A Reflection on the Security of Two-Party Key Establishment Protocols

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Abstract. Two-party key establishment has been a very fruitful research area in cryptography, with many security models and numerous protocols proposed. In this paper, we take another look at the YAK protocol and the HMQV protocols and present some extended analysis. Motivated by our analysis, we reflect on the security properties that are desired by two-party key establishment protocols, and their formalizations. In particular, we take into account the interface between a key establishment protocol and the applications which may invoke it, and emphasize the concept of *session* and the usage of *session identifier*. Moreover, we show how to design a two-party key establishment protocol to achieve both key authentication and entity authentication properties in our security model.

Keywords: key establishment, key authentication, entity authentication

1 Introduction

The history of two-party key establishment goes back a long way, although the modern study of key establishment protocols can be traced back to the seminal work of Needham and Schroeder [24]. Particularly, since the seminal work of Diffie and Hellman [11], key establishment has been a very fruitful area in cryptography. So far, numerous protocols have been proposed, as surveyed in Chapter 3 of Tang's PhD thesis [27] and the book by Boyd and Mathuria [5]. The standardization bodies, such as ISO and IEEE, have also created a number of key establishment standards [15,16,17,18,19]. Among these protocols, a large proportion are based on the concept of Diffie-Hellman key exchange [11].

Before the pioneering work by Bellare and Rogaway [3], who first analysed key establishment protocols using complexity-theoretic methods, the security of key establishment protocols was typically evaluated using only heuristic techniques. As a result, many protocols have been proposed which contain subtle vulnerabilities, only discovered after the schemes were published. The Bellare-Rogaway model [3] was designed to enable the analysis of entity authentication and two-party key establishment in the shared secret key setting. Subsequently, a number of variants of this model have been proposed. Blake-Wilson, Johnson, and Menezes [4] extended the model to the public key setting. Bellare, Canetti, and Krawczyk [2] provided a modular approach for the construction of authenticated key establishment protocols. They also proposed the first simulatability-based security model for key establishment. Shoup [25] refined the simulatability-based security model and proposed a model which works under three different corruption assumptions. Later, Bellare, Canetti, and Krawczyk [2,6] further extended the concept, and proposed another security model for two-party key establishment protocols in the universally composable security framework. Hitchcock, Boyd, and Nieto [14] optimised the Bellare-Canetti-Krawczyk model. A number of papers have been devoted to discussing the validation of, and relationships between, these various security models, e.g. [7,8,9].

1.1 Motivation and Contribution

In the literature, there have been a lot attacks on two-party key establishment protocols. Some of them are feasible because the target protocols lack rigorous security analysis, while others are feasible because they fall beyond the security model or are carried out in some unperceived execution scenarios. Motivated by some recent attacks against the provably secure HMQV protocols [13], we consider it to be an interesting task to reflect on the security of two-party key establishment protocols.

In this paper, we revisit two-party key establishment protocols, which exploit asymmetric key cryptography for security guarantees. This category of two-party protocols is of interest to us, because it provides better scalability than the category of protocols using symmetric key cryptography for security guarantees while provides stronger security than the category of protocols using passwords for security guarantees. Specifically, our contribution can be summarized as follows.

1. First, we take another look at the YAK protocol proposed by Feng [13] and HMQV protocols [20,21] and present some extended analysis. In particularly, we show that, contradicting Feng's claim, the YAK protocol does not achieve forward secrecy. The YAK protocol and the HMQV protocols do not achieve unknown key share resilience in some execution scenarios. In addition, we show that the 3-pass HMQV protocol is vulnerable to a key compromise impersonation attack, when the adversary has access to a user's long-term and short-term private keys.
2. Second, we re-examine two-party key establishment protocols from a number of aspects: the workflow, the involved secrets, the interface with invoking applications. In particular, we emphasize that the invoking application should provide application identifier to the key establishment protocol, which should return a (session identifier, session key) pair. To our knowledge, such extended examination has not been done in the literature of two-party key establishment protocols.
3. Third, we categorize the security properties into two categories, namely key authentication and entity authentication, and formalize them accordingly. The key authentication property guarantees that only the intended user is capable of computing a session key, while the entity authentication property guarantees that the intended user has actually involved in a session. Both properties are crucial for enabling a security service by running a key establishment protocol. Most existing attack scenarios have been covered in our security model.

4. Last, we present a two-party key establishment protocol which achieves both key authentication and entity authentication properties in our security model.

Arguably, by requiring the underlying applications to adopt some extra security mechanisms, a protocol proven secure in any existing security model could possibly achieve the same level of security in practice as a protocol proven secure in our security model. However, we believe it is important to use a comprehensive security model without relying on the underlying applications.

1.2 Organization

The rest of the paper is organised as follows. In Section 2, we analyse the YAK protocol and the HMQV protocols. In Section 3, we look at different aspects of two-party key establishment protocols. In Section 4, we categorize the security properties for two-party key establishment protocols and formalize them. In Section 5, we present a two-party key establishment protocol which is secure in our security model. In Section 6, we conclude the paper.

2 Review of YAK and HMQV

Throughout the paper, we use $x \in_R X$ to denote that x is chosen from the set X uniformly at random, and use $x||y$ to denote the concatenation of x and y.

2.1 Review of the YAK Protocol

The YAK protocol, proposed by Feng [13], is a two-party key establishment protocol. Note that this protocol has not been rigorously analysed in a security model. Let Alice and Bob be two users who trust TTP in common. The algorithms are defined as follows.

1. System Setup: Alice and Bob agree on a group $\mathbb{G}$ of prime order q, a generator g of $\mathbb{G}$, and a hash function H. Alice selects $a \in_R \mathbb{Z}_q$ as her private key. Alice sends g^a to the TTP with a proof about her knowledge on a, and receives a certificate on her public key g^a from the TTP if the proof is valid. The proof of knowledge is a non-interactive protocol described in [23], defined as follows.
   (a) Alice sends $X = g^a$ and $\{\text{Alice}, \text{OtherInfo}, V = g^v, r = v - a \cdot h\}$ to the TTP, where $v \in_R \mathbb{Z}_q$, $h = \mathsf{H}(g, V, X, \text{“Alice”}, \text{OtherInfo})$, and OtherInfo includes some auxiliary information.
   (b) The TTP checks X has the prime order q and $V = g^r X^h$.

   In a similar way, Bob generates his private key $b \in_R \mathbb{Z}_q$ and obtains a certificate on his public key g^b from the TTP.

2. Key establishment: The key establishment protocol is as follows.
   (a) Alice selects $x \in_R \mathbb{Z}_q$, and sends g^x and prf_1 to Bob. The prf_1 is a proof of knowledge of x, where

$$prf_1 = \{\text{“Alice”}, \mathit{OtherInfo}_1, V_1 = g^{v_1}, r_1 = v_1 - x \cdot h_1\},$$

$$v_1 \in_R \mathbb{Z}_q,\;\; h_1 = \mathsf{H}(g, V_1, g^x, \text{“Alice”}, \mathit{OtherInfo}_1).$$

   In the meantime, Bob selects $y \in_R \mathbb{Z}_q$, and sends g^y and prf_2 to Alice. The prf_2 is a proof of knowledge of y, where

$$prf_2 = \{\text{“Bob”}, \mathit{OtherInfo}_2, V_2 = g^{v_2}, r_2 = v_2 - y \cdot h_2\},$$

$$v_2 \in_R \mathbb{Z}_q,\;\; h_2 = \mathsf{H}(g, V_2, g^y, \text{“Bob”}, \mathit{OtherInfo}_2).$$

   $\mathit{OtherInfo}_1$ and $\mathit{OtherInfo}_2$ include some auxiliary information.
   (b) If Alice accepts Bob's proof, she computes the session key as $\mathsf{H}((g^y \cdot g^b)^{a+x})$. Similarly, if Bob accepts Alice's proof, he computes the session key as $\mathsf{H}((g^x \cdot g^a)^{b+y})$.

Vulnerability against forward secrecy. From the description, we note that the key establishment protocol is one round and the proof of knowledge of ephemeral private key is non-interactive. In addition, Bob is not required by Alice to prove his knowledge of the long-term private key b. This results in the fact that, in a session, an adversary can select $r \in_R \mathbb{Z}_q$, and send g^r and prf_2 to Alice. The prf_2 is a proof of knowledge of r, where

$$prf_2 = \{\text{“Bob”}, \text{OtherInfo}, V_2 = g^{v_2}, r_2 = v_2 - r \cdot h_2\},$$

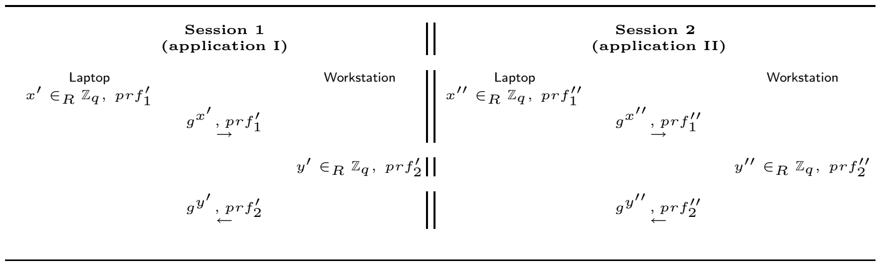
$$v_2 \in_R \mathbb{Z}_q,\;\; h_2 = \mathsf{H}(g, V_2, g^r, \text{“Bob”}, \mathit{OtherInfo}_2).$$

Clearly, Alice will accept the adversary's message and generate a session key $K = \mathsf{H}((g^r \cdot g^b)^{a+x})$. Consequently, we have the following attack scenario.

Suppose Alice has initiated the key agreement process to generate a session key to send Bob some data M. Then, after the key establishment process successfully finishes, Alice may just send $\mathsf{Encrypt}(M, K)$ *to Bob and end the session afterwards, where* $\mathsf{Encrypt}$ *is a symmetric key encryption algorithm. Later on, at any time, if the adversary can corrupt Bob's long-term private key, it can generate the same session key K and recover M.*

In the above attack scenario, Alice has infact established a session key with the adversary, instead of with Bob who has actually not been involved at all in the key establishment process. Moreover, Bob is not able to compute K since he has no knowledge about the ephemeral secret r. This attack violates the forward secrecy property, as defined in Section 4. In fact, this attack coincides with Krawcyzk's statement [20,21] that *2-pass key establishment protocol authenticated via public keys and with no secure shared state previously established between the parties will be vulnerable to the attacks against forward secrecy.*

Vulnerability against unknown key share resilience. Suppose Alice has two devices, say Laptop and Workstation, and two applications which result in communications between these two devices. Consider a scenario where the applications run the YAK protocol to establish session keys to protect the communications simultaneously. In the absence of attacks, the protocol executions will perform as in Fig. 1.



Session 1 (application I)
Session 2 (application II)
Laptop
Workstation
$x' \in_R \mathbb{Z}_q,\ prf'_1$
$g^{x'}, prf'_1$
$y' \in_R \mathbb{Z}_q,\ prf'_2$
$g^{y'}, prf'_2$
Laptop
Workstation
$x'' \in_R \mathbb{Z}_q,\ prf''_1$
$g^{x''}, prf''_1$
$y'' \in_R \mathbb{Z}_q,\ prf''_2$
$g^{y''}, prf''_2$

Fig. 1. Concurrent Sessions of YAK

In more details, the four knowledge proofs, namely prf'_i $(1 \leq i \leq 2)$ and prf''_i $(1 \leq i \leq 2)$ are defined as follows.

- $prf'_1 = \{\mathsf{Laptop}, OtherInfo'_1, V'_1 = g^{v'_1}, r'_1 = v'_1 - x' \cdot h'_1\}$, where $v'_1 \in_R \mathbb{Z}_q,\ h'_1 = \mathsf{H}(g, V'_1, g^{x'}, \mathsf{Laptop}, OtherInfo'_1)$.
- $prf'_2 = \{\mathsf{Workstation}, OtherInfo'_2, V'_2 = g^{v'_2}, r'_2 = v'_2 - y' \cdot h'_2\}$, where $v'_2 \in_R \mathbb{Z}_q,\ h'_2 = \mathsf{H}(g, V'_2, g^{y'}, \mathsf{Workstation}, OtherInfo'_2)$.
- $prf''_1 = \{\mathsf{Laptop}, OtherInfo''_1, V''_1 = g^{v''_1}, r''_1 = v''_1 - x'' \cdot h''_1\}$, where $v''_1 \in_R \mathbb{Z}_q,\ h''_1 = \mathsf{H}(g, V''_1, g^{x''}, \mathsf{Laptop}, OtherInfo''_1)$.
- $prf''_2 = \{\mathsf{Workstation}, OtherInfo''_2, V''_2 = g^{v''_2}, r''_2 = v''_2 - y'' \cdot h''_2\}$, where $v''_2 \in_R \mathbb{Z}_q,\ h''_2 = \mathsf{H}(g, V''_2, g^{y''}, \mathsf{Workstation}, OtherInfo''_2)$.

With respect to the above descriptions, we note that Laptop's proofs, namely prf'_1 and prf''_1, are generated based on the same identity Laptop. Similarly, we note that Workstation's proofs, namely prf'_2 and prf''_2, are generated based on the same identity Workstation. Therefore, an adversary can swap messages in the two concurrent sessions and mount an *unknown key share attack*. The attack is shown in Fig. 2.

It is straightforward to verify that Laptop's session 1 and Workstation's session 2 will compute the same session key, namely $\mathsf{H}((g^{x'} \cdot g^a)^{b+y''})$. While, Laptop's session 2 and Workstation's session 1 will compute the same session key, namely $\mathsf{H}((g^{x''} \cdot g^a)^{b+y'})$.

The consequence of the attack is that the adversary can swap messages in the two applications without being noticed. For example, if in application I, Laptop sends $\mathsf{Encrypt}(M, K)$ to Workstation where Encrypt is a symmetric key encryption algorithm, then the adversary can forward it to Workstation in application II which will successfully decrypt the ciphertext. As a result, messages have been delivered to the unintended recipient applications, and this may cause subtle attacks depending on the applications.

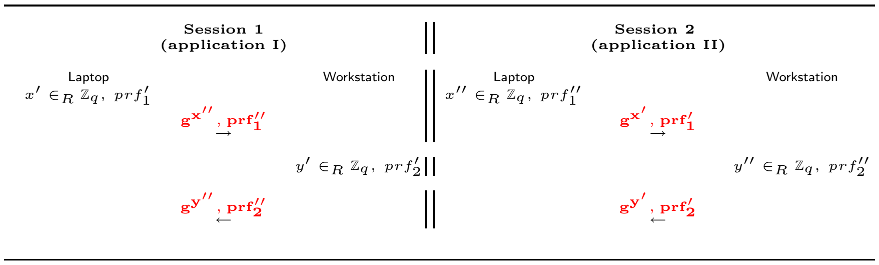

Session 1
(application I)
Laptop
Workstation
$x' \in_R \mathbb{Z}_q,\ prf'_1$
$\mathbf{g^{x''}}, \mathbf{prf''_1}$
$y' \in_R \mathbb{Z}_q,\ prf'_2$
$\mathbf{g^{y''}}, \mathbf{prf''_2}$
Session 2
(application II)
Laptop
Workstation
$x'' \in_R \mathbb{Z}_q,\ prf''_1$
$\mathbf{g^{x'}}, \mathbf{prf'_1}$
$y'' \in_R \mathbb{Z}_q,\ prf''_2$
$\mathbf{g^{y'}}, \mathbf{prf'_2}$

Fig. 2. *unknown key share attack* against YAK

2.2 Review of the HMQV Protocols

The HMQV family consists of a number of protocols, and we describe the 2-pass and 3-pass protocols below. Let Alice and Bob be two users who trust TTP in common. Alice and Bob agree on a group $\mathbb{G}$ of prime order q, a generator g of $\mathbb{G}$, and a hash function H. Alice selects her long-term private key $a \in_R \mathbb{Z}_q$ and lets the TTP certify the public key g^a, and Bob selects his long-term private key $b \in_R \mathbb{Z}_q$ and lets the TTP certify the public key g^b. The protocols are shown in Fig. 3.

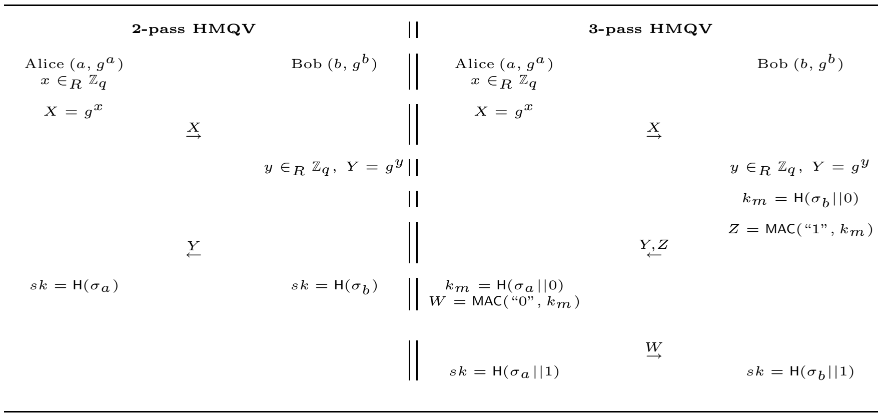

2-pass HMQV
Alice (a, g^a)
Bob (b, g^b)
$x \in_R \mathbb{Z}_q$
$X = g^x$
$\xrightarrow{X}$
$y \in_R \mathbb{Z}_q,\ Y = g^y$
$\xleftarrow{Y}$
$sk = \mathsf{H}(\sigma_a)$
$sk = \mathsf{H}(\sigma_b)$
3-pass HMQV
Alice (a, g^a)
Bob (b, g^b)
$x \in_R \mathbb{Z}_q$
$X = g^x$
$\xrightarrow{X}$
$y \in_R \mathbb{Z}_q,\ Y = g^y$
$k_m = \mathsf{H}(\sigma_b\|0)$
$Z = \mathsf{MAC}(\text{“1”}, k_m)$
$\xleftarrow{Y,Z}$
$k_m = \mathsf{H}(\sigma_a\|0)$
$W = \mathsf{MAC}(\text{“0”}, k_m)$
$\xrightarrow{W}$
$sk = \mathsf{H}(\sigma_a\|1)$
$sk = \mathsf{H}(\sigma_b\|1)$

Fig. 3. The HMQV Protocols

The value σ_a and σ_b are defined as follows.

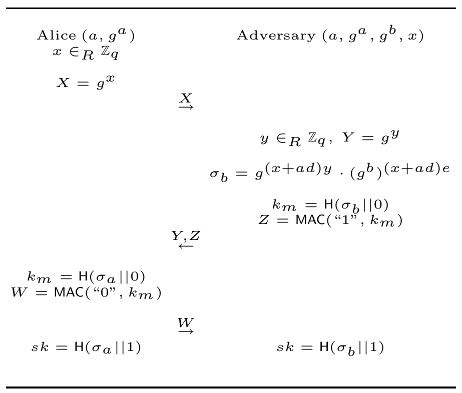
$$d = \bar{\mathsf{H}}(X\|\text{“}Bob\text{”}), e = \bar{\mathsf{H}}(Y\|\text{“}Alice\text{”}), \sigma_a = (Yg^{be})^{x+da}, \sigma_b = (Xg^{ad})^{y+eb},$$

where $\bar{\mathsf{H}}$ outputs the first ℓ bit of the input of the hash function H given that the security parameter is ℓ. It is straightforward to verify that

$$\sigma_a = \sigma_b = g^{(x+ad)(y+be)}.$$

Menezes [22] showed that both protocols might be vulnerable to so called “small subgroup attacks” and “lattice attacks”, through which Alice, if being malicious, could obtain Bob's long-term private key b. These attacks have exploited the fact that, in the protocol specification, it is not mandatory for Alice and Bob to verify that both long-term and short-term keys are generated from the right subgroup. Feng [13] showed that the 2-pass protocol is vulnerable to unknown share attacks if the same user executes the protocol between different devices. Furthermore, we have the following observations.

1. We show a key compromise impersonation attack in which, if the adversary knows Alice's long-term and ephemeral private keys a and x, it can impersonate Bob to Alice. We skip the details since it is straightforward to verify that the attack works.



Alice (a, g^a)
$x \in_R \mathbb{Z}_q$
$X = g^x$
Adversary (a, g^a, g^b, x)
$\xrightarrow{X}$
$y \in_R \mathbb{Z}_q,\ Y = g^y$
$\sigma_b = g^{(x+ad)y} \cdot (g^b)^{(x+ad)e}$
$k_m = \mathsf{H}(\sigma_b || 0)$
$Z = \mathsf{MAC}(\text{“1”}, k_m)$
$\xleftarrow{Y,Z}$
$k_m = \mathsf{H}(\sigma_a || 0)$
$W = \mathsf{MAC}(\text{“0”}, k_m)$
$\xrightarrow{W}$
$sk = \mathsf{H}(\sigma_a || 1)$
$sk = \mathsf{H}(\sigma_b || 1)$

Fig. 4. Attack against the 3-pass HMQV Protocol

2. Both protocols are vulnerable to the type of unknown key share attacks shown in the case of the YAK protocol. Nonetheless, we note that this kind of attack is beyond the security model used in [20,21].
3. Feng [13] presented a so called "invalid public key attack". To mount the attack, Bob lets the TTP certify a long-term public key without generating any private key. Due to the deliberate selection of his public key, Bob can still successfully establish a session key with Alice. Since the public key is weak, any entity can impersonate Bob to Alice so that it causes authentication and repudiation "problems". However, this kinds of problems apply to all protocols, if a user does not follow the protocol specification and chooses weak keys on purpose.
4. An execution of the 3-pass protocol requires 3 sequential message exchanges: Bob can only send his first message after he receives Alice's first message, Alice can only send her second message after she receives Bob's first message, and Bob can only end the execution after he receives Alice's second message. The process can be improved by turning it into a 4-pass protocol which requires only 2 sequential message exchanges. To achieve this, Alice and Bob first simultaneously send X and Y to each other, then simultaneously send Z and W to each other.

3 Overview of Two-Party Key Establishment

3.1 The workflow

A two-party key establishment protocol generally involves two types of entities.

– One type is users, any two of which may run the key establishment protocol to establish a session key with each other. Throughout the paper, we assume that the users that may run a key establishment protocol are denoted as U_i $(1 \leq i \leq N)$, where N is the maximum of user population. Furthermore, we assume that U_i possesses the identity ID_i. If $i \neq j$, U_i and U_j are different users.
– The other type of entity is a Trusted Third Party (TTP) which help users validate their long-term public keys, usually of some public-key encryption or digital signature schemes.

Generally, we consider the following execution scenarios for a two-party key establishment protocol.

1. Two users, say U_i and U_j $(i \neq j)$, may use the same protocol and the same long-term key to negotiate session keys for many applications. The applications may reside in different devices of U_i and U_j. For example, they may run the protocol to between their laptops to protect their chatting contents and between their work PCs to protect their work documents.
2. The same user, say U_i, may have multiple devices, any two of which use the same protocol and the same long-term key to negotiate session keys for for many applications. For example, Alice may have separate applications to remotely transfer medical files and sensitive work documents from her laptop to her PC in her company.
3. For each application, the protocol may be executed multiple times to establish cryptographic keys. We denote each execution of the protocol to be a session. We further assume that multiple sessions can be initiated simultaneously.

We use $\mathcal{S}$ to denote the application identifier set which contains the application identifiers of all users.

In our setting, a *session* denotes an execution of the key establishment protocol, invoked by an application of two users. Informally, a session is associated with a set $\{\mathsf{ID}_i, \mathsf{ID}_j, app_{id}\}$ and some execution-specific information (such as the message exchange transcript) which distinguish different protocol executions invoked by the same application of the two users. Formally, we use a *session identifier* to identify a session, and formally define its property under Definition 1 in Section 4. This concept is important not only for theoretically formalizing the security of a key establishment protocol, but also for applications to appropriately use the session keys.

Remark 1. In fact, the concept of *session* has been used in the existing models, e.g. [3,6]. However, it has not been advocated that a session identifier should be outputted to the invoking application, together with the session key, e.g. in [3]. In [6], it assumed that a session identifier should be supplied by the invoking application as an input, however, this will require the two participants of an application to negotiate an identifier in advance securely. Arguably, this is an undesirable assumption.

$$\begin{array}{ccc}
\mathsf{U}_i\ (PK_i, SK_i; \mathsf{ID}_j, PK_j, app_{id}) & & \mathsf{U}_j\ (PK_j, SK_j; \mathsf{ID}_i, PK_i, app_{id}) \\
& \xrightarrow{m_1} & \\
& \xleftarrow{m_2} & \\
& \cdots & \\
& \xrightarrow{m_{x-1}} & \\
& \xleftarrow{m_x} & \\
(sid, sk) & & (sid, sk)
\end{array}$$

Fig. 5. Workflow of a Two-party Key Establishment Protocol

The workflow of a two-party key establishment protocol follows the style shown in Fig. 5. The inclusion of identifiers ID_i and ID_j in the input to the key establishment protocol states that the users know their intended partners, and this is always implicitly assumed in other works. Here, we have two new features.

1. One is that the key establishment protocol takes an application identifier app_{id} as input. This reflects our consideration that the protocol can be invoked by multiple applications from the same user. We require that, for a user, its different applications hold different identifiers. Furthermore, if an application is between U_i and U_j (say, transfering friles between these users), then these two users hold the same identifier for the application.
2. The key establishment protocol returns a (session identifier, session key) pair, instead of a session key only as in most existing protocols.

These features are used to distinguish the key establishment sessions invoked by different applications, and subsequently they enable us to avoid the type of unknown key share attacks shown in Section 2.

Remark 2. About the protocol execution, we also make the following assumptions. Both U_i and U_j know how to validate the public keys of each other. At the beginning of a session, it may be required that U_i sends m_1 to U_j first before U_j sends m_2 back, namely there is an order of message exchange. In such case, we say U_i is the initiator while U_i is the responder. If there is no such order requirement, we simply say both users are initiator.

3.2 The long-term and ephemeral private keys

In a session of a two-party key establishment protocol, each participant makes use of two types of secrets.

1. One is its long-term private key[1], which guarantees the integrity and/or confidentiality of messages sent to the other participant. In some protocols, the long-term private key is also used to derive the session key. Without using a long-term private key, a two-party key establishment protocol will suffer from a man-in-the-middle attack, for example in the case of Diffie-Hellman [11].
2. The other is the ephemeral private key, which is the randomness locally generated by a random number generator or a pseudo-random number generator. The ephemeral private key is mainly used to derive a fresh session key, and it makes the session keys unique.

In practice, both types of private keys could be compromised. For the long-term private key, the user must store it somewhere and may also make some backups. Therefore, an adversary may obtain the long-term private key if it gains access to any one of the storage devices. With respect to the ephemeral private key, since it is generated locally by a computing device (normally a computer), an adversary may gain access to it by a number of means. For example, the adversary could install some trojan horse in the participant's computing device to steal the secrets, or it could make use of the imperfectness of the pseudo-random number generator in use.

In practice, a user may attempt to use its long-term private key for other purposes. For example, a user may use the signature key pair (assuming such a key pair is used in the key establishment protocol) to authenticate itself to a website. This kind of extended usage of the same private key is extremely risky. To demonstrate our point, we present a toy example of a key establishment protocol in Fig. 6 and a toy authentication protocol in Fig. 7, where r is a random string, (pk_2, sk_2) is a sign/verify key pair of U_2.

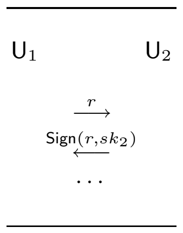


Fig. 6. Key Establishment Protocol

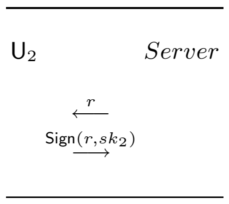


Fig. 7. Authentication Protocol

Since U_1 can make U_2 sign a nonce in the key establishment protocol, so that U_1 can impersonate U_2 in the authentication protocol trivially. We emphasize that

[1] The same long-term private key is used in all executions of the protocol.

extended usage of the same private key should be avoided in practice. To formulate the security properties, we assume the user's long-term private key is only used in executing a two-party key establishment protocol.

4 Formalization of Security Properties

In this section, we formalize the security properties desired by a two-party key establishment protocol. Further remarks are provided in the Appendix A.

4.1 The threat model

With respect to the relevant trust relationships in a two-party key establishment protocol, we make the following assumptions.

1. If a TTP is required, it is semi-trusted in the following sense. The TTP will honestly follow the protocol specification. For example, it will honestly certify users' public keys, but will not generate a forged certificate for either an existing identity or a new identity (which is different from ID_i $(1 \leq i \leq N)$).
2. Any user, say U_i, is semi-trusted by any other user U_j $(j \neq i)$ in the following sense.
   (a) Suppose U_j is engaged in a session with U_i, it may be possible that U_i will deviate from the protocol specification in order to obtain more information about U_j's long-term and short-term private keys. For example, U_i may try to impersonate U_j to another user in another session, by taking advantage of the harvested information.
   (b) Suppose U_j is engaged in a session with U_i, besides the possible malicious activities described in (a), U_i will do nothing else malicious. For example, U_i will not disclose the shared session key with U_j to any other entity.
   (c) Suppose U_j is engaged in a session with U_k, U_i is treated as a malicious adversary.

Since a key establishment protocol is normally used in a distributed environment, namely the two participants communicate through an open network. Therefore, it is essential to assume the adversary to be an active one. In other words, the adversary can not only eavesdrop on the communication network but also manipulate (delete, insert, replace, delay, etc) the messages at its will.

As a basic requirement, a two-party key establishment protocol should always be sound. Formally the soundness property is defined as follows.

Definition 1. *A two-party key agreement protocol is sound if the following requirements are satisfied.*

1. *In the absence of an adversary, the two users obtain the same (session identifier, session key) pair at end of a session.*
2. *For any two sessions of* U_i*, carried out with* U_j *for an application* app_{id}*, the probability that the session identifiers are identical is negligible, regardless of the coin flips of* U_j*.*
3. *Suppose that* U_i *successfully ended a session with* U_i *for the application* app_{id} *and* U_k *successfully ended a session with* U_l *for the application* app'_{id}*. Given that the sets* $\{\mathsf{ID}_i, \mathsf{ID}_j, app_{id}\}$ *and* $\{\mathsf{ID}_k, \mathsf{ID}_l, app'_{id}\}$ *are not identical, the probability that the session identifier of* ID_i *and the session identifier of* ID_k *are identical is negligible, regardless of the coin flips of all users.*

In the literature, many properties have been identified for key agreement protocols, and a summary of them can be found in the work by Menezes, Oorschot, and Vanstone [23], Boyd and Mathuria [5], and Tang [27]. In this paper, we categarize the desirable properties for a key establishment protocol into two categories, namely key authentication and entity authentication.

4.2 The Key Authentication Property

The *Key authentication* property[2] states that a user U_i has completed a successful protocol execution in one session (let it be denoted with session identifier sid^*) with U_j, it can be assured that only U_j is capable of compute the same session key. It is worth noting that this property does not guarantee that U_j actually possess the same session key. In fact, it does not even guarantee that U_j actually knows there is such a key establishment session.

When analysing the key authentication property, in the above threat model, we need to assume an active adversary which can access to the long-term private keys of all users except U_i and U_j. In fact, we often need to assume a more powerful adversary in the following attack scenarios.

1. *known session key attack*: In this case, the adversary is assumed to be able to compromise the session keys in all sessions except that identified by sid^*. The concept was first mentioned by Denning and Sacco [10] in a limited way, in which the adversary can compromise all past session keys.
2. *forward secrecy attack*: In this case, the adversary is assumed to be able to compromise the long-term private keys of U_i or U_j after the session with identifier sid^* has ended. If both private keys are compromised by the adversary, it is said to be a *perfect forward secrecy attack*.
3. *known master key attack*: In this case, the adversary is assumed to be able to compromise TTP and obtain its long-term private keys. In addition, if the TTP stores also other secret information then the adversary can obtain it as well.
4. *unknown key-share attack*: The notion of unknown key-share attacks were first discussed by Diffie, van Oorschot and Wiener [12]. It is defined to be the property that an honest user U_i never ends up believing it shares a key with user U_j,

[2] It is also referred to as the key secrecy property in the literature.

although it actually shares the session key with another user U_u $(u \neq j)$. We further require that one application should not share the same the session key with another application when the protocol is run by the two applications of the same pair of users.

5. *key-compromise impersonation attack*: This attack makes sense only when $i \neq j$, namely the two users in the session are different. In this case, the adversary is assumed to be able to compromise the long-term private key of U_i. Note that this attack scenario is also considered in evaluating the entity authentication property.

Formally, the key authentication property is evaluated by the attack game between a challenger and an adversary, as shown in Fig. 8, where the adversary's advantage is defined to be $|\Pr[b = b'] - \frac{1}{2}|$. For the simplicity of description, we use the notations U_i for some $1 \leq i \leq N$ in a few places, although the challenger will actually simulate all these activities.

Definition 2. *A two-party key establishment protocol achieves the key authentication property, if any polynomial-time adversary has only negligible advantage in the key authentication game.*

4.3 The Entity Authentication Property

The *entity authentication* property states that, if from the view of a user U_j it has successfully completed a key establishment session (identified by the session identifier sid^*) with U_j, then U_j should have actually engaged in a key establishment session (identified by the session identifier sid^*) with U_i. Typically, we need to assume an active adversary which can access to the long-term private keys of all users except U_i and U_j. Similar to the situation in analysing the key authentication property, we often also consider some extended attack scenarios.

1. *key-compromise impersonation attack*: This attack makes sense only when $i \neq j$, namely the two users in the session are different. In this case, the adversary is assumed to be able to compromise the long-term private key of U_i. In addition, we also allow the adversary to compromise all the ephemeral secrets of U_i.
2. *key confirmation*: In this case, if, from the view of a user U_i, it has successfully completed a key establishment session (identified by the session identifier sid^*) with U_j, then U_j should have engaged in a key establishment session (identified by the session identifier sid^*) with U_i and has computed the same session key as that of U_i.

Formally, the entity authentication property is evaluated by the attack game between a challenger and an adversary, as shown in Fig. 9, where the advantage is defined to be the probability that the user U_v does not have a session with the identifier sid^* and session key sk^*. For the simplicity of description, we use the notations U_i for some $1 \leq i \leq N$ in a few places, although the challenger will actually simulate all these activities.

1. Setup: the challenger generates the parameters for the TTP and publishes the public parameter.
2. Phase 1: Besides delivering messages for all sessions, the adversary is allowed to issue the following types of queries for any $1 \leq i, j \leq N$.
   (a) $\mathsf{Register}_h(\mathsf{U}_i)$: The user U_i selects its identity ID_i, generates a key pair (PK_i, SK_i), and let the challenger certify PK_i.
   (b) $\mathsf{Register}_m(\mathsf{U}_i)$: The adversary selects an identity ID_i, generates a key pair (PK_i, SK_i), and let the challenger certify PK_i.
   (c) $\mathsf{Invoke}(\mathsf{U}_i, \mathsf{U}_j, app_{id}, \mathsf{role})$: The user U_i initiates a new session with U_j for an application identified by $app_{id} \in \mathcal{S}$. We require that both users have registered their public keys. The session is locally indexed by
   $$(L_i, \mathsf{ID}_i, \mathsf{ID}_j, app_{id}),$$
   where the value L_i means that it is $\mathsf{U}_i's$ L_i-th session. In addition, U_i does the following
      – If $\mathsf{role} = \mathsf{initiator}$, U_i starts sending a message to U_j by following the protocol specification.
      – If $\mathsf{role} = \mathsf{responder}$, U_i waits for U_j's message by following the protocol specification.
   (d) $\mathsf{Corrupt_e}(L_i, \mathsf{ID}_i)$: The user U_i sends the ephemeral secrets of its L_i-th session to the adversary.
   (e) $\mathsf{Corrupt_l}(\mathsf{ID}_i)$: The user U_i sends its long-term private key SK_i to the adversary.
   (f) $\mathsf{Corrupt_k}(L_i, \mathsf{ID}_i)$: If the $\mathsf{U}_i's$ L_i-th session has successfully ended, which means a (sid, sk) pair should have been generated, the user U_i sends sk to the adversary. Otherwise, U_i returns nothing.
   (g) $\mathsf{Corrupt_t}(\mathsf{TTP})$: The challenger returns the private information of the TTP to the adversary.

   At some point, the adversary chooses $1 \leq u \leq N$ and challenges U_u's L_u^*-th session, which has successfully ended with (sid^*, sk^*). Suppose the session has been initiated to negotiate a session key with U_v for an application identified by $app_{id}^* \in \mathcal{S}$. This is subjected to the following restrictions.
   – The public keys PK_u and PK_v have been certified through $\mathsf{Register}_h$ queries.
   – There has been no $\mathsf{Corrupt_l}(\mathsf{ID}_v)$ query and no $\mathsf{Corrupt_l}(\mathsf{ID}_u)$ query if $u = v$. There has been no $\mathsf{Corrupt_t}$ query before U_i and U_j register their public keys.
   – There has been neither $\mathsf{Corrupt_e}(L_u^*, \mathsf{ID}_u)$ nor $\mathsf{Corrupt_k}(L_u^*, \mathsf{ID}_u)$ queries.
   – There has been neither $\mathsf{Corrupt_e}(L_v^*, \mathsf{ID}_v)$ nor $\mathsf{Corrupt_k}(L_v^*, \mathsf{ID}_v)$ query, where U_v's L_v^*-th session possesses the session identifier sid^*.
3. Challenge: Select $b \in_R \{0, 1\}$. If $b = 0$, send sk^* to the adversary, otherwise send $r \in_R \mathcal{K}$ to the adversary where $\mathcal{K}$ is the session key domain.
4. Phase 2: The adversary is allowed to issue the same types of queries as in Phase 1, with the following restrictions.
   – There has been neither $\mathsf{Corrupt_e}(L_u^*, \mathsf{ID}_u)$ nor $\mathsf{Corrupt_k}(L_u^*, \mathsf{ID}_u)$ queries.
   – There has been neither $\mathsf{Corrupt_e}(L_v^*, \mathsf{ID}_v)$ nor $\mathsf{Corrupt_k}(L_v^*, \mathsf{ID}_v)$ query, where U_v's L_v^*-th session possesses the session identifier sid^*.

   At some point, the adversary terminates by outputting a guess bit b'.

Fig. 8. The Key Authentication Game

1. Setup: the challenger generates the parameters for the TTP and publishes the public parameter.
2. Challenge: The adversary is allowed to issue the following type of queries: $\mathsf{Register}_h$, $\mathsf{Register}_m$, Invoke, $\mathsf{Corrupt_e}$, $\mathsf{Corrupt_l}$, and $\mathsf{Corrupt_k}$. The queries are answered in the same way as in the key authentication game, as shown in Fig. 8. At some point, the adversary chooses $1 \leq u \leq N$ and challenges U_u's L_u^*-th session, which has successfully ended with (sid^*, sk^*). Suppose the session has been initiated to negotiate a session key with U_v for an application identified by $app_{id}^* \in \mathcal{S}$. This is subjected to the following restrictions.
   – The public keys PK_u and PK_v have been certified through $\mathsf{Register}_h$ queries.
   – There has been no $\mathsf{Corrupt_l}(\mathsf{ID}_v)$ query and no $\mathsf{Corrupt_l}(\mathsf{ID}_u)$ query if $u = v$. There has been no $\mathsf{Corrupt_t}$ query before U_i and U_j register their public keys.

Fig. 9. The Entity Authentication Game

Definition 3. *A two-party key establishment protocol achieves the entity authentication property, if any polynomial-time adversary has only negligible advantage in the entity authentication game.*

5 A Secure Two-Party Key Establishment Protocol

5.1 The Proposed Protocol

We assume the users U_i $(1 \leq i \leq N)$ agree on a group $\mathbb{G}$ of prime order q, a generator g of $\mathbb{G}$, and a hash function $\mathsf{H} : \{0,1\}^* \to \{0,1\}^L$ where L is a polynomial in the security parameter ℓ. We assume $\mathcal{K} = \{0,1\}^L$ is the session key domain. Every user U_i generates a key pair (PK_i, SK_i) for a digital signature scheme $(\mathsf{KeyGen}, \mathsf{Sign}, \mathsf{Verify})$ which is existentially unforgeable under an adaptive chosen message attack [22], and lets the TTP certify PK_i. The application identifier set is $\mathcal{S} = \{0,1\}^*$. If U_i and U_j want to establish a session key, they follow the protocol shown in Fig. 10.

$$
\begin{array}{ccc}
\mathsf{U}_i\ (PK_i, SK_i, \mathsf{ID}_j, PK_j, app_{id}) & & \mathsf{U}_j\ (PK_j, SK_j, \mathsf{ID}_i, PK_i, app_{id}) \\
x \in_R \mathbb{Z}_q,\ X = g^x & & y \in_R \mathbb{Z}_q,\ Y = g^y \\
 & \xrightarrow{X} & \\
 & \xleftarrow{Y} & \\
W = PK_i||\mathsf{ID}_i||PK_j||\mathsf{ID}_j||app_{id}||X||Y & & \\
\sigma_1 = \mathsf{Sign}(0||W||\mathsf{H}(Y^x), SK_i) & & \sigma_2 = \mathsf{Sign}(1||W||\mathsf{H}(X^y), SK_j) \\
 & \xrightarrow{\sigma_1} & \\
 & \xleftarrow{\sigma_2} & \\
sid = \mathsf{H}(W), sk = \mathsf{H}(W||Y^x) & & sid = \mathsf{H}(W), sk = \mathsf{H}(W||X^y)
\end{array}
$$

Fig. 10. The Proposed Key Establishment Protocol

In the protocol execution, U_i validates PK_j and σ_2 and aborts if any of the validations fails, and U_j validates PK_i and σ_1 similarly.

The proposed protocol achieves all properties defined in our security model, namely soundness, key authentication, and entity authentication. The prrofs appear in Appendix B.

Theorem 1. *The proposed protocol is sound if the hash function* H *is collision resistant.*

Theorem 2. *The proposed protocol achieves entity authentication property, given that the digital signature scheme* $(\mathsf{KeyGen}, \mathsf{Sign}, \mathsf{Verify})$ *is existentially unforgeable under an adaptive chosen message attack.*

Corollary 1. *Suppose that the digital signature scheme* $(\mathsf{KeyGen}, \mathsf{Sign}, \mathsf{Verify})$ *is existentially unforgeable under an adaptive chosen message attack. In the entity authen-*

tication game, the probability that the Diffie-Hellman parameter Y is not generated by U_v is negligible.

Recall that the computational Diffie-Hellman (CDH) assumption holds for the group $\mathbb{G}$ if given g^x, g^y where $x, y \in_R \mathbb{Z}_q$ an adversary can only output g^{xy} with a negligible probability.

Theorem 3. *The proposed protocol achieves key authentication property based on the CDH assumption in the random oracle model, given that the digital signature scheme* $(\mathsf{KeyGen}, \mathsf{Sign}, \mathsf{Verify})$ *is existentially unforgeable under an adaptive chosen message attack.*

5.2 A Further Remark

In Section 3.1, we state a special case of executing a two-party key establishment protocol, i.e. a user, say U_i, runs the protocol between its devices while using the same long-term key pair for security guarantee. Though the proposed protocol in Section 5.1 is secure in this case, there is a higher probability that the long-term private key could be compromised since it needs to be deployed in many devices. To mitigate such risks, we can apply the following trick. Suppose that U_i has two devices Laptop and Workstation, it can use the long-term key pair to certify two signature key pairs for (PK'_i, SK'_i) and (PK''_i, SK''_i) for the devices, respectively. Then Laptop and Workstation can use these key pairs to run the key establishment protocol.

6 Conclusion

In this paper, we have revisited the YAK protocol and the HMQV protocols and shown their potential weaknesses. We have presented a comprehensive model for key establishment protocols, which has taken into account the interface between a key establishment protocol and the applications which may invoke it. We have emphasized the concept of *session* and the usage of *session identifier*. Moreover, we have shown how to design a two-party key establishment protocol to achieve both key authentication and entity authentication properties in our security model. We foresee the following interesting future works. The security model can be made even stronger by allowing the adversary to access more ephemeral secrets. For example, in the key authentication game, the adversary is allowed to obtain the ephemeral secrets in section sid^*. The security model can be extended in other ways. For example, similar privileges can be given to the adversary as in the key-leakage resilient key establishment protocols [1]. Instead of using digital signature for authentication purpose, it is worth trying to explore public key encryption techniques. By doing so, the efficiency could be improved.

References

1. J. Alwen, Y. Dodis, and D. Wichs. Leakage-resilient public-key cryptography in the bounded-retrieval model. In S. Halevi, editor, *Advances in Cryptology — CRYPTO 2009*, volume 5677 of *Lecture Notes in Computer Science*, pages 36–54. Springer, 2009.
2. M. Bellare, R. Canetti, and H. Krawczyk. A modular approach to the design and analysis of authentication and key exchange protocols (extended abstract). In *Proceedings of the thirtieth annual ACM symposium on Theory of computing*, pages 419–428, 1998.
3. M. Bellare and P. Rogaway. Entity authentication and key distribution. In D. R. Stinson, editor, *Advances in Cryptology — CRYPTO 1993*, volume 773 of *Lecture Notes in Computer Science*, pages 110–125. Springer, 1993.
4. S. Blake-Wilson, D. Johnson, and A. Menezes. Key agreement protocols and their security analysis. In M. Darnell, editor, *Proceedings of Cryptography and Coding, 6th IMA International Conference*, volume 1355 of *Lecture Notes in Computer Science*, pages 30–45. Springer, 1997.
5. C. Boyd and A. Mathuria. *Protocols for Authentication and Key Establishment.* Springer, 2004.
6. R. Canetti and H. Krawczyk. Analysis of key-exchange protocols and their use for building secure channels. In B. Pfitzmann, editor, *Advances in Cryptology — EUROCRYPT 2001*, volume 2045 of *Lecture Notes in Computer Science*, pages 453–474. Springer, 2001.
7. K. R. Choo, C. Boyd, and Y. Hitchcock. Errors in computational complexity proofs for protocols. In B. Roy, editor, *Advances in Cryptology — ASIACRYPT 2005*, volume 3788 of *Lecture Notes in Computer Science*, pages 624–643. Springer, 2005.
8. K. R. Choo, C. Boyd, and Y. Hitchcock. Examining indistinguishability-based proof models for key establishment protocols. In B. Roy, editor, *Advances in Cryptology — ASIACRYPT 2005*, volume 3788 of *Lecture Notes in Computer Science*, pages 585–604. Springer, 2005.
9. K. R. Choo and Y. Hitchcock. Security requirements for key establishment proof models: Revisiting bellare-rogaway and Jeong-Katz-Lee protocols. In C. Boyd and J. Nieto, editors, *Information Security and Privacy, 10th Australasian Conference, Proceedings*, volume 3574 of *Lecture Notes in Computer Science*, pages 429–442. Springer, 2005.
10. D. Denning and G. Sacco. Timestamps in key distribution protocols. *Commun. ACM*, 24(8):533–536, 1981.
11. W. Diffie and M. Hellman. New directions in cryptography. *IEEE Transactions on Information Theory*, IT-22(6):644–654, 1976.
12. W. Diffie, P. Oorschot, and M. Wiener. Authentication and authenticated key exchanges. *Des. Codes Cryptography*, 2(2):107–125, 1992.
13. H. Feng. On robust key agreement based on public key authentication (short paper). In *Proceedings of 14th International Conference on Financial Cryptography and Data Security, FC 2010*, 2010.
14. Y. Hitchcock, C. Boyd, and J. Manuel González Nieto. Modular proofs for key exchange: rigorous optimizations in the canetti-krawczyk model. *Appl. Algebra Eng. Commun. Comput.*, 16(6):405–438, 2006.
15. Institute of Electrical and Electronics Engineers, Inc. *IEEE P1363 Standard Specifications for Public-Key Cryptography*, 2000.
16. Institute of Electrical and Electronics Engineers, Inc. *IEEE P1363.2 draft D20, Standard Specifications for Password-Based Public-Key Cryptographic Techniques*, March 2005.
17. International Organization for Standardization. *ISO/IEC FCD 11770–2, Information technology — Security techniques — Key management — Part 2: Mechanisms Using Symmetric Techniques*, 1996.
18. International Organization for Standardization. *ISO/IEC FCD 11770–3, Information technology — Security techniques — Key management — Part 2: Mechanisms Using Asymmetric Techniques*, 1999.

19. International Organization for Standardization. *ISO/IEC 11770–4, Information technology — Security techniques — Key management — Part 4: Mechanisms based on weak secrets*, 2006.
20. H. Krawczyk. HMQV: A high-performance secure diffie-hellman protocol. In Victor Shoup, editor, *Advances in Cryptology — CRYPTO 2005*, volume 3621 of *Lecture Notes in Computer Science*, pages 546–566. Springer, 2005.
21. H. Krawczyk. HMQV: A high-performance secure diffie-hellman protocol. Cryptology ePrint Archive: Report 2005/176, 2005.
22. A. Menezes. Another look at HMQV. *Journal of Mathematical Cryptology*, 1:47–64, 2005.
23. A. J. Menezes, P. C. van Oorschot, and S. A. Vanstone. *Handbook of Applied Cryptography.* CRC Press, 1997.
24. R. Needham and M. Schroeder. Using encryption for authentication in large networks of computers. *Commun. ACM*, 21(12):993–999, 1978.
25. V. Shoup. On formal models for secure key exchange. Technical report, IBM Research Report RZ 3120, 1998.
26. V. Shoup. Sequences of games: a tool for taming complexity in security proofs. http://shoup.net/papers/, 2006.
27. Q. Tang. Key establishment protocols and timed-release encryption schemes. Royal Holloway, University of London, Mathematics Department Technical Report RHUL-MA-2007-9, 2007. PhD Thesis.

Appendix A. Remarks on the Security Model

The key authentication game, shown in Fig. 8, covers the extended attack scenarios listed in Section 4.2.

1. A *known session key attack* is covered because the adversary is allowed to obtain the available session keys (except that in the session sid^*) through $\mathsf{Corrupt_k}$ queries.
2. A *perfect forward secrecy attack* is covered because the adversary is allowed to obtain both SK_i and SK_j through $\mathsf{Corrupt_l}$ queries in Phrase 2.
3. A *known master key attack* is covered because the adversary is allowed to obtain the TTP's private information through a $\mathsf{Corrupt_t}$ query.
4. An *unknown key-share attack* is covered because the adversary is allowed to obtain the available session keys (except that in the session sid^*) through $\mathsf{Corrupt_k}$ queries.
5. A *key-compromise impersonation attack* is covered because the adversary is allowed to obtain both SK_i through $\mathsf{Corrupt_l}$ queries in Phrase 1.

In addition, the possibility of ephemeral secret leakage has been covered since the adversary is allowed to issue $\mathsf{Corrupt_e}$ queries

The key authentication game, shown in Fig. 9, covers the extended attack scenarios listed in Section 4.3.

1. A *key-compromise impersonation attack* is covered because the adversary is allowed to obtain both SK_i through $\mathsf{Corrupt_l}$ queries and the ephemeral secret through a $\mathsf{Corrupt_e}$ query.
2. A attack against *key confirmation* is covered because the adversary wins if the two users do not have the same session identifier and share the same session key.

Appendix B. Proof of Theorems

Proof sketch of Theorem 1. Briefly, we show that the three requirements in Definition 1 are satisfied.

1. It is straightforward to verify that, at the end of a session, the two users obtain the same (sid, sk) pair.
2. For U_i, the session identifier is computed partially with the input g^x, where x is randomly chosen in every session. Clearly, given that the hash function H is collision resistant, two sessions of U_i, carried out with U_j for an application app_{id}, the probability that the session identifiers are identical is negligible.
3. For a session between U_i and U_j for the application app_{id}, the session identifier is computed partially with the input
$$PK_i||\mathsf{ID}_i||PK_j||\mathsf{ID}_j||app_{id}.$$
For a session between U_k and U_l for the application app'_{id}, the session identifier is computed partially with the input
$$PK_k||\mathsf{ID}_k||PK_l||\mathsf{ID}_l||app'_{id}.$$
Clearly, given that the sets $\{\mathsf{ID}_i, \mathsf{ID}_j, app_{id}\}$ and $\{\mathsf{ID}_k, \mathsf{ID}_l, app'_{id}\}$ are not identical and the hash function H is collision resistant, the probability that the session identifier of ID_i and the session identifier of ID_k are identical is negligible, regardless of the coin flips of all users.

The theorem now follows. □

Proof sketch Theorem 2. Suppose that U_u successfully ended with a session with with U_v for the application app^*_{id}. Suppose also that U_u obtains (sid^*, sk^*) at the end of the session. Based on the protocol specification, U_u has received a valid signature of the form
$$\sigma_2 = \mathsf{Sign}(1||W||\mathsf{H}(Y^{\log_g^X}), SK_v), \text{ where}$$
$$sid^* = \mathsf{H}(W) \text{ and } W = PK_u||\mathsf{ID}_u||PK_v||\mathsf{ID}_v||app_{id}||X||Y.$$
Now, suppose the user U_v does not have a session with the session identifier sid^* and session key $sk^* = \mathsf{H}(W||X^{\log_g^Y})$, then the probability that U_v has generated a signature equalling to σ_2 is 0. Note that the adversary is not allowed to compromise SK_v in the game. As a result, the adversary has forged the signature σ_2, which has a negligible probability since the digital signature scheme $(\mathsf{KeyGen}, \mathsf{Sign}, \mathsf{Verify})$ is

existentially unforgeable under an adaptive chosen message attack. The theorem now follows. □

Proof sketch Theorem 3. Suppose that there are at most T Invoke oracle queries in Phase 1 of the key authentication game. Since we only consider a polynomial-time adversary, T is a polynomial in the security parameter ℓ. Let's the adversary's advantage be ϵ in the key authentication game, shown in Fig. 8. The rest of the security proof is done through a sequence of games [26].

Game_0: At the beginning of this game, the challenger makes a prediction on the session that the adversary would challenge in the game. The guess is correct implies that it is a U_u's L_u^*-th session with U_v for an application app_{id}^*. If the guess is incorrect, the challenger aborts. Otherwise, the challenger faithfully simulates the protocol execution and answers the oracle queries from the adversary $\mathcal{A}$. Let ϵ_1 be the probability that the challenger does not abort and the adversary wins the game. Clearly, the probability that the challenger does not abort is $\frac{1}{T}$, in which case the adversary's advantage is ϵ (in this case, Game_0 is equivalently the key authentication game). Therefore, $\epsilon_1 = \frac{\epsilon}{T}$.

Game_1: In this game, the challenger perform in the same way as in Game_0 except the session key is computed as

$$sid^* = \mathsf{H}(W),\ sk^* = R,\ R \in \{0,1\}^L,\ W = PK_u||\mathsf{ID}_u||PK_v||\mathsf{ID}_v||app_{id}^*||X||Y.$$

Let ϵ_2 be the probability that the challenger does not abort the game and the adversary wins the game. Clearly, the probability that the challenger does not abort is $\frac{1}{T}$, in which case the adversary's advantage is exactly 0 since the challenger returns a random message, regardless the coin flip b. Therefore, $\epsilon_2 = \frac{0}{T}$.

If the hash function is modeled as a random oracle, then Game_1 is identical to Game_0 unless the adversary has queries the random oracle H with the input $W||Y^{\log_g^X}$, which has a negligible probability based on Corollary 1 and the CDH assumption. We have $|\epsilon_2 - \epsilon_1|$, namely $\frac{\epsilon}{T}$, is negligible from the Difference Lemma in [26]. Since T is a polynomial in the security parameter ℓ, the value ϵ is negligible. The theorem now follows. □