
  $X_i \coloneqq w_0^i +_p x_i \times_p \Delta$
return $\hat{X} \coloneqq (X_1, \ldots, X_{|f.\mathsf{input\text{-}wires}|})$

$\mathsf{Ev}\ (\hat{F}, \hat{X})$

$(X_1, \ldots, X_{|f.\mathsf{input\text{-}wires}|}) \coloneqq \hat{X}$
$(\boldsymbol{c}^1, \ldots, \boldsymbol{c}^{|f.\mathsf{Proj}|}) \coloneqq \hat{F}$
for $I_i \in f.\mathsf{input\text{-}wires}$ **do**
  $w^{I_i} \coloneqq X_i$
for $g \in f.\mathsf{gates}$ **do**
  if g is Add-gate **then**
    $\{I_1, I_2\} \coloneqq g.\mathsf{input\text{-}wires}$
    $O \coloneqq g.\mathsf{output\text{-}wire}$
    $w^O \coloneqq w^{I_1} +_p w^{I_2}$
  else if g is Proj_ϕ-gate **then**
    $I \coloneqq g.\mathsf{input\text{-}wire}$
    $O \coloneqq g.\mathsf{output\text{-}wire}$
    $\mathbb{Z}_\zeta \coloneqq g.\mathsf{domain}$
    $\{c_0, \ldots, c_{\zeta-1}\} \coloneqq \boldsymbol{c}^g$
    for $t \in \mathbb{Z}_\zeta$ **do**
      if $\mathbf{Dec}_w(g, c_t) \neq \perp$ **then**
        $w^O \coloneqq \mathbf{Dec}_w(g, c_t^g)$
for $O_i \in f.\mathsf{output\text{-}wires}$ **do**
  $Y_i \coloneqq w^{O_i}$
$\hat{Y} \coloneqq (Y_1, \ldots, Y_{|f.\mathsf{output\text{-}wires}|})$
return $\hat{Y}$

$\mathsf{De}(\hat{d}, \hat{Y})$

$(Y_1, \ldots, Y_{|f.\mathsf{output\text{-}wires}|}) \coloneqq \hat{Y}$
for $d_i \in \hat{d}$ **do**
  $\mathbb{Z}_\zeta \coloneqq i.\mathsf{domain}$
  $\{d_0, \ldots, d_{\zeta-1}\} \coloneqq \boldsymbol{d}^i$
  for $t \in \mathbb{Z}_\zeta$ **do**
    if $\mathbf{Dec}_{Y_i}(d_k) = \mathsf{out} \| k$ **then**
      $y_i \coloneqq k$
return $\hat{y} \coloneqq (y_1, \ldots, y_{|f.\mathsf{output\text{-}wires}|})$

Fig. 2: The garbling scheme

protocol with 88×3 input bits (88×2 bits from the garbler and 88 bit from the evaluator), an 88-bit mod-p secure subtraction, and an 88-bit mod-p secure multiplication. Since it only depends on the computational security parameter rather than the range of the plaintext values, it fits better when the range of a can be very big (e.g., more than 2^{17}).

With either approach, we stress that the authenticity of the final output-wire labels holds if $a \ll p$, because without knowing w_0 and Δ, for any $a, b \in \mathbb{Z}_p$,

$$(w_0 +_p a \times_p \Delta, w_0 +_p b \times_p \Delta) \approx (X, Y)$$

where X, Y are uniform random samples from $0^{40} \| \mathbb{Z}_p$. So for example, when it is known that $a \leq 2^{32}$ from the application context, our approach can offer at least $87 - 32 = 55$ bits authenticity.

Binary Circuit Wire-labels to Arithmetic Wire-labels. Converting wire-labels from traditional binary circuit garbling to arithmetic wire-labels used in ours is more straightforward:

$\mathsf{Sim}_{\mathsf{prv}}(1^k, f, f(x))$

$\Delta \leftarrow \{0,1\}^k$

for $I \in f.\mathsf{input\text{-}wires}$ **do**

$w_0^I \leftarrow \mathbb{Z}_p$

$\hat{e} := (w_0^1, \ldots, w_0^{|f.\mathsf{input\text{-}wires}|}, \Delta)$

for $g \in f.\mathsf{gates}$ **do**

if g is Add-gate **then**

$\{I_1, I_2\} := g.\mathsf{input\text{-}wires}$

$O := g.\mathsf{output}$

$w_0^O := w_0^{I_1} +_p w_0^{I_2}$

else if g is Proj_ϕ-gate **then**

$I := g.\mathsf{input\text{-}wire}$

$O := g.\mathsf{output\text{-}wire}$

$\mathbb{Z}_\zeta := g.\mathsf{domain}$

for $t \in \mathbb{Z}_\zeta$ **do**

$w_t^I := w_0^I +_p t \times_p \Delta$

$w_t^O := w_0^O +_p \phi(t) \times_p \Delta$

$c_t^g \leftarrow \mathbf{Enc}_{w_t^I}(g, w_t^O)$

$\boldsymbol{c}^g := \{c_1^g, \ldots, c_{\zeta-1}^g\}$

$F' := (\boldsymbol{c}^1, \ldots, \boldsymbol{c}^{|f.\mathsf{Proj}|})$

for $O_i \in f.\mathsf{output\text{-}wires}$ **do**

$\mathbb{Z}_\zeta \leftarrow i.\mathsf{domain}$

for $t \in \mathbb{Z}_\zeta$ **do**

$w_t^{O_i} := w_0^{O_i} +_p t \times_p \Delta$

$d_t^{O_i} \leftarrow \mathbf{Enc}_{w_t^{O_i}}(\mathsf{out} \| f(x)_i)$ {$f(x)_i$ denotes the value of $f(x)$ on the i^{th} output-wire.}

$\boldsymbol{d}^i := \{d_0^O, \ldots, d_{\zeta-1}^O\}$

$d' := (\boldsymbol{d}^1, \ldots, \boldsymbol{d}^{|f.\mathsf{output\text{-}wires}|})$

$X' := \mathsf{En}(\hat{e}, x_0)$ {$x_0 \in f.\mathsf{domain}$ denotes a legitimate plaintext input.}

return (F', X', d')

Fig. 3: The Simulator for Proving Privacy

the garbler only needs to send a randomly permuted pair of ciphertext

$$\left[\mathbf{Enc}_{w_0'}(w_0),\ \mathbf{Enc}_{w_1'}(w_1)\right]$$

per wire in the binary circuit, where w_0', w_1' are wire-labels conforming to the format required by the traditional garbling (e.g., $\forall b \in \{0,1\}, w_b' = w_0' \oplus b\Delta, \Delta \in \{0,1\}^{128}$), and w_0, w_1 are freshly sampled labels based on our garbling scheme (e.g., $\forall b \in \{0,1\}, w_b = 0^{40} \| k_b, k_b = k_0 +_p b \times_p \Delta, \Delta \in \mathbb{Z}_p$). So the evaluator can decrypt the ciphertext corresponding to the binary circuit wire-labels it learns from the evaluation.

To derive an arithmetic wire-label w_a that encodes

$$a = a_0 + a_1 \times 2 + \cdots + a_n \times 2^{n-1}, \quad a_i \in \{0,1\},$$

from binary wire-labels $w_{a_0}', \ldots, w_{a_n}'$, it suffices to first convert $w_{a_0}', \ldots, w_{a_n}'$ to $w_{a_0}, \ldots, w_{a_n}$ that conform to wire-labels in our scheme, then w_a can be locally derived from $w_{a_0}, \ldots, w_{a_n}$ using the (almost) free addition and constant multiplication.

B. Stronger Computational Security

The scheme as we described thus far can guarantee 87 bits computational security. Next, we show that it is possible to modify our scheme to provide 127 bits computational security.

The key idea is to set p to be a 128-bit prime (in doing so, we abandon the idea of using 40-bit all-zero tags to identify successful decryptions) and retrofit each garbled row $\mathbf{Enc}_{w_{in}}(i, w_{out})$ with an additional 128-bit authentication tag. That is, we redefine

$$\mathbf{Enc}_{w_{in}}(i, w_{out}) = (C_1, C_2)$$

where

$$\begin{aligned} C_1 &= \pi(K) \oplus K \oplus w_{out} \\ C_2 &= \pi(K \oplus 1) \oplus K \oplus w_{out} \\ K &= 2w_{in} \oplus i \end{aligned}$$

where $2w_{in}$ refers to doubling w_{in} in the finite field GF(2^{128}) and π can be instantiated as a fixed-key AES.

Symmetrically, we can define

$$\mathbf{Dec}_{w_{in}}(i, (C_1, C_2)) = \begin{cases} m_1, & m_1 = m_2 \\ \perp, & \text{otherwise} \end{cases}$$

where

$$\begin{aligned} m_1 &= \pi(K) \oplus K \oplus C_1 \\ m_2 &= \pi(K \oplus 1) \oplus K \oplus C_2, \end{aligned}$$

and K is as was defined above. Thus, the evaluator, who obtains w_{out}' by trial decrypting garbled rows in the i-th gate with wire-label w_{in}', can verify whether

$$\pi(2w_{in}' \oplus i \oplus 1) \oplus 2w_{in}' \oplus i \oplus w_{out}' = C_2$$

to tell if the decryption was successful. The intuitive reason behind this is that if w_{in}' is not equal to w_{in} (the key used to generate (C_1, C_2), then $w_{out}' \neq w_{out}$ and

$$\pi(2w_{in}' \oplus i \oplus 1) \oplus 2w_{in}' \oplus i \oplus w_{out}' \neq \pi(2w_{in} \oplus i \oplus 1) 2w_{in} \oplus i \oplus w_{out},$$

except for a negligible probability.

V. Results

We have implemented our garbling scheme and applied it to several applications whose problem structures resemble that of edit distance. In this section, we evaluate our approach by implementing a number of private string-metrics mentioned in the introduction. We also provide micro-benchmarks that helps to explain the high performance of the above applications.

Experiment Setup. Our experiments are conducted on two Amazon EC2 free-tier `t2.micro` instances [27] running Ubuntu 14.04. The instances are connected over a 1 GB LAN with $0.031ms$ latency.

We implemented our scheme in C/C++, using Intel AESNI intrinsic instructions to realize the fixed block cipher π. We use the very recent C/C++ library emp-tool [14]'s realization of the Half-Gates [13] garbling scheme and efficient OT extension [28], [29] to construct the baseline protocols to compare with. For fair comparison, all baseline protocols are carefully designed with Boolean circuits optimized to take advantage of free-XOR [11] benefits.

A. Application Performance

We applied the proposed garbling scheme to implementing five applications: edit distance, weighted edit distance, Needleman-Wunsch, longest common subsequence (LCS), and heaviest common subsequence. Figure 4 and Figure 5 delineate the time and bandwidth costs of these end-to-end applications over input strings of lengths 800–4000 characters. The curves all show a quadratic shape, which is consistent with the theoretical complexity of the underlying dynamic programming algorithms. We used $c_{ins} = c_{del} = 5$ and $c_{sub} = 1$ in the weighted edit distance experiments and the weight tables of Figure 1 in the Needleman-Wunsch experiments.

We show performance numbers for realizing 87-bit and 127-bit computational security using both the proposed approach and the baseline approach using Half-Gates. With the baseline approach, the computational overheads are the same for both 87-bit and 127-bit security as they involve exact the same amount of computation. With our approach, however, we observed 22% (for Needleman-Wunsch) to 50% (for basic Edit Distance) slowdowns and a uniform 100% increase in bandwidth for these applications. The variation of the slowdown in time is due to the fact that the proportion of CPU time spent on sampling wire-labels (which also requires AESNI calls) varies from applications to applications, but increasing the computational security parameter from 87 to 127 does not affect the amount of time spent on generating fresh wire-labels.

Comparison with [16] and [17]. One may wonder how our protocols compare with the estimation algorithms of [16] and [17]. However, in addition to the dramatic differences in the security model, their approaches do not work for anything beyond the basic edit distance metric. Further, even with respect to the basic edit distance, we note that all our experiments above were conducted over randomly sampled strings while [16] and [17] work only on low-entropy human genomes. For example, although the "goodness"[2] of the public reference string affects not only the accuracy but also the performance of their approach, neither [16] nor [17] describes how to pick good reference strings without leaking information about the secret inputs. We have done experiments (over 2000 pairs of sample strings, each of length 3500 characters) using randomly generated reference strings, and observe a root-mean-square relative-error (RMSRE)[3] of 75% and 59% with [16]'s and [17]'s approach, respectively. This shows the accuracy degradation can be unacceptable for many applications. Thus, without knowing how to pick good reference strings, it is difficult to draw meaningful comparisons between these works and ours.

B. Micro-benchmarks

We measured the performance of several basic operations under our garbling scheme. All experiments in this subsection are conducted with respect to 87-bit computational security.

Secure Addition. Table II shows the performance of secure addition in our approach. Recall that addition is (almost) free, our scheme is able to perform one addition every 2.8 nano-seconds, regardless of the bit-length of the numbers to add. This result is in line with the cost of computing a mod-p addition on this hardware. In contrast, costs of binary circuit based addition circuits (powered by Half-Gates) increase roughly linearly with the width of the adder. Ours are 500–40,000 times faster and consume no bandwidth.

Secure Table-Lookup. This is also the essential enabling primitive for secure comparison and bounded range minimum computations. Figure 6 shows the efficiency of secure table lookup with our scheme and compares it to the best existing garbled-circuit-based implementation. Two relevant parameters are used to describe the table: the table size (i.e., the number of entries in the table) and the bit-length of each entry. With our scheme, the cost of secure table-lookup grows linearly with the number of entries in the table, but not the bit-length of the entries.

In contrast, a garbled-circuit-based table-lookup costs more when the values in the table grow bigger, because the secure multiplexers has to take wider inputs. In our experiments, we assumed the table contains either 4-, 8-, or 12-bit values, representing the value range of constant tables used in many practical applications. On these table parameters, our approach is 3.6–20 times faster and 6–23 times more bandwidth-efficient.

Wire-label Conversions. Converting Boolean wire-labels from the Half-Gates binary circuit garbling scheme into arithmetic wire-labels in our scheme is highly efficient, at about $420ns$ (and $\sim$32 bytes bandwidth) per bit of Boolean wire-label, since it involves only two garbled rows per Boolean wire (Table III).

Converting arithmetic wire-labels into Boolean ones used in Half-Gates is comparatively more expensive. The generic method needs 9.6 millisecond and 2MB per arithmetic wire-label, mostly spent on oblivious mod-p multiplication under the Half-Gates garbling scheme. However, if the arithmetic wire-label is known to denote values of a smaller range

[2]We note that there were no discussion of how "goodness" should be formally defined given in [16] and [17].

[3]Let $v = \{v_1, v_2, ...\}$ be a family of approximations of a fixed value u. A common metric used to evaluate the quality of the estimation, known as the root-mean-square relative-error of the family of approximation is defined as $\sqrt{\frac{1}{n}\sum_{i=1}^{n}[(v_i - u)/u]^2}$. Obliviously, approximation with RMSRE of 50% or more will not be useful in most real-world string-comparison applications.

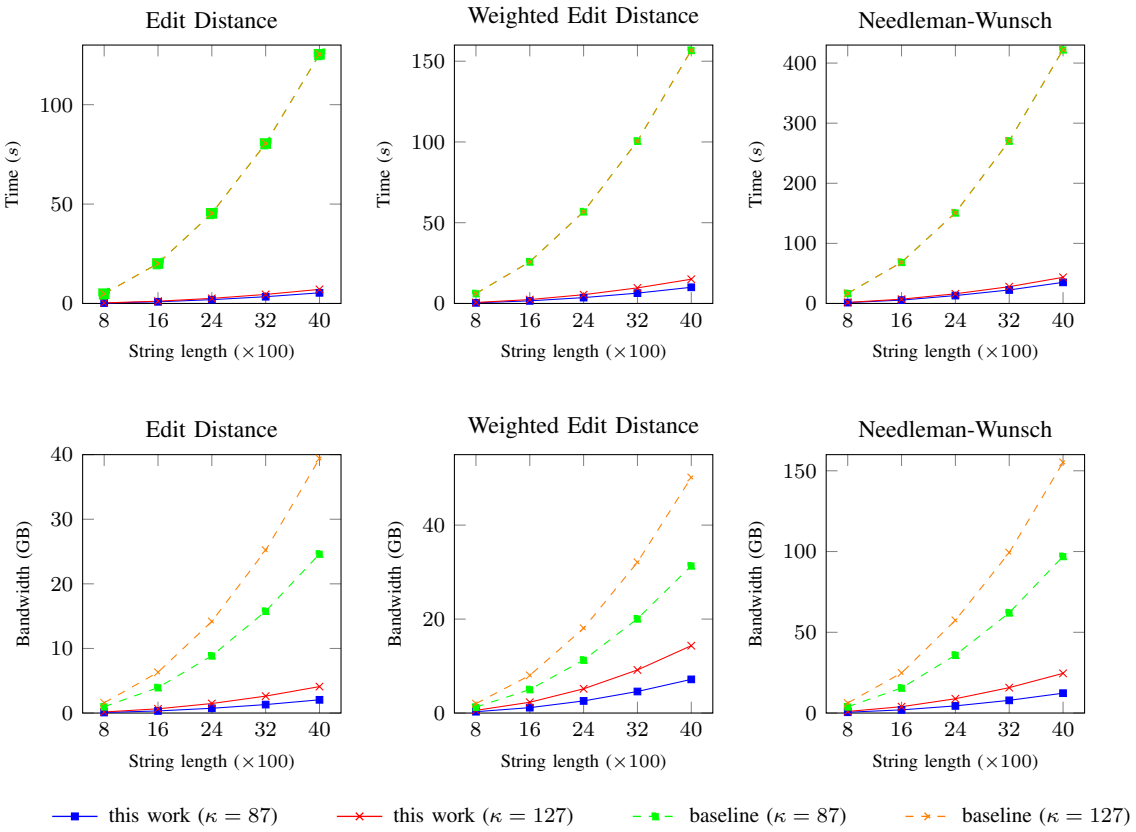


Fig. 4: Cost of edit distance and its variants. (Timings are averaged over 10 iterations.)

TABLE II: Costs of secure additions

	Time (ns)				Bandwidth (byte)			
	8-bit	16-bit	32-bit	64-bit	8-bit	16-bit	32-bit	64-bit
Half-Gates [13]	1420	2770	5520	11100	154	330	682	1386
This Work	**2.8**				**0**			

Above timings are in line with the cost of AESNI-based garbling ($\sim 45ns$ per garbled row) and the costs of modulo arithmetic with respect to an 88-bit prime ($2.8ns$ per modulo addition). Timings of Half-Gates are measured by averaging 10^6 iterations while those of this work are taken over 10^9 iterations.

(usually $< 2^{20}$ possibilities), the faster secret-sharing based label conversion method turns out very efficient. For example, if the range of the arithmetic signal is up to 2^8, the conversion an arithmetic wire-label takes only less than $11ns$ and 4.2KB bandwidth. We empirically find that the secret-sharing based conversion can outperform the generic method when the plaintext value is within 2^{16}.

VI. Other Related Works

Secure Garbling. Ball, Malkin and Rosulek [22] have recently proposed a seminal secure garbling scheme that could circumvent the lower bound provided by Zahur et al [13]. Their work bears some similarities with our work, e.g., both works address the semi-honest threat model and support "free" additions and constant multiplications. Nevertheless, the two works differ significantly in their goals, techniques and concrete results that have been achieved.

First, their work aimed to improve the bandwidth efficiency for certain types of computations, i.e., high fan-in thresholding and mod-m ring arithmetic operations. Instead, our scheme is designed with both time and bandwidth efficiency in mind while targeted directly at realistic security challenges in secure genome and string comparisons.

Technically speaking, their work extended the traditional binary circuit garbling scheme by generalizing the wire-labels from a single $GF(2^{127})$ element to a 128-bit vector divided into entries of small prime field (i.e., $\mathbb{Z}_p$ where almost always $p \leq 103$) elements. To encode a plaintext value (e.g., a 32-bit integer), their scheme decomposes the plaintext value into its CRT-representation (Chinese Remainder Theorem) and

TABLE III: Costs of label conversions.

	Time (μs)				Bandwidth (KB)			
	8-bit	16-bit	32-bit	64-bit	8-bit	16-bit	32-bit	64-bit
Boolean to Arithmetic	3.34	6.69	13.31	26.75	0.26	0.51	1.02	2.05
Arithmetic to Boolean (via secret-shares)	10.89	743.2	———		4.22	1048.83	———	
Arithmetic to Boolean (via generic secure modulo-arithmetic)	9628				2004.96			

Timings in the first two rows are averaged over 10^6 iterations while those in the third row are over 10^3 iterations.

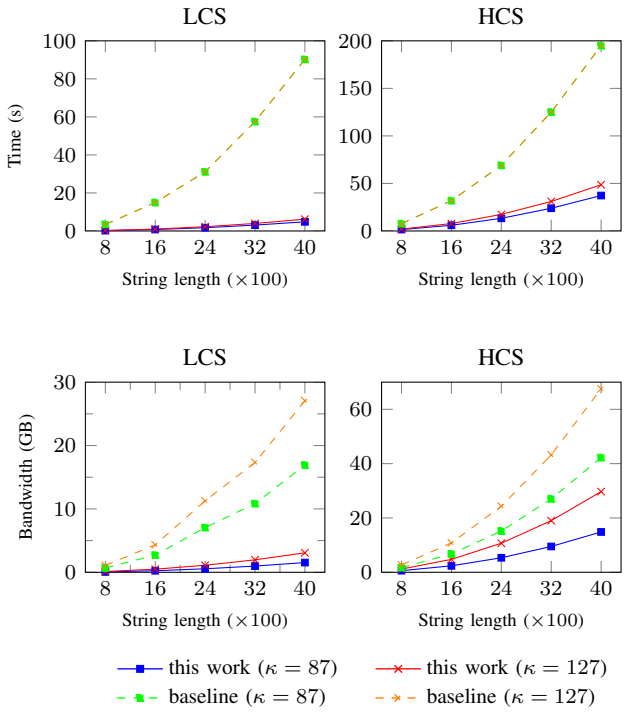


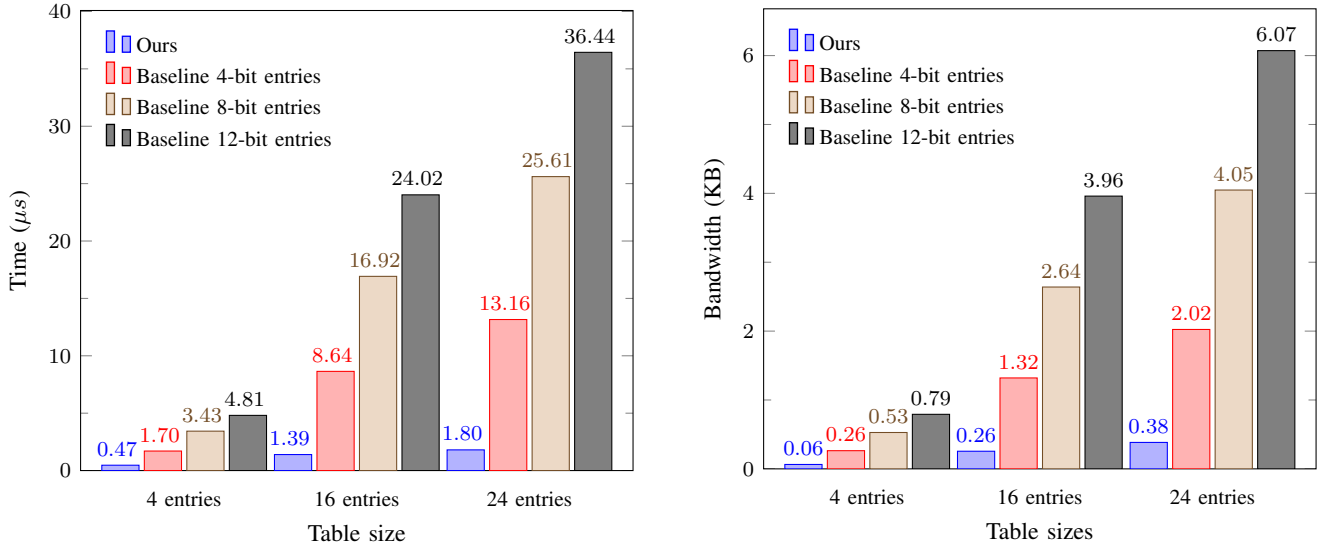
Fig. 5: Cost of LCS and HCS. (Timings are averaged over 10 iterations.)

requires a bundle of wire-labels each of which encodes one component of the CRT-representation. Hence, secure addition and constant multiplication in their scheme typically involve processing a bundle of wires and each wire-handling requires dozens of modulo additions/multiplications over small primes. In contrast, our approach is significantly simpler: we generalize the format of traditional binary field wire-labels directly to one over an extremely large prime field so that overflow will never be an issue when operating values in the realistic problem context. As a result, it requires only a single modulo operation to accomplish a secure addition or constant multiplication.

Moreover, their garbling scheme relies on the generalized point-and-permute technique for the evaluator to identify the "right" ciphertext to decrypt, whereas ours uses authentication tags to identify successful decryptions from failed trials. Although their approach saves the evaluator some CPU cycles, we stress that their point-and-permute trick is incompatible with the bounded-difference minimum gadgets that we introduce, since an adversary can infer the secret permutation by simply observing how the gap entries (i.e., the entries that do not correspond to a possible plaintext signal) move before and after the permutation, unless all gap entries are stuffed with dummy garbled rows (which will be unnecessarily more expensive).

Finally, while their garbling scheme is mostly of theoretical interest, without practical implementation, it is unclear whether it will actually bring performance benefit. In fact, garbling an entry with fixed-key AESNI instructions nowadays has become so cheap (roughly 10–50 ns) that even the so-called "free"-XOR gates (a few nano-seconds per gate) in traditional garbling schemes [13] are not really free. Computing dozens of mod-p additions/multiplications per "free" addition (or constant multiplication) is even much slower. The "free" additions and constant multiplications in their scheme is significantly more expensive than other non-free gadgets that requires just a few AESNI calls. In comparison, we experimentally verified our approach with a class of useful applications that our scheme is indeed more than an order-of-magnitude more efficient in both time and bandwidth than the state-of-the-art garbling schemes.

Private Edit Distance. Wang et al. [16] and Asharov et al. [17] proposed highly efficient secure estimation protocols for the basic edit distance metric. Both works were targeted at low-entropy human genome strings and relied on a public reference genome to reduce the entropy in the private input genome strings. But these works used very different heuristics: Wang et al. [16] transformed the low-entropy edit distance problem into a set-difference size problem while Asharov et al. [17] (over)-approximated edit distance by first breaking the input strings into segments and summing up edit distances of corresponding segments. These works can't be proved secure with respect to the standard ideal definition of edit distance functionality [19], both because they are more leaky than necessary and they lack the kind of *composability* offered by generic protocols like garbled circuits.



(a) Time

(b) Bandwidth

Fig. 6: Costs of secure table lookup. (Timings are measured by averaging 10^6 iterations.)

In comparison, our approach may not outperform theirs when computing the edit-distance between certain low-entropy input strings. However, our protocols are provably secure with respect to the standard definition of private edit distance. More specifically, our approach offers a number of advantages:

1) it works for arbitrary input strings;
2) it always outputs accurate results;
3) it is generically composable with surrounding secure computations to realize more powerful applications;
4) it applies to securely compute a broad class of string metrics like Needleman-Wunsch, LCS and HCS.
5) it doesn't require any public reference string to work.

To appreciate the value of our approach not requiring a good public reference string, we stress that prior works did not actually describe how to pick "good" reference strings to guarantee the expected accuracy and performance, and can't give useful estimations if random reference strings are used (see experiments in the end of Section V-A).

VII. Conclusion

Customizing a secure garbling scheme to leverage publicly exploitable traits of target computations can lead to secure computation protocols that are dramatically more efficient than traditional Boolean circuit based protocols. We have taken a first step to explore this methodology in the realm of constructing semi-honest protocols for obliviously computing several widely-used dynamic-programming-based string metrics including generalized edit-distance and weighted LCS. Our resulting protocols are an order-of-magnitude more efficient than their comparable state-of-the-art alternatives. We hope our findings shed some light on designing efficient application-specific secure computation protocols in the future.

Appendix

Let $\mathtt{s}, \mathtt{t}, D_{i,j}, c_{ins}, c_{del}, c_{sub}$ be defined as in Section II-B, where c_{ins}, c_{del} are generalized to one-dimensional tables and c_{sub} is generalized to a two-dimensional table. Let

$$m_{i,j} = \min\left(D_{i,j-1} + c_{del}\left[\mathtt{t}[j]\right],\ D_{i-1,j-1} + c_{sub}\left[\mathtt{s}[i], \mathtt{t}[j]\right]\right)$$
$$u_{i,j} = \left(D_{i,j-1} + c_{del}\left[\mathtt{t}[j]\right]\right) - \left(D_{i-1,j-1} + c_{sub}\left[\mathtt{s}[i], \mathtt{t}[j]\right]\right)$$
$$v_{i,j} = \left(D_{i-1,j} + c_{ins}\left[\mathtt{s}[i]\right]\right) - m_{i,j}$$

Then, there exist $D_{i,j}$-independent public constants C_1, C_2, C_3, C_4 such that for all valid indices i, j.

$$C_1 \le u_{i,j} \le C_2, \qquad C_3 \le v_{i,j} \le C_4.$$

Proof Because $|D_{i,j-1} - D_{i-1,j-1}| \le c_{ins}\left[\mathtt{s}[i]\right]$, therefore

$$D_{i-1,j-1} - c_{ins}\left[\mathtt{s}[i]\right] \le D_{i,j-1} \le D_{i-1,j-1} + c_{ins}\left[\mathtt{s}[i]\right]$$

so,

$$D_{i,j-1} + c_{del}\left[\mathtt{t}[j]\right] \ge D_{i-1,j-1} - c_{ins}\left[\mathtt{s}[i]\right] + c_{del}[\mathtt{t}[j]]$$
$$D_{i,j-1} + c_{del}\left[\mathtt{t}[j]\right] \le D_{i-1,j-1} + c_{ins}\left[\mathtt{s}[i]\right] + c_{del}\left[\mathtt{t}[j]\right]$$

hence,

$$\begin{aligned} u_{i,j} =& D_{i,j-1} + c_{del}\left[\mathtt{t}[j]\right] - \left(D_{i-1,j-1} + c_{sub}\left[\mathtt{s}[i], \mathtt{t}[j]\right]\right) \\ \ge& c_{del}\left[\mathtt{t}[j]\right] - c_{ins}\left[\mathtt{s}[i]\right] - c_{sub}\left[\mathtt{s}[i], \mathtt{t}[j]\right] \end{aligned} \tag{5}$$
$$\begin{aligned} u_{i,j} =& D_{i,j-1} + c_{del}\left[\mathtt{t}[j]\right] - \left(D_{i-1,j-1} + c_{sub}\left[\mathtt{s}[i], \mathtt{t}[j]\right]\right) \\ \le& c_{ins}\left[\mathtt{s}[i]\right] + c_{del}\left[\mathtt{t}[j]\right] - c_{sub}\left[\mathtt{s}[i], \mathtt{t}[j]\right] \end{aligned} \tag{6}$$

So we can set

$$C_1 \coloneqq \min_{i,j}\left(c_{del}\left[\mathtt{t}[j]\right] - c_{ins}\left[\mathtt{s}[i]\right] - c_{sub}\left[\mathtt{s}[i], \mathtt{t}[j]\right]\right),$$
$$C_2 \coloneqq \max_{i,j}\left(c_{ins}\left[\mathtt{s}[i]\right] + c_{del}\left[\mathtt{t}[j]\right] - c_{sub}\left[\mathtt{s}[i], \mathtt{t}[j]\right]\right),$$

and we have $C_1 \le u_{i,j} \le C2$.

Symmetrically, we can derive that

$$\begin{aligned}&\big(D_{i-1,j}+c_{ins}\big[\mathtt{s}[i]\big]\big)-\big(D_{i-1,j-1}+c_{sub}\big[\mathtt{s}[i],\,\mathtt{t}[j]\big]\big)\\ \geq& c_{ins}\big[\mathtt{s}[i]\big]-c_{sub}\big[\mathtt{s}[i],\,\mathtt{t}[j]\big]-c_{del}\big[\mathtt{t}[j]\big] \qquad (7)\\ &\big(D_{i-1,j}+c_{ins}\big[\mathtt{s}[i]\big]\big)-\big(D_{i-1,j-1}+c_{sub}\big[\mathtt{s}[i],\,\mathtt{t}[j]\big]\big)\\ \leq& c_{ins}\big[\mathtt{s}[i]\big]+c_{del}\big[\mathtt{t}[j]\big]-c_{sub}\big[\mathtt{s}[i],\,\mathtt{t}[j]\big] \qquad (8)\end{aligned}$$

(8) − (5) yields

$$\big(D_{i-1,j}+c_{ins}\big[\mathtt{s}[i]\big]\big)-\big(D_{i,j-1}+c_{del}\big[\mathtt{t}[j]\big]\big)\geq -2c_{del}\big[\mathtt{t}[j]\big] \qquad (9)$$

(7) − (6) yields

$$\big(D_{i-1,j}+c_{ins}\big[\mathtt{s}[i]\big]\big)-\big(D_{i,j-1}+c_{del}\big[\mathtt{t}[j]\big]\big)\leq 2c_{ins}\big[\mathtt{s}[i]\big] \qquad (10)$$

Thus, we know from (7) and (9) that

$$v_{i,j}\geq \max\big(c_{ins}\big[\mathtt{s}[i]\big]-c_{sub}\big[\mathtt{s}[i],\mathtt{t}[j]\big]-c_{del}\big[\mathtt{t}[j]\big],\\ -2c_{del}\big[\mathtt{t}[j]\big]\big)$$

and from (8) and (10) that

$$v_{i,j}\leq \max\big(c_{ins}\big[\mathtt{s}[i]\big]+c_{del}\big[\mathtt{t}[j]\big]-c_{sub}\big[\mathtt{s}[i],\mathtt{t}[j]\big],\\ 2c_{ins}\big[\mathtt{s}[i]\big]\big)$$

Finally, by defining

$$C_3\coloneqq\min_{i,j}\Big(\max\big(c_{ins}\big[\mathtt{s}[i]\big]-c_{sub}\big[\mathtt{s}[i],\mathtt{t}[j]\big]-c_{del}\big[\mathtt{t}[j]\big],-2c_{del}\big[\mathtt{t}[j]\big]\big)\Big)$$

$$C_4\coloneqq\max_{i,j}\Big(\max\big(c_{ins}\big[\mathtt{s}[i]\big]+c_{del}\big[\mathtt{t}[j]\big]-c_{sub}\big[\mathtt{s}[i],\mathtt{t}[j]\big],2c_{ins}\big[\mathtt{s}[i]\big]\big)\Big)$$

we proved $C_3\leq v_{i,j}\leq C_4$. ■

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