# A Differential Fault Attack on Grain-128a using MACs

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Abstract. The 32-bit MAC of Grain-128a is a linear combination of the first 64 and then the alternative keystream bits. In this paper we describe a successful differential fault attack on Grain-128a, in which we recover the secret key by observing the correct and faulty MACs of certain chosen messages. The attack works due to certain properties of the Boolean functions and corresponding choices of the taps from the LFSR. We present methods to identify the fault locations and then construct set of linear equations to obtain the contents of the LFSR and the NFSR. Our attack requires less than  $2^{11}$  fault injections and invocations of less than  $2^{12}$  MAC generation routines.

Keywords: Grain v1, Grain-128, Grain-128a, LFSR, MAC, NFSR, Stream Cipher.

# 1 Introduction

The Grain-128a authentication scheme was proposed in SKEW 2011 by Ågren et al. Any message in  $\{0,1\}^*$  can be mapped to a 32-bit tag using this authenticated-encryption scheme. Grain-128a is essentially part of the Grain family which was first proposed by Hell, Johansson and Meier in 2005 [13] as a part of the eStream project. The physical structure of the Grain family is simple as well as elegant and has been designed so as to require low hardware complexity. In response to cryptanalysis against the initial design of the cipher, the modified versions Grain v1 [13], Grain-128 [14] and Grain-128a [2] were proposed in due course. Analysis of this cipher is an area of recent interest as evident from number of cryptanalytic results [3–12, 16–19, 22, 23].

Fault attacks are known to be very efficient against stream ciphers in general, and have received attention in recent cryptographic literature [15]. For differential fault attack scenario in stream ciphers, the attacker is allowed to inject faults in the internal state, and then by analyzing the difference in the faulty and the fault-free keystreams, one should be able to deduce the complete or partial information about the internal state/secret key. The most common method of injecting faults is by using laser shots or clock glitches [20, 21]. Though the fault attacks usually rely on optimistic assumptions and study the cipher in a model that is weaker than the original version, they are not unrealistic as evident from literature. In this paper too, the model we study is a follow up of existing state-of-the-art literature [4, 6, 16]. A detailed justification of the feasibility of such fault model is presented in [6, Section IIIB].

Grain-128 has been successfully cryptanalyzed by employing fault attacks [6, 16]. In this case, the attacker has the advantage of accessing and analyzing the entire fault-free and faulty keystreams. In Grain-128a, this is not the case as it accommodates authentication too. The scheme does not make the first 64 keystream bits available to the attacker. Thereafter the keystream bits are used for encryption and authentication alternatively. The scheme outputs 32-bit MAC of any message and this can be used by the attacker. In our work, we have described an approach to find the secret key used in the authentication scheme by observing the correct and faulty MACs of certain specific messages.

We proceed with the description of the Grain family, and in particular Grain-128a, in this section. The implementation of the attack on Grain-128a along with the fault location identification routine is explained in Section 2.

### 1.1 Brief description of Grain family

The exact structure of the Grain family is explained in this section. It consists of an *n*-bit LFSR and an *n*-bit NFSR. Certain bits of both the shift registers are taken as inputs to a combining Boolean function, whence the keystream is produced. The update function of the LFSR is given by the equation  $y_{t+n} = f(Y_t)$ , where  $Y_t = [y_t, y_{t+1}, \ldots, y_{t+n-1}]$  is an *n*-bit vector that denotes the LFSR state at the  $t^{th}$ clock interval and  $\phi$  is a linear function on the LFSR state bits obtained from a primitive polynomial in GF(2) of degree *n*.

We abuse the + notation for Boolean XOR, i.e., GF(2) addition as well as standard arithmetic addition. However, that will be clear from the context.

The NFSR state is updated as  $x_{t+n} = y_t + g(X_t)$ . Here,  $X_t = [x_t, x_{t+1}, \dots, x_{t+n-1}]$  is an *n*-bit vector that denotes the NFSR state at the  $t^{th}$  clock interval and g is a non-linear function of the NFSR state bits.

The output keystream is produced by combining the LFSR and NFSR bits as  $z_t = h'(X_t, Y_t) = \bigoplus_{a \in A} x_{t+a} + h(X_t, Y_t)$ , where A is some fixed subset of  $\{0, 1, 2, \ldots, n-1\}$ .

The Grain family uses an *n*-bit key K, and an *m*-bit initialization vector IV, with m < n. The key is loaded in the NFSR and the IV is loaded in the  $0^{th}$  to the  $(m-1)^{th}$  bits of the LFSR. The remaining  $m^{th}$  to  $(n-1)^{th}$  bits of the LFSR are loaded with some fixed pad  $P \in \{0,1\}^{n-m}$ . Hence at this stage, the 2*n* bit initial state is of the form K||IV||P. Then, for the first 2*n* clocks, the keystream produced at the output point of the function h' is XOR-ed to both the LFSR and NFSR update functions, i.e., during the first 2*n* clock intervals, the LFSR and the NFSR bits are updated as  $y_{t+n} = z_t + f(Y_t)$ ,  $x_{t+n} = y_t + z_t + g(X_t)$ . This is the Key Scheduling Algorithm (KSA).

After the completion of the KSA,  $z_t$  is no longer XOR-ed to the LFSR and the NFSR but it is used as the Pseudo-Random keystream bit. This is the Pseudo-Random Generation Algorithm (PRGA). Therefore during this phase, the LFSR and NFSR are updated as  $y_{t+n} = f(Y_t), x_{t+n} = y_t + g(X_t)$ .

For Grain-128a authenticated encryption scheme the exact parameters are as follows. The size of Key n = 128 bits and the IV is of size m = 96 bits. The value of pad used is P = 0xFFFF FFFE. The LFSR update rule is given by

$$y_{t+128} \stackrel{\Delta}{=} f(Y_t) = y_{t+96} + y_{t+81} + y_{t+70} + y_{t+38} + y_{t+7} + y_t.$$

The NFSR state is updated as follows

$$x_{t+128} = y_t + g(x_{t+96}, x_{t+95}, x_{t+93}, x_{t+92}, x_{t+91}, x_{t+88}, x_{t+84}, x_{t+82}, x_{t+78}, x_{t+70}, x_{t+68}, x_{t+67}, x_{t+65}, x_{t+61}, x_{t+59}, x_{t+48}, x_{t+40}, x_{t+27}, x_{t+26}, x_{t+25}, x_{t+24}, x_{t+22}, x_{t+13}, x_{t+11}, x_{t+3}, x_t),$$

where  $g(x_{t+96}, x_{t+95}, \dots, x_t)$ 

$$\stackrel{\Delta}{=} g(X_t) = x_t + x_{t+26} + x_{t+56} + x_{t+91} + x_{t+96} + x_{t+3}x_{t+67} + x_{t+11}x_{t+13} + x_{t+17}x_{t+18} + x_{t+27}x_{t+59} + x_{t+40}x_{t+48} + x_{t+61}x_{t+65} + x_{t+68}x_{t+84} + x_{t+88}x_{t+92}x_{t+93}x_{t+95} + x_{t+22}x_{t+24}x_{t+25} + x_{t+70}x_{t+78}x_{t+82}.$$

The pre-output function  $z_t$  is defined as

$$z_t = \bigoplus_{j \in A} x_{t+j} + y_{t+93} + h(x_{t+12}, y_{t+8}, y_{t+13}, y_{t+20}, x_{t+95}, y_{t+42}, y_{t+60}, y_{t+79}, y_{t+94})$$

where  $A = \{2, 15, 36, 45, 64, 73, 89\}$  and  $h(s_0, \ldots, s_8) = s_0 s_1 + s_2 s_3 + s_4 s_5 + s_6 s_7 + s_0 s_4 s_8$ . The output function is defined as  $y_t = z_{64+2t}$ .

Authentication Assume that we have a message of length L defined by the bits  $m_0, \ldots, m_{L-1}$ . Set  $m_L = 1$  as padding. To provide authentication, two registers, called accumulator and shift register

of size 32 bits each, are used. The content of accumulator and shift register at time t is denoted by  $a_t^0, \ldots, a_t^{31}$  and  $r_t, \ldots, r_{t+31}$ . The accumulator is initialized through  $a_0^t = z_t, 0 \le t \le 31$  and the shift register is initialized through  $r_t = z_{32+t}, 0 \le t \le 31$ . The shift register is updated as  $r_{t+32} = z_{64+2t+1}$ . The accumulator is updated as  $a_{t+1}^j = a_t^j + m_t r_{t+j}$  for  $0 \le j \le 31$  and  $0 \le t \le L$ . The final content of accumulator,  $a_{L+1}^0, \ldots, a_{L+1}^{31}$  is used for authentication.

#### $\mathbf{2}$ Differential Fault Analysis on Grain-128a

We like to point out that to the best of our knowledge there is no existing fault attack on Grain-128a available in literature. Moreover, our attack strategy works using the MAC of certain messages instead of exploiting the keystream bits directly. Before proceeding further, let us now formalize the fault model.

- 1. The attacker is able to reset the system with the original Key-IV (as in [4, 6]) or the original Key and different IVs (as in [16]) and start the cipher operations again.
- 2. The attacker can inject a fault at any one random bit location of the LFSR or NFSR. As a result of the fault injection, the binary value in the bit-location (where the fault has been injected) is toggled. The attacker is not allowed to choose the location where he wants to inject the fault. However, as assumed in both [4, 6, 16] the fault in any bit may be reproduced at any later stage of operation, once injected.
- 3. Similar to [4, 6], the attacker can inject faults in the LFSR only, whereas the NFSR has been used for fault injection in [16].
- 4. The attacker has full control over the timing of fault injection, i.e., it is possible to inject the fault precisely at any stage of the cipher operation.
- 5. The attacker can obtain the MAC of any message of his choice including the empty message.

#### 2.1**Obtaining the Location of the Fault**

Our attack model assumes that the attacker is allowed to togel the value at exactly one random location of the LFSR. The attacker, however can not explicitly choose the location where the fault is to be injected. In order for the attack to succeed, it is very important that it will be possible to identify the location of the LFSR where the fault has been induced.

Let  $S_0 \in \{0,1\}^{256}$  be the initial state of the Grain-128a PRGA, and  $S_{0,\Delta_{\phi}}$  be the initial state resulting after injecting fault in LFSR location  $\phi \in [0, 127]$ . Let  $Z = [z_0, z_1, \ldots, z_{65}]$  and  $Z^{\phi} = [z_0^{\phi}, z_1^{\phi}, \ldots, z_{65}^{\phi}]$  be the first 66 keystream bits produced by  $S_0$  and  $S_{0,\Delta_{\phi}}$  respectively. Then as per the authentication scheme the MAC  $\sigma(\emptyset)$  of the empty message  $\emptyset$  is given by the vector  $[z_0 + z_{32}, z_1 +$  $z_{33}, \ldots, z_{31} + z_{63}$  and similarly the MAC for the singular message bit 0 will be given by  $\sigma(0) =$  $[z_{0} + z_{33}, z_{1} + z_{34}, \dots, z_{30} + z_{63}, z_{31} + z_{65}].$  The corresponding faulty MACs are  $\sigma^{\phi}(\emptyset) = [z_{0}^{\phi} + z_{32}^{\phi}, z_{1}^{\phi} + z_{33}^{\phi}, \dots, z_{31}^{\phi} + z_{63}^{\phi}]$  and  $\sigma^{\phi}(0) = [z_{0}^{\phi} + z_{33}^{\phi}, z_{1}^{\phi} + z_{34}^{\phi}, \dots, z_{30}^{\phi} + z_{63}^{\phi}, z_{31}^{\phi} + z_{65}^{\phi}].$ The task for the fault location identification routine is to determine the fault location  $\phi$  by analyzing

the difference between  $[\sigma(\emptyset), \sigma(0)]$  and  $[\sigma^{\phi}(\emptyset), \sigma^{\phi}(0)]$ .

**Definition 1.** We define a 64-bit vector  $E_{\phi}$  over GF(2) defined as follows. Let  $E_{\phi}^{1}$  be the bitwise  $\textit{logical XNOR (complement of XOR) of the MACs of } \sigma(\emptyset) \textit{ and } \sigma^{\phi}(\emptyset), \textit{ i.e., } E^1_{\phi} = 1 + \sigma(\emptyset) + \sigma^{\phi}(\emptyset),$ (here + should be interpreted as  $\oplus$ ) and similarly  $E_{\phi}^2 = 1 + \sigma(0) + \sigma^{\phi}(0)$ . Then  $E_{\phi} = E_{\phi}^1 ||E_{\phi}^2$ .

Since  $S_0$  can have  $2^{224}$  values (each arising from a different combination of the 128 bit key and 96 bit IV, rest 32 padding bits are fixed), each of these choices of  $S_0$  may lead to different patterns of  $E_{\phi}$ . The bitwise logical AND of all such vectors  $E_{\phi}$  is denoted as the Signature vector  $Sgn_{\phi}$  for the fault location  $\phi$ .

Since it is computationally infeasible to generate  $2^{224}$  patterns and AND them, below we present a clever idea to achieve this efficiently. Whenever  $Sgn_{\phi}(i)$  is 1 for  $0 \le i \le 31$ , this implies that the  $i^{th}$ MAC bit produced by  $S_0$  and  $S_{0,\Delta_{\phi}}$  for the empty message is equal for all choices of  $S_0$ . Similarly if  $Sgn_{\phi}(i)$  is 1 for  $32 \le i \le 63$  this implies that the  $(i - 32)^{th}$  MAC bit produced by  $S_0$  and  $S_{0,\Delta_{\phi}}$  for the zero message is equal.

For Grain-128a, two initial states of the PRGA  $S_0, S_{0,\Delta_{127}} \in \{0,1\}^{256}$  which differ only in the 127<sup>th</sup> position of the LFSR, produce identical output bits in 62 specific positions among the initial 66 keystream bits produced during the PRGA. If an input differential is introduced in the 127<sup>th</sup> LFSR position, then at all rounds numbered  $k \in [0, 65] \setminus \{33, 34, 48, 65\}$ , the difference exists in positions that do not provide input to the Boolean function h and hence at these clocks the keystream bit produced by the two states are essentially the same. At all other clock rounds the difference appears at positions which provide input to h. Hence the keystream produced at these clocks may be different. Since

$$\sigma(\emptyset) = [z_0 + z_{32}, z_1 + z_{33}, \dots, z_{31} + z_{63}] \text{ and } \sigma^{\phi}(\emptyset) = [z_0^{\phi} + z_{32}^{\phi}, z_1^{\phi} + z_{33}^{\phi}, \dots, z_{31}^{\phi} + z_{63}^{\phi}]$$

this implies that all bits of  $\sigma(\emptyset)$  and  $\sigma^{127}(\emptyset)$  are equal except for the bits indexed by 1, 2, 16. Also since

$$\sigma(0) = [z_0 + z_{33}, z_1 + z_{34}, \dots, z_{30} + z_{63}, z_{31} + z_{65}] \text{ and } \sigma^{\phi}(0) = [z_0^{\phi} + z_{33}^{\phi}, z_1^{\phi} + z_{34}^{\phi}, \dots, z_{30}^{\phi} + z_{63}^{\phi}, z_{31}^{\phi} + z_{65}^{\phi}],$$

we can say that all bits of  $\sigma(0)$  and  $\sigma^{127}(0)$  are equal except for the bits indexed by 0, 1, 15, 31. Following the explanation given above, we can write  $Sgn_{127}$  in hexadecimal notation,  $Sgn_{79} = 9FFF$ 7FFF 3FFE FFFE, which has 64 - 3 - 4 = 57 many 1's and rest 0's.

Generalizing the above idea, for two PRGA initial states  $S_0, S_{0,\Delta_{\phi}} \in \{0,1\}^{256}$  which differ only in the  $\phi^{th}$  LFSR location, an analysis of the differential trails shows that out of the first 66 keystream bits produced by them, the bits at a certain fixed rounds are guaranteed to be equal. Thus by performing the above analysis for all fault locations  $\phi$  ( $0 \le \phi \le 127$ ), it is possible to calculate all the Signature vectors. Table 1 presents the vectors for each fault location  $\phi$ , where the Fault Signature Vectors  $Sgn_{\phi}$ for  $0 \le \phi \le 127$  are written in hexadecimal notation.

Steps for location Identification As mentioned above, the task for the fault identification routine is to determine the value of  $\phi$  given the vector  $E_{\phi}$ , i.e., obtaining a unique  $Sgn_{\phi}$ . For any *l*-bit vector V, let  $B_V = \{i : 0 \leq i < l, V(i) = 1\}$ . Now define a relation  $\preceq$  in  $\{0,1\}^l$  such that for 2 elements  $V_1, V_2 \in \{0,1\}^l$ , we will have  $V_1 \preceq V_2$  if  $B_{V_1} \subseteq B_{V_2}$ .

So we start with a Key-IV pair  $K, IV_0$  and record the MACs of the empty and zero messages. We then reset the cipher with  $K, IV_0$  and apply a fault at some location  $\phi$  (that is selected randomly and not known at this point) at the beginning of the PRGA, and obtain the corresponding faulty MACs of the empty and zero message. Using these we compute the  $E_{\phi}$  vector as given in Definition 1. The entire process requires 4 invocations of the MAC routine. Now we check the elements in  $B_{E_{\phi}}$ . By the definition of Signature vector proposed above, we know that for the correct value of  $\phi$ ,  $B_{Sgn_{\phi}} \subseteq B_{E_{\phi}}$ and hence  $Sgn_{\phi} \leq E_{\phi}$ . So our strategy would be to search all the Signature vectors and formulate the candidate set  $\Psi_0 = \{\psi : 0 \leq \psi \leq 127, Sgn_{\psi} \leq E_{\phi}\}$ . If  $|\Psi_0|$  is 1, then the single element in  $\Psi_0$  will give us the fault location  $\phi$ . However, this may not necessarily be the case always. If  $|\Psi_0| > 1$ , we will be unable to decide conclusively at this stage.

In such a scenario we reset the cipher with K,  $IV_1$  ( $IV_1$  different from  $IV_0$ ) and record the fault-free MAC of the empty and zero messages. We then reset the cipher with K,  $IV_1$  again and apply the fault at the location  $\phi$  (our fault model considers that the fault can be applied at the same location without knowing it) at the beginning of the PRGA round and record the corresponding faulty MACs. Now we recalculate the vector  $E_{\phi}$  as defined previously. We now search over the Signature vectors in the candidate set  $\Psi_0$  and narrow down the set of possible candidates to  $\Psi_1 = \{\psi : \psi \in \Psi_0, Sgn_{\psi} \leq E_{\phi}\}$ . Clearly,  $|\Psi_1| \leq |\Psi_0|$ , and so if  $|\Psi_1| = 1$  then the fault location  $\phi$  is the single element in  $\Psi_1$ . If not, we

$\phi$	$Sgn_{\phi}$		$\phi$ $Sgn_{\phi}$					$\phi$		Sg	$n_{\phi}$		$\phi$	$Sgn_{\phi}$			
0	8EFF BEFF 1DFF	7DFE	32	FFF7	EF67	FFF7	EF4E	64	F7F7	ED73	F7EF	DCE7	96	D7FF	9DF3	8FFF	3BE7
1	C77F DF7F 8EFF	BEFE	33	FFFB	F7B3	FFFB	F7A7	65	FBFB	F6B9	FBF7	EE73	97	EBFF	CEF9	C7FF	9DF2
2	E3BF EFBF C77F	DF7F	34	FFFD	FBD9	FFFD	FBD3	66	FDFD	FB5C	FDFB	F739	98	F5FF	E77C	E3FF	CEF9
3	F1DF F7DF E3BF	EFBF	35	FFFE	FDEC	FFFE	FDE9	67	FEFE	FDAE	FEFD	FB9D	99	FAFF	F3BE	F1FF	E77D
4	F8EF FBEF F1DF	F7DF	36	FFFF	7EF6	FFFF	7EF5	68	FF7F	7ED7	FF7E	FDCE	100	FD7F	F9DF	F8FF	F3BE
5	FC77 FDF7 F8EF	FBEF	37	FFFF	BF7B	FFFF	BF7A	69	FFBF	BF6B	FFBF	7EE6	101	FEBF	FCEF	FC7F	F9DE
6	FE3B FEFB FC77	FDF7	38	CFFF	9FBD	9FFF	5FBC	70	CFDF	9FB5	9FDF	3F73	102	FF5F	FE77	FE3F	FCEF
7	CF1D BF7D 9E3B	7EFB	39	E7FF	CFDE	CFFF	AFDE	71	E7EF	CFDA	CFEF	9FB9	103	FFAF	FF3B	FF1F	FE77
8	678E DFBE 4F1D	BF7D	40	73FF	E7EF	E7FF	D7EF	72	F3F7	E7ED	E7F7	CFDD	104	FFD7	FF9D	FF8F	FF3B
9	B3C7 6FDF A78E	DFBF	41	B9FF	F3F7	73FF	EBF7	73	F9FB	F3F6	F3FB	E7EE	105	FFEB	FFCE	FFC7	FF9D
10	D9E3 B7EF D3C7	6FDE	42	5CFF	F9FB	39FF	F5FB	74	7CFD	F9FB	F9FD	F3F7	106	FFF5	FFE7	FFE3	FFCF
11	ECF1 DBF7 E9E3	B7EF	43	AE7F	FCFD	9CFF	FAFD	75	BE7E	FCFD	7CFE	F9FB	107	FFFA	FFF3	FFF1	FFE6
12	F678 EDFB F4F1	DBF7	44	D73F	FE7E	CE7F	FD7E	76	DF3F	7E7E	BE7F	7CFD	108	FFFD	7FF9	FFF8	FFF3
13	7B3C 76FD 7A78	EDFB	45	6B9F	FF3F	E73F	FEBF	77	EF9F	BF3F	DF3F	BE7F	109	FFFE	BFFC	FFFC	7FF9
14	BD9E 3B7E BD3C	76FD	46	B5CF	FF9F	739F	FF5F	78	F7CF	DF9F	EF9F	DF3E	110	FFFF	5FFE	FFFE	3FFC
15	DECF 1DBF DE9E	3B7F	47	DAE7	FFCF	B9CF	FFAF	79	7BE7	EFCF	77CF	EF9F	111	7FFF	AFFF	FFFF	1FFE
16	EF67 8EDF EF4F	1DBE	48	ED73	FFE7	DCE7	FFD7	80	BDF3	F7E7	BBE7	F7CF	112	BFFF	D7FF	7FFF	8FFE
17	F7B3 C76F F7A7	8EDF	49	F6B9	FFF3	EE73	FFEB	81	CEF9	BBF3	9DF3	7BE7	113	DFFF	EBFF	BFFF	C7FF
18	FBD9 E3B7 FBD3	C76F	50	FB5C	FFF9	F739	FFF5	82	E77C	DDF9	CEF9	BDF3	114	EFFF	F5FF	DFFF	E3FF
19	FDEC F1DB FDE9	E3B7	51	FDAE	7FFC	FB9C	FFFA	83	F3BE	6EFC	E77C	DEF9	115	F7FF	FAFF	EFFF	F1FF
20	7EF6 78ED 7EF4	F1DB	52	7ED7	3FFE	FDCE	7FFD	84	F9DF	377E	F3BE	6F7D	116	FBFF	FD7F	F7FF	F8FF
21	BF7B 3C76 BF7A	78ED	53	BF6B	9FFF	7EE7	3FFF	85	FCEF	9BBF	F9DF	37BE	117	FDFF	FEBF	FBFF	FC7F
22	DFBD 9E3B DFBD	3C77	54	DFB5	CFFF	BF73	9FFE	86	FE77	CDDF	FCEF	9BDE	118	FEFF	FF5F	FDFF	FE3F
23	EFDE CF1D EFDE	9E3A	55	EFDA	E7FF	DFB9	CFFF	87	FF3B	E6EF	FE77	CDEF	119	FF7F	FFAF	FEFF	FF1F
24	F7EF 678E F7EF	4F1D	56	F7ED	73FF	EFDC	E7FF	88	FF9D	F377	FF3B	E6F7	120	FFBF	FFD7	FF7F	FF8F
25	FBF7 B3C7 FBF7	A78F	57	FBF6	B9FF	F7EE	73FF	89	FFCE	F9BB	FF9D	F37B	121	FFDF	FFEB	FFBF	FFC7
26	FDFB D9E3 FDFB	D3C6	58	FDFB	5CFF	FBF7	39FF	90	FFE7	7CDD	FFCE	F9BD	122	FFEF	FFF5	FFDF	FFE3
27	FEFD ECF1 FEFD	E9E3	59	FEFD	AE7F	FDFB	9CFF	91	FFF3	BE6E	FFE7	7CDE	123	FFF7	FFFA	FFEF	FFF1
28	FF7E F678 FF7E	F4F1	60	7F7E	D73F	7EFD	CE7F	92	7FF9	DF37	FFF3	BE6F	124	FFFB	FFFD	FFF7	FFF8
29	FFBF 7B3C FFBF	7A79	61	BFBF	6B9F	BF7E	E73F	93	3FFC	EF9B	7FF9	DF37	125	7FFD	FFFE	FFFB	FFFC
30	FFDF BD9E FFDF	BD3C	62	DFDF	B5CF	DFBF	739F	94	1FFE	77CD	<b>3FFC</b>	EF9B	126	3FFE	FFFF	7FFD	FFFE
31	FFEF DECF FFEF	DE9E	63	EFEF	DAE7	EFDF	B9CF	95	8FFF	3BE6	9FFE	77CD	127	9FFF	7FFF	<b>3FFE</b>	FFFE

 Table 1. Signature Vectors for different fault locations.

repeat the above process for another round for a different Key-IV pair  $K, IV_2$ . If after k rounds of this process,  $|\Psi_{k-1}| = 1$ , then the single element in  $\Psi_{k-1}$  gives us the desired location  $\phi$ .

With detailed experiments taking an average over  $2^{20}$  uniformly randomly chosen Key-IV pairs, it we found that the average value of k is 1.31 to uniquely identify a fault location in the LFSR. Since we are working with the MAC of empty and zero message, thus, for each location we need to inject  $\mu = 2 \cdot 1.31 = 2.62$  faults.

Now let us argue that the LFSR fault location can be uniquely identified by the signature scheme proposed here. The signature scheme is based on both the empty and the zero message. Now a simple exhaustive search, through the Signature vectors for all fault locations, will show that  $Sgn_{\phi_1} \not\leq Sgn_{\phi_2}$ for any two fault locations  $0 \leq \phi_1 \neq \phi_2 \leq 127$ . This implies that for any value of the fault location  $\phi \in [0, 127]$  the fault identification scheme will eventually narrow down the candidate set  $\Psi_{k-1}$  to just one element for some value of k.

One may wonder if the Signature vector were to be based on the difference of MAC of just the empty or the 0 message, whether a location identification scheme could have been proposed. The answer is no. Take the signature scheme based on the MAC difference of just the empty message in which l = 32. Studying the Signature vectors, one can check that the first 32 bits of  $Sgn_{21} = BF7B$  3C76 and  $Sgn_{36} = FFFF$  7EF6. Note that, for all locations  $i \in [0, 31]$  such that  $Sgn_{21}(i) = 1$ , the value of  $Sgn_{36}(i)$  is also 1. This implies that  $Sgn_{21} \preceq Sgn_{36}$ . Now consider the case with the fault location  $\phi = 36$ . Then by the definition of the signature vector we have  $Sgn_{36} \preceq E_{\phi}$ . Since  $\preceq$  is a partial order on  $\{0,1\}^l$ , this implies that  $Sgn_{21} \preceq E_{\phi}$  and so whenever  $\phi = 36$  the fault location identification routine will never be able to narrow down the set of possible candidates  $\Psi_k$  to only  $\{36\}$  for any value

of k. So the signature scheme can not be based on the MAC difference of the empty message only. If we were to base the signature scheme on the MAC difference of the 0 message bit, then too a look at the signature tables will show us that  $Sgn_{16} \leq Sgn_{111}$  and the scheme would fail by the above argument. It will be very interesting to find out a message for which the signature scheme will work just looking at the fault-free and faulty MACs on it.

## 2.2 Determining the LFSR State

Towards this, let us present a few more notations at this point.

- 1.  $S_t = [x_0^t, x_1^t, \dots, x_{127}^t, y_0^t, y_1^t, \dots, y_{127}^t]$  is used to denote the internal state of the cipher at the beginning of round t of the PRGA when initialized with the Key-IV pair  $K, IV_0$ . Thus  $x_i^t$   $(y_i^t)$  denotes the  $i^{th}$  NFSR (LFSR) bit at the start of round t of the PRGA. When t = 0, we use  $S_0 = [x_0, x_1, \dots, x_{127}, y_0, y_1, \dots, y_{127}]$  to denote the internal state for convenience.
- 2.  $S_t^{\phi}$  is used to denote the internal state of the cipher at the beginning of round t of the PRGA when initialized with the Key-IV pair  $K, IV_0$ , when a fault has been injected in LFSR location  $\phi$  at the beginning of the PRGA round.
- 3.  $z_i^{\phi}$  denotes the keystream bit produced in the *i*<sup>th</sup> PRGA round, after faults have been injected in LFSR location  $\phi$  at the beginning of the PRGA round.  $z_i$  is the fault-free *i*<sup>th</sup> keystream bit.

We start by making the following observations about the output Boolean function h in Grain-128a:

$$h(s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8) + h(s_0, s_1, 1 + s_2, s_3, s_4, s_5, s_6, s_7, s_8) = s_3$$

$$(1)$$

$$h(s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8) + h(s_0, s_1, s_2, 1 + s_3, s_4, s_5, s_6, s_7, s_8) = s_2$$

$$(2)$$

$$h(s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8) + h(s_0, s_1, s_2, s_3, s_4, s_5, 1 + s_6, s_7, s_8) = s_7$$
(3)

$$h(s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8) + h(s_0, s_1, s_2, s_3, s_4, s_5, s_6, 1 + s_7, s_8) = s_6$$

$$\tag{4}$$

Let us now explain in detail how we can obtain the bit-value at a specific location of the LFSR, say for example  $y_{108}$ . Note that  $s_0, s_4$  correspond to the NFSR locations 12,95 respectively and  $s_1, s_2, s_3, s_5, s_6, s_7, s_8$  correspond to the LFSR locations 8, 13, 20, 42, 60, 79, 94 respectively. Now look at (1) above and note that  $s_2$  corresponds to the LFSR location 13. If two internal states S and  $S_{\Delta}$ be such that they differ in the LFSR location 13 (and in no other tap locations that contribute to the keystream bit generation), then the difference of the keystream bit produced by them will be equal to the value in LFSR location 20. Similar analysis can be done corresponding to (2), (3), (4).

Assume that the attacker has injected a fault at location 127 of the LFSR at the beginning of the PRGA. Then at round 48 of the PRGA the input differential travels to location 79 of the LFSR, i.e., at round 48 the original state  $S_{48}$  and the faulty state  $S_{48}^{127}$  differ in location 79 of the LFSR and in no other location that contributes inputs to the output keystream bit at round 48. Then by equation (4), the sum of the corresponding fault-free and faulty bits produced at round 48 is given by  $z_{48} + z_{48}^{127} = y_{60}^{48} = y_{108}$ .

At round 16 of the PRGA, the differential does not sit on any LFSR location that contributes input to the output keystream bit at that round. Hence  $z_{16} = z_{16}^{127}$ .

Now consider the fault-free and faulty MAC (due to the fault at  $\phi = 127$  at the beginning of the PRGA) of the empty message  $\sigma(\emptyset)$  and  $\sigma^{127}(\emptyset)$ . From the definition of the MAC of empty message, it can be deduced that the bit number 16 of  $\sigma(\emptyset) \oplus \sigma^{127}(\emptyset)$  is given by  $z_{16} + z_{48} + z_{16}^{127} + z_{48}^{127} = y_{108}$ .

Hence by looking at the difference in the correct and faulty MACs of the empty messages one can deduce the LFSR state bit  $y_{108}$  at the beginning of the PRGA.

In Table 2 we give a list of 115 LFSR state bits  $y_i$  that can be recovered by observing the difference of the faulty and correct  $d^{th}$  ( $0 \le d \le 31$ ) MAC bit of the empty message for different values of the fault location  $\phi$ . There are 174 (more than 115) entries in the table and this is due to the multiple

$\phi$	d	State bit $y_i$	$\phi$	d	State bit $y_i$	$\phi$	d	State bit $y_i$	$\phi$	d	State bit $y_i$	$\phi$	d	State bit $y_i$	$\phi$	d	State bit $y_i$
0	17	$y_{109}$	21	31	$y_{123}$	65	5	$y_{84}$	75	15	$y_{94}$	83	23	$y_{102}$	97	5	$y_{116}$
1	18	$y_{110}$	38	17	$y_{109}$	65	20	$y_{72}$	75	22	$y_{114}$	83	30	$y_{122}$	98	6	$y_{117}$
2	19	$y_{111}$	39	18	$y_{110}$	66	6	$y_{85}$	75	23	$y_{68}$	83	31	$y_{76}$	99	7	$y_{118}$
3	20	$y_{112}$	40	19	$y_{111}$	66	21	$y_{73}$	75	30	$y_{82}$	84	20	$y_{112}$	100	8	$y_{119}$
4	21	$y_{113}$	41	20	$y_{112}$	67	7	$y_{86}$	76	16	$y_{95}$	84	24	$y_{103}$	101	9	$y_{120}$
5	22	$y_{114}$	42	21	$y_{113}$	67	22	$y_{74}$	76	23	$y_{115}$	84	31	$y_{123}$	102	10	$y_{121}$
6	23	$y_{115}$	43	22	$y_{114}$	68	8	$y_{87}$	76	24	$y_{69}$	85	21	$y_{113}$	103	11	$y_{122}$
7	17	$y_{109}$	44	23	$y_{115}$	68	23	$y_{75}$	76	31	$y_{83}$	85	25	$y_{104}$	104	12	$y_{123}$
7	24	$y_{116}$	45	24	$y_{116}$	69	9	$y_{88}$	77	17	$y_{96}$	86	22	$y_{114}$	105	13	$y_{124}$
8	18	$y_{110}$	46	25	$y_{117}$	69	24	$y_{76}$	77	24	$y_{116}$	86	26	$y_{105}$	106	14	$y_{125}$
8	25	$y_{117}$	47	26	$y_{118}$	70	10	$y_{89}$	77	25	$y_{70}$	87	23	$y_{115}$	107	15	$y_{126}$
9	19	$y_{111}$	48	27	$y_{119}$	70	17	$y_{109}$	78	18	$y_{97}$	87	27	$y_{106}$	108	16	$y_{127}$
9	26	$y_{118}$	49	28	$y_{120}$	70	25	$y_{77}$	78	25	$y_{117}$	88	24	$y_{116}$	111	0	$y_{92}$
10	20	$y_{112}$	50	29	$y_{121}$	71	11	$y_{90}$	78	26	$y_{71}$	88	28	$y_{107}$	112	1	$y_{93}$
10	27	$y_{119}$	51	30	$y_{122}$	71	18	$y_{110}$	79	19	$y_{98}$	89	25	$y_{117}$	113	2	$y_{94}$
11	21	$y_{113}$	52	31	$y_{123}$	71	19	$y_{64}$	79	26	$y_{118}$	89	29	$y_{108}$	114	3	$y_{95}$
11	28	$y_{120}$	57	12	$y_{64}$	71	26	$y_{78}$	79	27	$y_{72}$	90	26	$y_{118}$	115	4	$y_{96}$
12	2 22	$y_{114}$	58	13	$y_{65}$	72	12	$y_{91}$	80	20	$y_{99}$	90	30	$y_{109}$	116	5	$y_{97}$
12	2 2 9	$y_{121}$	59	14	$y_{66}$	72	19	$y_{111}$	80	27	$y_{119}$	91	27	$y_{119}$	117	6	$y_{98}$
13	3 23	$y_{115}$	60	0	$y_{79}$	72	20	$y_{65}$	80	28	$y_{73}$	91	31	$y_{110}$	118	7	$y_{99}$
13	30	$y_{122}$	60	15	$y_{67}$	72	27	$y_{79}$	81	17	$y_{109}$	92	0	$y_{111}$	119	8	$y_{100}$
14	24	$y_{116}$	61	1	$y_{80}$	73	13	$y_{92}$	81	21	$y_{100}$	92	28	$y_{120}$	120	9	$y_{101}$
14	31	$y_{123}$	61	16	$y_{68}$	73	20	$y_{112}$	81	28	$y_{120}$	93	1	$y_{112}$	121	10	$y_{102}$
15	525	$y_{117}$	62	2	$y_{81}$	73	21	$y_{66}$	81	29	$y_{74}$	93	29	$y_{121}$	122	11	$y_{103}$
16	5 26	$y_{118}$	62	17	$y_{69}$	73	28	$y_{80}$	82	18	$y_{110}$	94	2	$y_{113}$	123	12	$y_{104}$
17	27	$y_{119}$	63	3	$y_{82}$	74	14	$y_{93}$	82	22	$y_{101}$	94	30	$y_{122}$	124	13	$y_{105}$
18	8 28	$y_{120}$	63	18	$y_{70}$	74	21	$y_{113}$	82	29	$y_{121}$	95	3	$y_{114}$	125	14	$y_{106}$
19	29	$y_{121}$	64	4	$y_{83}$	74	22	$y_{67}$	82	30	$y_{75}$	95	31	$y_{123}$	126	15	$y_{107}$
$2\overline{0}$	30	$y_{122}$	$ 6\bar{4} $	19	$y_{71}$	74	29	$y_{81}$	$ 8\overline{3} $	19	$y_{111}$	96	4	$y_{115}$	127	16	$y_{108}$

 Table 2. LFSR state bits recovered

fault options for identifying some of the LFSR bits. The LFSR state bits not present in Table 2 are  $y_0, y_1, \ldots, y_{12}$ . However it can be verified that  $\forall i \in [0, 12]$ , by applying a fault at location  $\phi = 109 + i$  the  $(17+i)^{th}$  bit in difference of  $\sigma(\emptyset)$  and  $\sigma^{109+i}(\emptyset)$  is equal to the state bit  $y_{127}^{1+i}$ . Since  $y_{127}^{1+i}$  is a linear function of  $y_0, y_1, \ldots, y_{127}$ , we can derive  $y_0$  to  $y_{12}$  as follows. By the LFSR update rule of Grain-128a, we have the following 13 equations

$$y_{127}^{1+i} = y_{96+i} + y_{81+i} + y_{70+i} + y_{38+i} + y_{7+i} + y_i, \quad \forall i \in [0, 12].$$

In the last equation  $y_{12}$  is the only unknown and its value can be calculated easily. Similarly  $y_{11}$  is the only unknown in the previous equation. Solving the equations in this manner one can obtain the entire LFSR state at the beginning of the PRGA.

### 2.3 Determining the NFSR State

Once the LFSR internal state of the initial PRGA round is known, one can then proceed to determine the NFSR internal state. In [5] it was shown, that this could have been done efficiently for the initial version of the cipher i.e. Grain v0. After the attack in [5] was reported, the designers made the necessary changes to Grain v1, Grain-128 and Grain-128a so that for these new ciphers, determining the NFSR state form the knowledge of the LFSR state was no longer straightforward. In order to determine the NFSR bits, we look into the decomposition of the Boolean function h in more detail.

One may note that for Grain-128a,  $h(\mathbf{s}) = s_0 \cdot u(\mathbf{s}) + v(\mathbf{s})$ , where  $u(\mathbf{s}) = s_1 + s_4 s_8$ , and  $v(\mathbf{s}) = s_2 s_3 + s_4 s_5 + s_6 s_7$ . Thus we note that

(i) 
$$u(s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8) + u(s_0, 1 + s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8) = 1$$

$$(ii) \ v(s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8) + v(s_0, 1 + s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8) = 0.$$

Also h can be written as  $h(\mathbf{s}) = s_4 \cdot U(\mathbf{s}) + V(\mathbf{s})$ , where  $U(\mathbf{s}) = s_5 + s_0 s_8$ , and  $V(\mathbf{s}) = s_2 s_3 + s_4 s_5 + s_6 s_7$ . We also have

$$\begin{aligned} (i) \ U(s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8) + U(s_0, s_1, s_2, s_3, s_4, 1 + s_5, s_6, s_7, s_8) &= 1, \\ (ii) \ V(s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8) + V(s_0, s_1, s_2, s_3, s_4, 1 + s_5, s_6, s_7, s_8) &= 0. \end{aligned}$$

Since  $s_0, s_4$  correspond to NFSR variables, h satisfies all the properties listed above.

As before, assume the scenario in which the attacker has injected a fault at location 8 of the LFSR at the beginning of the PRGA. Then at this round of the PRGA the input differential travels sits on location 8 of the LFSR i.e. at round 0 of the PRGA the original state  $S_0$  and the faulty state  $S_0^8$  differ in location 8 of the LFSR and in no other location that contributes inputs to the output keystream bit at round 0. Then by the previous relation equation, the sum of the corresponding fault-free bits produced at round 0 is given by  $z_0 + z_0^8 = x_{12}^0 \cdot 1 + 0 = x_{12}$ .

Also note that at round 32 of the PRGA the differential does not sit on any LFSR location that contributes input to the output keystream bit at that round. Hence  $z_{32} = z_{32}^8$ .

Now consider the fault-free and faulty MAC (due to fault at  $\phi = 8$  at the beginning of the PRGA) of the empty message  $\sigma(\emptyset)$  and  $\sigma^8(\emptyset)$ . From the definition of MAC of empty message it can be deduced that the bit number 16 of  $\sigma(\emptyset) \oplus \sigma^8(\emptyset)$  is given by  $z_0 + z_{32} + z_0^8 + z_{32}^8 = x_{12}$ .

Hence by looking at the difference in the correct and faulty MACs of the empty messages one is able to deduce the NFSR state bit  $x_{12}$  at the beginning of the PRGA. In Table 3 we give an exhaustive list of the NFSR state bits  $x_i$  that can be recovered by observing the difference of the faulty and correct  $d^{th}$  MAC bit of the empty message for different values of the fault location  $\phi$ .

$\phi$	d	State bit $x_i$	$\phi$	d	State bit $x_i$	$\phi$	d	State bit $x_i$	$\phi$	d	State bit $x_i$	$\phi$	d	State bit $x_i$	$\phi$	d	State bit $x_i$
8	0	$x_{12}$	24	16	$x_{28}$	40	0	$x_{44}$	49	7	$x_{102}$	57	15	$x_{110}$	65	23	x118
9	1	$x_{13}$	25	17	$x_{29}$	41	1	$x_{45}$	49	9	$x_{53}$	57	17	$x_{61}$	65	25	$x_{69}$
10	2	$x_{14}$	26	18	$x_{30}$	42	0	$x_{95}$	50	8	$x_{103}$	58	16	$x_{111}$	66	24	$x_{119}$
11	3	$x_{15}$	27	19	$x_{31}$	42	2	$x_{46}$	50	10	$x_{54}$	58	18	$x_{62}$	66	26	$x_{70}$
12	4	$x_{16}$	28	20	x <sub>32</sub>	43	1	$x_{96}$	51	9	$x_{104}$	59	17	$x_{112}$	67	25	$x_{120}$
13	5	$x_{17}$	29	21	$x_{33}$	43	3	$x_{47}$	51	11	$x_{55}$	59	19	$x_{63}$	67	27	x <sub>71</sub>
14	6	$x_{18}$	30	22	$x_{34}$	44	2	$x_{97}$	52	10	$x_{105}$	60	18	$x_{113}$	68	26	$x_{121}$
15	7	$x_{19}$	31	23	$x_{35}$	44	4	$x_{48}$	52	12	$x_{56}$	60	20	$x_{64}$	68	28	$x_{72}$
16	8	$x_{20}$	32	24	$x_{36}$	45	3	$x_{98}$	53	11	$x_{106}$	61	19	$x_{114}$	69	27	$x_{122}$
17	9	$x_{21}$	33	25	$x_{37}$	45	5	$x_{49}$	53	13	$x_{57}$	61	21	$x_{65}$	69	29	$x_{73}$
18	10	$x_{22}$	34	26	$x_{38}$	46	4	$x_{99}$	54	12	$x_{107}$	62	20	$x_{115}$	70	28	$x_{123}$
19	11	$x_{23}$	35	27	$x_{39}$	46	6	$x_{50}$	54	14	$x_{58}$	62	22	$x_{66}$	70	30	$x_{74}$
20	12	$x_{24}$	36	28	$x_{40}$	47	5	$x_{100}$	55	13	$x_{108}$	63	21	$x_{116}$	71	29	$x_{124}$
21	13	$x_{25}$	37	29	$x_{41}$	47	7	$x_{51}$	55	15	$x_{59}$	63	23	$x_{67}$	71	31	$x_{75}$
22	14	$x_{26}$	38	30	$x_{42}$	48	6	$x_{101}$	56	14	$x_{109}$	64	22	$x_{117}$	72	30	$x_{125}$
23	15	$x_{27}$	39	31	$x_{43}$	48	8	$x_{52}$	56	16	$x_{60}$	64	$\overline{24}$	$x_{68}$	73	31	$x_{126}$
															74	0	$x_{127}$

Table 3. NFSR state bits recovered

Finding the Remaining Bits From the table given above all state bits of the NFSR can be found except  $x_0, x_1, \ldots, x_{11}$  and  $x_{76}, x_{77}, \ldots, x_{94}$ . These bits may be found as follows. It can be verified that  $\forall i \in [0, 8]$ , by applying a fault at location  $\phi = 73 + 2i$  at the beginning of the PRGA, the difference travels to the LFSR location 8 at round 65 + 2i. It can also be checked that at this PRGA round the differential does not affect any other location that contributes to the output bit i.e. the states  $S_{65+2i}$  and  $S_{65+21}^{73+2i}$  differ in only the LFSR location 8 and no other location that affects the output bit at this round. Then by the previous relation

$$z_{65+2i} + z_{65+2i}^{73+2i} = x_{12}^{65+2i} \cdot 1 + 0 = x_{77+2i}, \quad \forall i \in [0,8].$$

It can also be verified that as a result of applying the fault at 73 + 2i,  $\forall i \in [0, 8]$  at round 31 of the PRGA, the differential does not affect any location that provides inputs to the output bit. Hence,  $z_{31} = z_{31}^{73+2i}$ . Now consider the fault-free and faulty MAC of the message  $0^{i+1}$  (string of i + 1 zeros) obtained by faulting LFSR location 73 + 2i at the beginning of the PRGA. From definition

$$\sigma(0^{i+1}) = [z_0 + z_{33+i}, z_1 + z_{34+i}, \dots, z_{30-i} + z_{63}, z_{31-i} + z_{65}, z_{32-i} + z_{67}, \dots, z_{31} + z_{65+2i}], \quad \forall i \in [0, 8]$$

Hence the last bit in difference of  $\sigma(0^{i+1})$  and  $\sigma^{73+2i}(0^{i+1})$  is equal to

$$z_{31} + z_{65+2i} + z_{31}^{73+2i} + z_{65+2i}^{73+2i} = x_{77+2i}, \ \forall i \in [0,8].$$

Thus far we have recovered 106 of the 128 NFSR state bits. Consider the  $0^{th}$  bit of  $\sigma(\emptyset)$  given by

$$z_0 + z_{32} = \bigoplus_{t \in B} x_t + y_{93} + y_{125} + h(x_{12}, y_8, y_{13}, y_{20}, x_{95}, y_{42}, y_{60}, y_{79}, y_{94}) + h(x_{44}, y_{40}, y_{55}, y_{75}, x_{127}, y_{74}, y_{92}, y_{111}, y_{126})$$

Here  $B = \{2, 15, 36, 45, 64, 73, 89, 34, 47, 68, 77, 96, 105, 121\}$ . Note that  $x_2$  is the only unknown linear term in the above equation, and so its value can be calculated immediately.

Define  $x_{127+i} = x_{127}^i$ ,  $y_{127+i} = y_{127}^i$  for all  $i \ge 1$ . Then by using a similar analysis, it can be verified that the  $i^{th}$  bit of

$$\sigma(\emptyset) + \sigma^{74+i}(\emptyset) = x_{127+i}, \ \forall i \in [0, 31].$$
(5)

Again consider the 0<sup>th</sup> bit of  $\sigma(0^{2j+1})$  for  $0 \le j \le 8$ , given by  $z_0 + z_{33+2j}$ 

$$z_0 + z_{33+2j} = \bigoplus_{t \in B_j} x_t + y_{93} + y_{126+2j} + h(x_{12}, y_8, y_{13}, y_{20}, x_{95}, y_{42}, y_{60}, y_{79}, y_{94}) \\ + h(x_{45+2j}, y_{41+2j}, y_{56+2j}, y_{76+2j}, x_{128+2j}, y_{75+2j}, y_{93+2j}, y_{112+2j}, y_{127+2j}), \ \forall j \in [0, 8].$$

Here  $B_j = \{2, 15, 36, 45, 64, 73, 89, 35 + 2j, 48 + 2j, 69 + 2j, 78 + 2j, 97 + 2j, 106 + 2j, 122 + 2j\}$ . In the above set of equations any  $x_k$  with k > 127 may be calculated from (5). Any  $y_k$  with k > 127 is a linear function of  $y_0, \ldots, y_{127}$  which are already known. Hence  $x_{78+2j}$  with  $0 \le j \le 8$  are the only unknown linear terms in each of these equations and their values are also immediately determined. At this point the only unknown state bits are  $x_0, x_1, x_3, \ldots, x_{11}, x_{76}$ . Consider the  $p^{th}$  bit of  $\sigma(\emptyset)$  given by  $z_p + z_{32+p}$  for  $p \in [1,9] \setminus \{3\}$ .

$$z_p + z_{32+p} = \bigoplus_{t \in B} x_{t+p} + y_{93+p} + y_{125+p} + h(x_{12+p}, y_{8+p}, y_{13+p}, y_{20+p}, x_{95+p}, y_{42+p}, y_{60+p}, y_{79+p}, y_{94+p}) + h(x_{44+p}, y_{40+p}, y_{55+p}, y_{75+p}, x_{127+p}, y_{74+p}, y_{92+p}, y_{111+p}, y_{126+p}), \ \forall p \in [1, 9] \setminus \{3\}$$

In all these equations  $x_{2+p}$  is the only unknown linear term and its value can also be determined immediately (the strategy does not work for p = 3 as  $x_{76}$  is still unknown). We are left with  $x_0, x_1, x_5, x_{76}$ . The values of  $x_0, x_1, x_5$  can be obtained from the expansion of  $x_{128}, x_{129}, x_{133}$  from the NFSR update rule. Now,  $x_{76}$  occurs as a linear term in bit number 3 of  $\sigma(\emptyset)$  and its value too is calculated immediately. Thus we have calculated all of  $S_0$ .

### 2.4 Finding the Secret Key and Complexity of the Attack

It is known that the KSA and PRGA routines in the Grain family are invertible. Once we have all the bits of  $S_0$ , by running the inverse KSA routine one can recover the secret key.

First we need to hit each of the locations of the LFSR. We inject the fault randomly in the LFSR locations and thus, we need  $\tau = 128 \cdot \sum_{i=1}^{128} \frac{1}{i} \approx 695.4$  expected number of fault injections. For each of these injected faults, we need to identify the fault locations. Taking the value of  $\mu$  from Section 2.1 that is the required number of expected faults for each LFSR location, the total number of faults to be injected =  $\tau \mu = 695.4 \cdot 2.62 \approx 1822$ . Additionally, as described in Section 2.3, 9 more fault injections are required for the locations  $\phi = 73, 75, \ldots, 89$  to recover certain NFSR bits. Therefore, the expected number of faults that our attack needs is  $1822 + 9 = 1831 < 2^{11}$ .

For each fault during the location identification stage, two MAC invocations are required, that amounts to  $1822 \cdot 2 = 3644$ . Additionally, 20 more invocations are required during some cases of NFSR bit recovery. Thus the total number of invocations is less than  $2^{12}$ .

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