Golay Complementary Sequences Over the QAM Constellation

Wenping Ma[1] Chen Yang[2] and Shaohui Sun[3]

[1] National Key Lab. of ISN, Xidian University , Xi'an 710071, P.R.China
wp_ma@hotmail.com
[2] China Electronic Standardization Institute, Beijing 100007, P.R.China
yangchenyf@163.com
[3] Datang Mobile Communications Equipment Co.,Lid,Beijing 100083, P.R.China

Abstract. In this paper,we present new constructions for M^2 -QAM and $2M$ Q-PAM Golay complementary sequences of length 2^n for integer n, where $M = 2^m$ for integer m. New decision conditions are proposed to judge whether the sequences with offset pairs proposed by Ying Li are Golay complementary, and with the new decision conditions, we prove the conjecture 1 and point out some drawbacks in conjecture 2 proposed by Ying Li. We describe a new offset pairs and construct new 64-QAM Golay sequences based on this new offset pairs. We also study the 128-QAM Golay complementary sequences, and propose a new decision condition to judge whether the sequences are 128-QAM Golay complementary.

Index Terms: Golay Complementary Sequences, Quadrature Amplitude Modulation(QAM), Orthogonal Frequency Division Multiplexing ($OFDM$), Quadrature Pulse Amplitude Modulation (Q-PAM).

1 Introduction

Complementary binary sequences were first introduced by Marcel Golay [1] to study problems in infrared multislit spectrometry. Nowadays,Golay sequences have many applications in communications, including peak power control for orthogonal frequency division multiplexing($OFDM$) signals, channel estimation, and complementary code-code division multiple access(CC-$CDMA$).

Multicarrier communications including orthogonal frequency division multiplexing ($OFDM$) has been receiving increasing attention [3]. However, a major drawback to $OFDM$ applications is the large peak to mean envelope power ratio ($PMEPR$). A large peak to mean power ratio ($PMEPR$) brings disadvantages such as an increased complexity of the analog-to-digital and digital-to-analog converters , a reduced efficiency of the RF power amplifier, and sometimes for certain applications, like ultra-wide-band communications, the peak transmit power is limited by regulations.

Coding techniques is one of the main techniques to reduce $PMEPR$. In 1999, J.A.Davis and J.Jedwab[2] discovered an important relation between Golay sequences and Reed-Muller codes, their method of generating binary and nonbinary Golay sequences is known as the GDJ construction. Davis and Jedwab made major progress in attacking the $PMEPR$ problem by coding techniques; they proposed the coding scheme for $OFDM$ transmission for 2^h-ary PSK modulation to reduce the $PMEPR$. In 2000, V.Tarokh and H.Jafarkhani[6] introduced a geometric approach to the offset selection problem for PSK modulation. In 2000, K.G.Paterson and V.Tarokh[5] found the lower bound on the achievable rate of a code of a given length, the minimum Euclidean distance and the maximum peak-to-average power ratio ($PAPR$). 1n 2001, Cornelai Robing and V.Tarokh[7] made significant progress on the construction of complementary sequences for both amplitude and phase modulation. 1n 2003, Chan Vee Chong,R. Venkataramani, and V.Tarokh[8] explicitly constructed 16-QAM complementary sequences using cosets of second order Reed-Muller codes by setting up the two coordinates. 1n 2003,B. Tarokh and H.R.Sadjadpour[9] derived the upper bound for the PEP for square M-QAM Golay sequences under the assumption that all the symbols are equiprobable. 1n 2006, Heekwan Lee and Solomon W.Golomb extended the constructions of 16-QAM Golay sequences to 64-QAM constellation using the offsets discovered in [10]. 1n 2008, M. Anand and P. Vijay Kumar[11] studied the low correlation sequences over the QAM constellation. In 2008, Ying Li [15, 16] gave some corrections for the sequence pairing descriptions of 16-QAM and 64-QAM, he proposed two conjectures to describe the new offset pairs and enumerate all known first order offset pairs.

In this paper, we extend the constructions of 16-QAM Golay sequences and 64-QAM Golay sequences to M^2-QAM constellations and $2M$ Q-PAM constellations using the offsets discovered in [7, 10]. We study the 64-QAM Golay sequences using the offsets discovered in [16], propose two sufficient conditions to judge whether the sequences are Golay complementary sequences,and as a result, we prove the conjectures in [16]. A new Golay complementary sequence constructed by new offset pairs is described in this paper. We also give a sufficient condition to judge whether a sequence over 128-QAM using the offsets in [16] is Golay complementary.

2 The Golay Complementary Sequences over M^2-QAM constellation

The M^2-QAM constellation is the set

$$\{a + bj| - M + 1 \leq a, b \leq M - 1, a, b\ odd\}.$$

Where $M = 2^m$,this constellation can alternately be described as

$$\{\sqrt{2j}(\sum_{k=0}^{m-1} 2^k j^{a_k})|a_k \in Z_4\}.$$

Where by $\sqrt{2j}$ we mean the element $1+j$. We now present new constructions of M^2-QAM Golay sequences that is similar to that of 16-QAM Golay sequences and 64-QAM Golay sequences as derived in [7, 10].

Theorem 1. *Let* $A(x) = 2\sum_{k=1}^{n-1} x_{\pi(k)}x_{\pi(k+1)} + \sum_{k=1}^{n} c_k x_k + c,$

$$a^{(k)}(x) = A(x) + s^{(k)}(x),$$

$$b^{(k)}(x) = A(x) + s^{(k)}(x) + \mu(x).$$

Where $c_k \in Z_4$, $k = 0, 1, \cdots, m-1$,$c \in Z_4$,π *is a permutation from* $\{1, 2, \cdots, n\}$ *to* $\{1, 2, \cdots, n\}$, $s^{(k)}(x)$ *and* $\mu(x)$ *satify the following cases.*

Case 1: $s^{(k)} = d_0^{(k)} + d_1^{(k)} x_{\pi(1)},$

$$\mu(x) = 2x_{\pi(n)}.$$

Case 2: $s^{(k)}(x) = d_0^{(k)} + d_1^{(k)} x_{\pi(n)},$

$$\mu(x) = 2x_{\pi(1)}.$$

Case 3: $s^{(k)} = d_0^{(k)} + d_1^{(k)} x_{\pi(\omega)} + d_2^{(k)} x_{\pi(\omega+1)}, 1 \le \omega \le n-1,$

$$2d_0^{(k)} + d_1^{(k)} + d_2^{(k)} = 0,$$

$$\mu(x) = 2x_{\pi(1)} \text{ or } 2x_{\pi(n)}.$$

Where $d_0^{(k)}, d_1^{(k)}, d_2^{(k)} \in Z_4$, $k = 0, 1, \cdots, m-1.$

Then the M^2-QAM *sequences*

$$c(x) = \sqrt{2j}\left(\sum_{k=0}^{m-1} 2^k j^{a^{(k)}(x)}\right) , \; d(x) = \sqrt{2j}\left(\sum_{k=0}^{m-1} 2^k j^{b^{(k)}(x)}\right) , \; M = 2^m$$

are Golay complementary sequences.

Proof:
Case 1: *The aperiodic autocorrelation function of sequences* $c(x)$, *where* $x = 0, 1, \cdots, 2^n - 1$,*at delay shift* τ *is*

$$C_c(\tau) = \sum_{i=0}^{2^n-1-\tau} c_i c_{i+\tau}^*$$

$$= 2\sum_{i=0}^{2^n-1-\tau} [\sum_{k=0}^{m-1} 2^k (j)^{a^{(k)}(i)}][\sum_{k=0}^{m-1} 2^k (j)^{-a^{(k)}(i+\tau)}]$$

$$= 2\{\sum_{k=0}^{m-1} 2^{2k} C_{a^{(k)}}(\tau) + \sum_{k,f,k\neq f} 2^{k+f} C_{a^{(k)},a^{(f)}}(\tau)\}$$

Similarly,

$C_d(\tau) = \sum_{i=0}^{2^n-1-\tau} d_i d_{i+\tau}^*$

$= 2\{\sum_{k=0}^{m-1} 2^{2k} C_{b^{(k)}}(\tau) + \sum_{k,f,k\neq f} 2^{k+f} C_{b^{(k)},b^{(f)}}(\tau)\}$

For $\tau > 0$,

$$C_c(\tau) + C_d(\tau) = 2\{\sum_{k=0}^{m-1} 2^{2k}[C_{a^{(k)}}(\tau) + C_{b^{(k)}}(\tau)] + \sum_{k,f,k\neq f} 2^{k+f}[C_{a^{(k)},a^{(f)}}(\tau) + C_{b^{(k)},b^{(f)}}(\tau)]\}$$

$$= 2\sum_{k,f,k\neq f} 2^{k+f}[C_{a^{(k)},a^{(f)}}(\tau) + C_{b^{(k)},b^{(f)}}(\tau)]$$

$$C_{a^{(k)},a^{(f)}}(\tau) + C_{a^{(f)},a^{(k)}}(\tau) + C_{b^{(k)},b^{(f)}}(\tau) + C_{b^{(f)},b^{(k)}}(\tau)$$

$$= \sum_{i=0}^{2^n-1-\tau} (j)^{A(i)+d_0^{(k)}+d_1^{(k)}(i)_{\pi(1)}-A(i+\tau)-d_0^{(f)}-d_1^{(f)}(i+\tau)_{\pi(1)}}$$

$$+ \sum_{i=0}^{2^n-1-\tau} (j)^{A(i)+d_0^{(f)}+d_1^{(f)}(i)_{\pi(1)}-A(i+\tau)-d_0^{(k)}-d_1^{(k)}(i+\tau)_{\pi(1)}}$$

$$+ \sum_{i=0}^{2^n-1-\tau} (j)^{A(i)+d_0^{(k)}+d_1^{(k)}(i)_{\pi(1)}+2(i)_{\pi(n)}-A(i+\tau)-d_0^{(f)}-d_1^{(f)}(i+\tau)_{\pi(1)}-2(i+\tau)_{\pi(n)}}$$

$$+ \sum_{i=0}^{2^n-1-\tau} (j)^{A(i)+d_0^{(f)}+d_1^{(f)}(i)_{\pi(1)}+2(i)_{\pi(n)}-A(i+\tau)-d_0^{(k)}-d_1^{(k)}(i+\tau)_{\pi(1)}-2(i+\tau)_{\pi(n)}}$$

$$= \sum_{i=0}^{2^n-1-\tau} (j)^{A(i)-A(i+\tau)}\big[(j)^{d_0^{(k)}+d_1^{(k)}(i)_{\pi(1)}-d_0^{(f)}-d_1^{(f)}(i+\tau)_{\pi(1)}}$$

$$+(j)^{d_0^{(f)}+d_1^{(f)}(i)_{\pi(1)}-d_0^{(k)}-d_1^{(k)}(i+\tau)_{\pi(1)}}\big]\times[1 + (-1)^{(i)_{\pi(n)}+(i+\tau)_{\pi(n)}}]$$

If $(i)_{\pi(n)} \neq (i+\tau)_{\pi(n)}$*, then* $1 + (-1)^{(i)_{\pi(n)}+(i+\tau)_{\pi(n)}} = 0$.

If $(i)_{\pi(n)} = (i+\tau)_{\pi(n)}$*, Let* ν *denote the largest index for which* $(i)_{\pi(\nu)} \neq (i+\tau)_{\pi(\nu)}$*, then* $(i)_{\pi(k)} = (i+\tau)_{\pi(k)}, \nu < k \leq n$. *Let* i' *and* j' *denote indexes whose binary representations differ from those of* i *and* j *only at position* $\pi(\nu+1)$.

Similar to the Proof in [1], we obtain $j^{A(i)-A(i+\tau)} = -j^{A(i')-A(i'+\tau)}$.

Obviously, $\nu + 1 \neq 1$ *, then*

$$(j)^{d_0^{(k)}+d_1^{(k)}(i)_{\pi(1)}-d_0^{(f)}-d_1^{(f)}(i+\tau)_{\pi(1)}} + (j)^{d_0^{(f)}+d_1^{(f)}(i)_{\pi(1)}-d_0^{(k)}-d_1^{(k)}(i+\tau)_{\pi(1)}}$$

$$= (j)^{d_0^{(k)}+d_1^{(k)}(i')_{\pi(1)}-d_0^{(f)}-d_1^{(f)}(i'+\tau)_{\pi(1)}} + (j)^{d_0^{(f)}+d_1^{(f)}(i')_{\pi(1)}-d_0^{(k)}-d_1^{(k)}(i'+\tau)_{\pi(1)}}$$

Thus,

$$\sum_{i=0}^{2^n-1-\tau}(j)^{A(i)-A(i+\tau)}\big[(j)^{d_0^{(k)}+d_1^{(k)}(i)_{\pi(1)}-d_0^{(f)}-d_1^{(f)}(i+\tau)_{\pi(1)}} + (j)^{d_0^{(f)}+d_1^{(f)}(i)_{\pi(1)}-d_0^{(k)}-d_1^{(k)}(i+\tau)_{\pi(1)}}\big] \times[1+(-1)^{(i)_{\pi(n)}+(i+\tau)_{\pi(n)}}] = 0.$$

We obtain,

$$C_c(\tau)+C_d(\tau)=0$$

Case 2: *The proof is similar to the proof in the case 1.*

Case 3: *In the Case 3, we have* $\pi(i)_\nu+\pi(i+\tau)_{\pi(\nu+1)}=1$,

$$(i)_{\pi(\nu+1)}=(i+\tau)_{\pi(\nu+1)}=1-(i')_{\pi(\nu+1)}=1-(i'+\tau)_{\pi(\nu+1)}$$

$$(j)^{d_0^{(k)}+d_1^{(k)}(i)_{\pi(\nu)}+d_2^{(k)}(i)_{\pi(\nu+1)}-d_0^{(f)}-d_1^{(f)}(i+\tau)_{\pi(\nu)}-d_2^{(f)}(i+\tau)_{\pi(\nu+1)}}$$
$$+(j)^{d_0^{(f)}+d_1^{(f)}(i)_{\pi(\nu)}+d_2^{(f)}(i)_{\pi(\nu+1)}-d_0^{(k)}-d_1^{(k)}(i+\tau)_{\pi(\nu)}-d_2^{(k)}(i+\tau)_{\pi(\nu+1)}}$$
$$=(j)^{d_0^{(f)}+d_1^{(f)}(i')_{\pi(\nu)}+d_2^{(f)}(i')_{\pi(\nu+1)}-d_0^{(k)}-d_1^{(k)}(i'+\tau)_{\pi(\nu)}-d_2^{(k)}(i'+\tau)_{\pi(\nu+1)}}$$
$$+(j)^{d_0^{(k)}+d_1^{(k)}(i')_{\pi(\nu)}+d_2^{(k)}(i')_{\pi(\nu+1)}-d_0^{(f)}-d_1^{(f)}(i'+\tau)_{\pi(\nu)}-d_2^{(f)}(i'+\tau)_{\pi(\nu+1)}}$$

Then,

$$\sum_{i=0}^{2^n-1-\tau}(j)^{A(i)-A(i+\tau)}[1+(-1)^{(i)_{\pi(n)}+(i+\tau)_{\pi(n)}}]$$
$$\times\big[(j)^{d_0^{(k)}+d_1^{(k)}(i)_{\pi(\nu)}+d_2^{(k)}(i)_{\pi(\nu+1)}-d_0^{(f)}-d_1^{(f)}(i+\tau)_{\pi(\nu)}-d_2^{(f)}(i+\tau)_{\pi(\nu+1)}}$$
$$+(j)^{d_0^{(f)}+d_1^{(f)}(i)_{\pi(\nu)}+d_2^{(f)}(i)_{\pi(\nu+1)}-d_0^{(k)}-d_1^{(k)}(i+\tau)_{\pi(\nu)}-d_2^{(k)}(i+\tau)_{\pi(\nu+1)}}\big]$$
$$=0$$

thus, they are also complementary sequences.

3 The Golay Complementary Sequences over Q-PAM constellation

The class of Q-PAM constellation considered in this paper is the subset of the M^2-QAM constellation of size $2M=2^{m+1}$ having representation

$$\{\sqrt{2j}(j^{a_0}+\sum_{k=1}^{m-1}2^k(j)^{a_0+2a_k})|a_0\in Z_4, a_k\in Z_2, k\geq 1\}$$

These representations suggest that quaternary sequences be used in the construction of Golay complementary sequences over these constellations.

Theorem 2. *Let* $A(x) = 2\sum_{k=1}^{n-1} x_{\pi(k)}x_{\pi(k+1)} + \sum_{k=1}^{n} c_k x_k + c$,

$$a^{(0)}(x) = A(x) + s^{(0)}(x),$$

$$b^{(0)}(x) = A(x) + s^{(0)}(x) + \mu(x),$$

Where $c_k \in Z_4$, $k = 0, 1, \cdots, m-1$,$c \in Z_4$,π *is a permutation from* $\{1, 2, \cdots, n\}$ *to* $\{1, 2, \cdots, n\}$, $s^{(k)}(x)$ *and* $\mu(x)$ *satify the following cases.*

Case 1: $s^{(k)} = d_0^{(k)} + d_1^{(k)} x_{\pi(1)}$,

$$\mu(x) = 2x_{\pi(n)}.$$

Case 2: $s^{(k)}(x) = d_0^{(k)} + d_1^{(k)} x_{\pi(n)}$,

$$\mu(x) = 2x_{\pi(1)}.$$

Case 3: $s^{(k)} = d_0^{(k)} + d_1^{(k)} x_{\pi(\omega)} + d_2^{(k)} x_{\pi(\omega+1)}$,$1 \leq \omega \leq n-1$,

$$2d_0^{(0)} + d_1^{(0)} + d_2^{(0)} = 0,\ d_1^{(k)} = d_2^{(k)},$$

$$\mu(x) = 2x_{\pi(1)} \text{ or } 2x_{\pi(n)}.$$

Where $d_0^{(0)}, d_1^{(0)}, d_2^{(0)} \in Z_4$,$d_0^{(k)}, d_1^{(k)}, d_2^{(k)} \in Z_2$,$k = 1, \cdots, m-1$.

Then the $2M$*-*$QPAM$ *sequences*

$c(x) = \sqrt{2j}(j^{a^0(x)} + \sum_{k=1}^{m-1} 2^k (j)^{a^0(x)+2s^{(k)}(x)})$,

$d(x) = \sqrt{2j}(j^{b^0(x)} + \sum_{k=1}^{m-1} 2^k (j)^{b^0(x)+2s^{(k)}(x)})$

are Golay complementary sequences.

The proof is similar to the proof in the theorem 1, we omit it.

Example 1: The 8-ary Q-PAM constellation is a subset of the 16-QAM constellation given by

$\{\sqrt{2j}(j^{a_0} + 2j^{a_0+2a_1}) | a_0 \in Z_4, a_1 \in Z_2\}$

Then as discussed above, let $a_0 = f(x_1, x_2, x_3) = x_1x_2 + x_2x_3$, $a_1 = 1 + x_3$, then the following sequences are Golay complementary sequences of length 8 over the 8-ary Q-PAM constellation.

$-\sqrt{2j}, -\sqrt{2j}, -\sqrt{2j}, \sqrt{2j}, 3\sqrt{2j}, 3\sqrt{2j}, -3\sqrt{2j}, 3\sqrt{2j},$

$-\sqrt{2j}, \sqrt{2j}, -\sqrt{2j}, -\sqrt{2j}, 3\sqrt{2j}, -3\sqrt{2j}, -3\sqrt{2j}, -3\sqrt{2j}.$

4 Golay Complementary Sequences over 64-QAM Using Offset Pairs Proposed By Ying Li

Based on offset pairs proposed by Ying Li[16], he proposed two constructions called modified case 4 and modified case 5 respectively, based on the constructions, he also proposed two conjectures. Now we study the constructions and prove the conjectures. We rewrite original modified case 4 and modified case 5 in[16] as case 4 and case 5 in this paper.

Case 4: Let $A(x) = 2\sum_{k=1}^{n-1} x_{\pi(k)}x_{\pi(k+1)} + \sum_{k=1}^{n} c_k x_k + c,$

$$a_1(x) = A(x) + s^{(1)}(x),$$

$$a_2(x) = A(x) + s^{(2)}(x),$$

$$B(x) = A(x) + \mu(x),$$

$$b_1(x) = a_1(x) + \mu(x),$$

$$b_2(x) = a_2(x) + \mu(x),$$

$$(s^{(1)}(x), s^{(2)}(x)) = (d_0 + d_1 x_{\pi(\omega)}, d_0' + d_1' x_{\pi(\omega)}), \text{ with } 2 \leq \omega \leq n-1.$$

$$\mu(x) = 2x_{\pi(1)} \text{ or } 2x_{\pi(n)},$$

$$c(x) = 4j^{A(x)} + 2j^{a_1(x)} + j^{a_2(x)}\ ,\ d(x) = 4j^{B(x)} + 2j^{b_1(x)} + j^{b_2(x)},$$

Then,

$$\begin{aligned}C_c(\tau) &= \sum_{i=0}^{2^n-1-\tau} c_i c_{i+\tau}^* \\ &= \sum_{i=0}^{2^n-1-\tau}(8j^{A(i)-A(i+\tau)} + 4j^{a_1(i)-a_1(i)-a_1(i+\tau)} + j^{a_2(i)-a_2(i+\tau)}) \\ &\quad + \sum_{i=0}^{2^n-1-\tau} 2[4(j^{A(i)-a_1(i+\tau)} + j^{a_1(i)-A(i+\tau)}) \\ &\qquad +2(j^{A(i)-a_2(i+\tau)} + j^{a_2(i)-A(i+\tau)}) + (j^{a_1(i)-a_2(i+\tau)} + j^{a_2(i)-a_1(i+\tau)})]\end{aligned}$$

$$C_d(\tau) = \sum_{i=0}^{2^n-1-\tau} d_i d_{i+\tau}^*$$

$$= \sum_{i=0}^{2^n-1-\tau}(8j^{B(i)-B(i+\tau)} + 4j^{b_1(i)-b_1(i+\tau)} + j^{b_2(i)-b_2(i+\tau)})$$

$$+ \sum_{i=0}^{2^n-1-\tau} 2[4(j^{B(i)-b_1(i+\tau)}+j^{b_1(i)-B(i+\tau)})+2(j^{B(i)-b_2(i+\tau)}+j^{b_2(i)-B(i+\tau)})+ (j^{b_2(i)-b_1(i+\tau)} + j^{b_1(i)-b_2(i+\tau)})]$$

The following three equations can be verified easily.

$$\sum_{i=0}^{2^n-1-\tau}(j^{A(i)-a_1(i+\tau)} + j^{a_1(i)-A(i+\tau)} + j^{B(i)-b_1(i+\tau)} + j^{b_1(i)-B(i+\tau)}) \quad (1)$$

$$= \sum_{i=0}^{2^n-1-\tau}\big[j^{A(i)-A(i+\tau)-d_0-d_1(i+\tau)_{\pi(\omega)}}+j^{A(i)-A(i+\tau)-d_0-d_1(i+\tau)_{\pi(\omega)}+2(i)_{\pi(1)}-2(i+\tau)_{\pi(1)}}\big]$$

$$+\sum_{i=0}^{2^n-1-\tau}\big[j^{-A(i+\tau)+A(i)+d_0+d_1(i)_{\pi(\omega)}}+j^{-A(i+\tau)+A(i)+d_0+d_1(i)_{\pi(\omega)}+2(i)_{\pi(1)}-2(i+\tau)_{\pi(1)}}\big]$$

$$= \sum_{i=0}^{2^n-1-\tau} j^{A(i+\tau)+A(i)}\big[j^{d_0+d_1(i)_{\pi(\omega)}} + j^{-d_0-d_1(i+\tau)_{\pi(\omega)}}\big][1 + (-1)^{i_{\pi(1)}+(i+\tau)_{\pi(1)}}\big]$$

$$\sum_{i=0}^{2^n-1-\tau}(j^{A(i)-a_2(i+\tau)} + j^{a_2(i)-A(i+\tau)} + j^{B(i)-b_2(i+\tau)} + j^{b_2(i)-B(i+\tau)}) \quad (2)$$

$$= \sum_{i=0}^{2^n-1-\tau}\big[j^{A(i)-A(i+\tau)-d'_0-d'_1(i+\tau)_{\pi(\omega)}}+j^{A(i)-A(i+\tau)-d'_0-d'_1(i+\tau)_{\pi(\omega)}+2(i)_{\pi(1)}-2(i+\tau)_{\pi(1)}}\big]$$

$$+\sum_{i=0}^{2^n-1-\tau}\big[j^{-A(i+\tau)+A(i)+d'_0+d'_1(i)_{\pi(\omega)}}+j^{-A(i+\tau)+A(i)+d'_0+d'_1(i)_{\pi(\omega)}+2(i)_{\pi(1)}-2(i+\tau)_{\pi(1)}}\big]$$

$$= \sum_{i=0}^{2^n-1-\tau} j^{A(i+\tau)+A(i)}\big[j^{d'_0+d'_1(i)_{\pi(\omega)}} + j^{-d'_0-d'_1(i+\tau)_{\pi(\omega)}}\big][1 + (-1)^{i_{\pi(1)}+(i+\tau)_{\pi(1)}}\big]$$

$$\sum_{i=0}^{2^n-1-\tau}(j^{a_1(i)-a_2(i+\tau)} + j^{a_2(i)-a_1(i+\tau)} + j^{b_2(i)-b_1(i+\tau)} + j^{b_1(i)-b_2(i+\tau)}) \quad (3)$$

$$= \sum_{i=0}^{2^n-1-\tau}\big[j^{A(i)+d_0+d_1(i)_{\pi(\omega)}-A(i+\tau)-d'_0+d'_1(i+\tau)_{\pi(\omega)}}$$

$$+j^{A(i)+d_0+d_1(i)_{\pi(\omega)}-A(i+\tau)-d'_0-d'_1(i+\tau)_{\pi(\omega)}+2(i)_{\pi(1)}-2(i+\tau)_{\pi(1)}}\big]$$

$$+\sum_{i=0}^{2^n-1-\tau}\big[j^{-A(i+\tau)-d_0-d_1(i+\tau)_{\pi(\omega)}+A(i)+d'_0+d'_1(i)_{\pi(\omega)}}$$

$$+j^{-A(i+\tau)-d_0+d_1(i)_{\pi(\omega)}+A(i)+d'_0+d'_1(i)_{\pi(\omega)}+2(i)_{\pi(1)}-2(i+\tau)_{\pi(1)}}\big]$$

$$= \sum_{i=0}^{2^n-1-\tau} j^{A(i+\tau)+A(i)}\big[j^{d'_0+d'_1(i)_{\pi(\omega)}-d_0-d_1(i+\tau)_{\pi(\omega)}}+j^{-d'_0-d'_1(i+\tau)_{\pi(\omega)}+d_0+d_1(i)_{\pi(\omega)}}\big]$$

$$\times[1 + (-1)^{i_{\pi(1)}+(i+\tau)_{\pi(1)}}\big]$$

Thus we obtain the following equation

$$4\times(1) + 2\times(2) + (3)$$

$$= \sum_{i=0}^{2^n-1-\tau}[4(j^{A(i)-a_1(i+\tau)} + j^{a_1(i)-A(i+\tau)} + j^{B(i)-b_1(i+\tau)} + j^{b_1(i)-B(i+\tau)})$$

$+2(j^{A(i)-a_2(i+\tau)}+j^{a_2(i)-A(i+\tau)}+j^{B(i)-b_2(i+\tau)}+j^{b_2(i)-B(i+\tau)})$

$+(j^{a_1(i)-a_2(i+\tau)}+j^{a_2(i)-a_1(i+\tau)}+j^{b_2(i)-b_1(i+\tau)}+j^{b_1(i)-b_2(i+\tau)})]$

$=\sum_{i=0}^{2^n-1-\tau} j^{A(i+\tau)+A(i)}Q(i)[1+(-1)^{i_{\pi(1)}+(i+\tau)_{\pi(1)}}]$

Where $Q(i)=4(j^{d_0+d_1(i)_{\pi(\omega)}}+j^{-d_0-d_1(i+\pi)_{\pi(\omega)}})+2(j^{d_0'+d_1'(i)_{\pi(\omega)}}+j^{-d_0'-d_1'(i+\pi)_{\pi(\omega)}})$

$+(j^{d_0'+d_1'(i)_{\pi(\omega)}-d_0-d_1(i+\tau)_{\pi(\omega)}}+j^{-d_0'-d_1'(i+\tau)_{\pi(\omega)}+d_0+d_1(i)_{\pi(\omega)}})$

Similar to the proof of theorem 1, we set $(i)_{\pi(\omega)}=(i+\tau)_{\pi(\omega)}=0$, then the above equation is

$4(j^{d_0}+j^{-d_0})+2(j^{d_0'}+j^{-d_0'})+(j^{d_0'-d_0}+j^{-d_0'+d_0}),$

and set $(i)_{\pi(\omega)}=(i+\tau)_{\pi(\omega)}=1$, then the above equation is

$4(j^{d_0+d_1}+j^{-d_0-d_1})+2(j^{d_0'+d_1'}+j^{-d_0'-d_1'})+(j^{d_0'+d_1'-d_0-d_1}+j^{-d_0'-d_1'+d_0+d_1}),$

then $c(x)$ and $d(x)$ are Golay complementary pair if

$$\begin{aligned}&4(j^{d_0}+j^{-d_0})+2(j^{d_0'}+j^{-d_0'})+(j^{d_0'-d_0}+j^{-d_0'+d_0}),\\&=4(j^{d_0+d_1}+j^{-d_0-d_1})+2(j^{d_0'+d_1'}+j^{-d_0'-d_1'})+(j^{d_0'+d_1'-d_0-d_1}+j^{-d_0'-d_1'+d_0+d_1}),\end{aligned}\tag{4}$$

By exhaust search, there are 32 sets of (d_0,d_1,d_0',d_1') satisfying the above equation(4), and the four sets $\{(0123),(0321),(1311),(3133)\}$ proposed by Ying Li in [16] also satisfy the equation.

Case 5: Let $A(x)=2\sum_{k=1}^{n-1}x_{\pi(k)}x_{\pi(k+1)}+\sum_{k=1}^{n}c_kx_k+c,$

$a_1(x)=A(x)+s^{(1)}(x),$

$a_2(x)=A(x)+s^{(2)}(x),$

$B(x)=A(x)+\mu(x),$

$b_1(x)=a_1(x)+\mu(x),$

$b_2(x)=a_2(x)+\mu(x),$

$(s^{(1)}(x),s^{(2)}(x))=(d_0+d_1x_{\pi(\omega)}+d_2x_{\pi(k)},d_0'+d_1'x_{\pi(\omega)}+d_2'x_{\pi(k)}),$

with $2\le\omega\le n-2$, $\omega+2\le k\le n.$

$$\mu(x) = 2x_{\pi(1)} \text{ or } 2x_{\pi(n)},$$

$$c(x) = 4j^{A(x)} + 2j^{a_1(x)} + j^{a_2(x)}\ ,\ d(x) = 4j^{B(x)} + 2j^{b_1(x)} + j^{b_2(x)}.$$

It is easy to check the following three equations hold.

$$\begin{aligned}&\textstyle\sum_{i=0}^{2^n-1-\tau}\big(j^{A(i)-a_1(i+\tau)} + j^{a_1(i)-A(i+\tau)} + j^{B(i)-b_1(i+\tau)} + j^{b_1(i)-B(i+\tau)}\big)\\ &= \textstyle\sum_{i=0}^{2^n-1-\tau} j^{A(i)-A(i+\tau)}\big[1+(-1)^{i_{\pi(1)}-(i+\tau)_{\pi(1)}}\big]\\ &\quad\times\big[j^{d_0+d_1(i)_{\pi(\omega)}+d_2(i)_{\pi(k)}} + j^{-d_0-d_1(i+\tau)_{\pi(\omega)}-d_2(i+\tau)_{\pi(k)}}\big]\end{aligned}$$

$$\begin{aligned}&\textstyle\sum_{i=0}^{2^n-1-\tau}\big(j^{A(i)-a_2(i+\tau)} + j^{a_2(i)-A(i+\tau)} + j^{B(i)-b_2(i+\tau)} + j^{b_2(i)-B(i+\tau)}\big)\\ &= \textstyle\sum_{i=0}^{2^n-1-\tau} j^{A(i)-A(i+\tau)}\big[1+(-1)^{i_{\pi(1)}-(i+\tau)_{\pi(1)}}\big]\\ &\quad\times\big[j^{d'_0+d'_1(i)_{\pi(\omega)}+d'_2(i)_{\pi(k)}} + j^{-d'_0-d'_1(i+\tau)_{\pi(\omega)}-d'_2(i+\tau)_{\pi(k)}}\big]\end{aligned}$$

$$\begin{aligned}&\textstyle\sum_{i=0}^{2^n-1-\tau}\big(j^{a_1(i)-a_2(i+\tau)} + j^{a_2(i)-a_1(i+\tau)} + j^{b_2(i)-b_1(i+\tau)} + j^{b_1(i)-b_2(i+\tau)}\big)\\ &= \textstyle\sum_{i=0}^{2^n-1-\tau} j^{A(i)-A(i+\tau)}\big[1+(-1)^{i_{\pi(1)}-(i+\tau)_{\pi(1)}}\big]\times\\ &\quad\big[j^{d'_0+d'_1(i)_{\pi(\omega)}+d'_2(i)_{\pi(k)}-d_0-d_1(i+\tau)_{\pi(\omega)}-d_2(i+\tau)_{\pi(k)}}\\ &\quad+j^{d_0+d_1(i)_{\pi(\omega)}+d_2(i)_{\pi(k)}-d'_0-d'_1(i+\tau)_{\pi(\omega)}-d'_2(i+\tau)_{\pi(k)}}\big]\end{aligned}$$

then as in case 4, we obtain $c(x)$ and $d(x)$ are Golay complementary pair if the following conditions hold.

$$\begin{aligned}&4\big(j^{d_0+d_1+d_2y} + j^{-d_0-d_1-d_2z}\big) + 2\big(j^{d'_0+d'_1+d'_2y} + j^{-d'_0-d'_1-d'_2z}\big)\\ &\qquad+\big(j^{d'_0+d'_1+d'_2y-d_0-d_1-d_2z} + j^{d_0+d_1+d_2y-d'_0-d'_1-d'_2z}\big)\\ &= 4\big(j^{d_0+d_2y}+j^{-d_0-d_2z}\big)+2\big(j^{d'_0+d'_2y}+j^{-d'_0-d'_2z}\big)+\big(j^{d'_0+d'_2y-d_0-d_2z}+j^{d_0+d_2y-d'_0-d'_2z}\big)\end{aligned}\tag{5}$$

$$\begin{aligned}&4\big(j^{d_0+d_1y} + j^{-d_0-d_1z}\big) + 2\big(j^{d'_0+d'_1y} + j^{-d'_0-d'_1z}\big)\\ &\qquad+\big(j^{d'_0+d'_1y-d_0-d_1z} + j^{d_0+d_1y-d'_0-d'_1z}\big)\\ &= 4\big(j^{d_0+d_1y+d_2}+j^{-d_0-d_1z-d_2}\big)+2\big(j^{d'_0+d'_1y+d'_2}+j^{-d'_0-d'_1z-d'_2}\big)\\ &\quad+\big(j^{d'_0+d'_1y+d'_2-d_0-d_1z-d_2} + j^{d_0+d_1y+d_2-d'_0-d'_1z-d'_2}\big)\end{aligned}\tag{6}$$

Where $y, z \in \{0, 1\}$.

By exhaust search, there are 52 sets of $(d_0, d_1, d_2, d'_0, d'_1, d'_2)$ satisfying the above equation(5)and equation (6), and the four sets $\{(013231), (031213), (133111), (311333)\}$ proposed by Ying Li in[16] also satisfy the equations.

In the paper[16], the author asserted that there only exist four sets of coefficients for the case 4 and case 5 respectively, but, based on the proposed judgment conditions above, we find there are 32 sets of coefficients for the case 4 and 52 sets of coefficients for the case 5 using exhaust search. We provide these detail materials in the appendix 1 and appendix 2 respectively.

Due to the discussion above, we have proved the conjecture 1 proposed in[16]by Ying Li. Because the conjecture 2 is based on conjecture 1 and there are some drawbacks existing in counting the set elements, some drawbacks can be found in conjecture 2.

Here, we extend the method proposed by Ying Li and introduce a new case called case 6 as follows:

Case 6: Let $A(x) = 2\sum_{k=1}^{n-1} x_{\pi(k)}x_{\pi(k+1)} + \sum_{k=1}^{n} c_k x_k + c$,

$$a_1(x) = A(x) + s^{(1)}(x),$$

$$a_2(x) = A(x) + s^{(2)}(x),$$

$$B(x) = A(x) + \mu(x),$$

$$b_1(x) = a_1(x) + \mu(x),$$

$$b_2(x) = a_2(x) + \mu(x),$$

$$(s^{(1)}(x), s^{(2)}(x)) =$$

$$(d_0 + d_1 x_{\pi(\omega)} + d_2 x_{\pi(k)} + d_3 x_{\pi(l)}, d'_0 + d'_1 x_{\pi(\omega)} + d'_2 x_{\pi(k)} + d'_3 x_{\pi(l)}),$$

$$\text{with } 2 \leq \omega \leq n-4 \text{ , } \omega+2 \leq k \leq n-2 \text{ , } k+2 \leq l \leq n.$$

$$\mu(x) = 2x_{\pi(1)} \text{ or } 2x_{\pi(n)},$$

$$c(x) = 4j^{A(x)} + 2j^{a_1(x)} + j^{a_2(x)} \text{ , } d(x) = 4j^{B(x)} + 2j^{b_1(x)} + j^{b_2(x)}.$$

It is easy to check the following three equations hold.

$$\sum_{i=0}^{2^n-1-\tau}(j^{A(i)-a_1(i+\tau)} + j^{a_1(i)-A(i+\tau)} + j^{B(i)-b_1(i+\tau)} + j^{b_1(i)-B(i+\tau)})$$

$$= \sum_{i=0}^{2^n-1-\tau} j^{A(i)-A(i+\tau)}[1 + (-1)^{i_{\pi(1)}-(i+\tau)_{\pi(1)}}]\times$$

$$[j^{d_0+d_1(i)_{\pi(\omega)}+d_2(i)_{\pi(k)}+d_3(i)_{\pi(l)}} + j^{-d_0-d_1(i+\tau)_{\pi(\omega)}-d_2(i+\tau)_{\pi(k)}-d_3(i+\tau)_{\pi(l)}}]$$

$$\sum_{i=0}^{2^n-1-\tau}\left(j^{A(i)-a_2(i+\tau)}+j^{a_2(i)-A(i+\tau)}+j^{B(i)-b_2(i+\tau)}+j^{b_2(i)-B(i+\tau)}\right)$$
$$=\sum_{i=0}^{2^n-1-\tau} j^{A(i)-A(i+\tau)}\left[1+(-1)^{i_{\pi(1)}-(i+\tau)_{\pi(1)}}\right]\times$$
$$\left[j^{d'_0+d'_1(i)_{\pi(\omega)}+d'_2(i)_{\pi(k)}+d'_3(i)_{\pi(l)}}+j^{-d'_0-d'_1(i+\tau)_{\pi(\omega)}-d'_2(i+\tau)_{\pi(k)}-d'_3(i+\tau)_{\pi(l)}}\right]$$

$$\sum_{i=0}^{2^n-1-\tau}\left(j^{a_1(i)-a_2(i+\tau)}+j^{a_2(i)-a_1(i+\tau)}+j^{b_2(i)-b_1(i+\tau)}+j^{b_1(i)-b_2(i+\tau)}\right)$$
$$=\sum_{i=0}^{2^n-1-\tau} j^{A(i)-A(i+\tau)}\left[1+(-1)^{i_{\pi(1)}-(i+\tau)_{\pi(1)}}\right]\times$$
$$\left[j^{d'_0+d'_1(i)_{\pi(\omega)}+d'_2(i)_{\pi(k)}+d'_3(i)_{\pi(l)}-d_0-d_1(i+\tau)_{\pi(\omega)}-d_2(i+\tau)_{\pi(k)}-d_3(i+\tau)_{\pi(l)}}\right.$$
$$\left.+j^{d_0+d_1(i)_{\pi(\omega)}+d_2(i)_{\pi(k)}+d_3(i)_{\pi(l)}-d'_0-d'_1(i+\tau)_{\pi(\omega)}-d'_2(i+\tau)_{\pi(k)}-d'_3(i+\tau)_{\pi(l)}}\right]$$

then as in case 4, we obtain $c(x)$ and $d(x)$ are Golay complementary pair if the following conditions hold.

$$\begin{aligned}
&4\left(j^{d_0+d_2y_1+d_3y_3}+j^{-d_0-d_2y_2-d_3y_4}\right)+2\left(j^{d'_0+d'_2y_1+d'_3y_3}+j^{-d'_0-d'_2y_2-d'_3y_4}\right)\\
&\quad+\left(j^{d'_0+d'_2y_1+d'_3y_3-d_0-d_2y_2-d_3y_4}+j^{d_0+d_2y_1+d_3y_3-d'_0-d'_2y_2-d'_3y_4}\right)\\
&=4\left(j^{d_0+d_1+d_2y_1+d_3y_3}+j^{-d_0-d_1-d_2y_2-d_3y_4}\right)+2\left(j^{d'_0+d'_1+d'_2y_1+d'_3y_3}+j^{-d'_0-d'_1-d'_2y_2-d'_3y_4}\right)\\
&\quad+\left(j^{d'_0+d'_1+d'_2y_1+d'_3y_3-d_0-d_1-d_2y_2-d_3y_4}+j^{d_0+d_1+d_2y_1+d_3y_3-d'_0-d'_1-d'_2y_2-d'_3y_4}\right)
\end{aligned}\tag{7}$$

$$\begin{aligned}
&4\left(j^{d_0+d_1y_1+d_3y_3}+j^{-d_0-d_1y_2-d_3y_4}\right)+2\left(j^{d'_0+d'_1y_1+d'_3y_3}+j^{-d'_0-d'_1y_2-d'_3y_4}\right)\\
&\quad+\left(j^{d'_0+d'_1y_1+d'_3y_3-d_0-d_1y_2-d_3y_4}+j^{d_0+d_1y_1+d_3y_3-d'_0-d'_1y_2-d'_3y_4}\right)\\
&=4\left(j^{d_0+d_1y_1+d_2+d_3y_3}+j^{-d_0-d_1y_2-d_2-d_3y_4}\right)+2\left(j^{d'_0+d'_1y_1+d'_2+d'_3y_3}+j^{-d'_0-d'_1y_2-d'_2-d'_3y_4}\right)\\
&\quad+\left(j^{d'_0+d'_1y_1+d'_2+d'_3y_3-d_0-d_1y_2-d_2-d_3y_4}+j^{d_0+d_1y_1+d_2+d_3y_3-d'_0-d'_1y_2-d'_2-d'_3y_4}\right)
\end{aligned}\tag{8}$$

$$\begin{aligned}
&4\left(j^{d_0+d_1y_1+d_2y_3}+j^{-d_0-d_1y_2-d_2y_4}\right)+2\left(j^{d'_0+d'_1y_1+d'_2y_3}+j^{-d'_0-d'_1y_2-d'_2y_4}\right)\\
&\quad+\left(j^{d'_0+d'_1y_1+d'_2y_3-d_0-d_1y_2-d_2y_4}+j^{d_0+d_1y_1+d_2y_3-d'_0-d'_1y_2-d'_2y_4}\right)\\
&=4\left(j^{d_0+d_1y_1+d_2y_3+d_3}+j^{-d_0-d_1y_2-d_2y_4-d_3}\right)+2\left(j^{d'_0+d'_1y_1+d'_2y_3+d'_3}+j^{-d'_0-d'_1y_2-d'_2y_4-d'_3}\right)\\
&\quad+\left(j^{d'_0+d'_1y_1+d'_2y_3+d'_3-d_0-d_1y_2-d_2y_4-d_3}+j^{d_0+d_1y_1+d_2y_3+d_3-d'_0-d'_1y_2-d'_2y_4-d'_3}\right)
\end{aligned}\tag{9}$$

Where $y_1, y_2, y_3, y_4 \in \{0, 1\}$.

By exhaust search, there are 76 sets of $(d_0, d_1, d_2, d_3, d'_0, d'_1, d'_2, d'_3)$ satisfying the above equation(7),equation (8) and equation (9), please see appendix 3.
Note. We can choose

$(s^{(1)}(x), s^{(2)}(x)) =$

$(d_0 + d_1 x_{\pi(\omega_1)} + d_2 x_{\pi(\omega_2)} + \cdots + d_l x_{\pi(\omega_l)}, d'_0 + d'_1 x_{\pi(\omega_1)} + d'_2 x_{\pi(\omega_2)} + \cdots + d'_l x_{\pi(\omega_l)}),$

there are somewhat difficult to obtain sufficient conditions for judging whether the sequence using above offset pairs is Golay Complementary sequence.

5 Golay Complementary Sequences over 128-QAM Constellation

It is easy to construct Golay complementary sequences over 128-QAM constellation using the offset pairs in the Case 1, Case 2 and Case 3.We consider the case 5 here.

Let $A(x) = 2\sum_{k=1}^{n-1} x_{\pi(k)} x_{\pi(k+1)} + \sum_{k=1}^{n} c_k x_k + c,$

$a_1(x) = A(x) + s^{(1)}(x),$

$a_2(x) = A(x) + s^{(2)}(x),$

$a_3(x) = A(x) + s^{(3)}(x),$

$b_1(x) = a_1(x) + \mu(x),$

$b_2(x) = a_2(x) + \mu(x),$

$b_3(x) = a_3(x) + \mu(x),$

$s^{(1)}(x) = d_0 + d_1 x_{\pi(\omega)} + d_2 x_{\pi(k)},$

$s^{(2)}(x) = d'_0 + d'_1 x_{\pi(\omega)} + d'_2 x_{\pi(k)},$

$s^{(3)}(x) = d^*_0 + d^*_1 x_{\pi(\omega)} + d^*_2 x_{\pi(k)},$

With $2 \le \omega \le n-2$, $\omega + 2 \le k \le n$.

$\mu(x) = 2x_{\pi(1)}$ or $2x_{\pi(n)},$

Then the 128-QAM sequences can be constructed as follows

$$c(x) = 8j^{A(x)} + 4j^{a_1(x)} + 2j^{a_2(x)} + j^{a_3(x)},$$

$$d(x) = 8j^{B(x)} + 4j^{b_1(x)} + 2j^{b_2(x)} + j^{b_3(x)}.$$

It is easy to check the following five equations hold.

$$\sum_{i=0}^{2^n-1-\tau}\big(j^{A(i)-a_1(i+\tau)} + j^{a_1(i)-A(i+\tau)} + j^{B(i)-b_1(i+\tau)} + j^{b_1(i)-B(i+\tau)}\big)$$
$$= \sum_{i=0}^{2^n-1-\tau} j^{A(i)-A(i+\tau)}\big[1 + (-1)^{i_{\pi(1)}-(i+\tau)_{\pi(1)}}\big]$$
$$\big[j^{d_0+d_1(i)_{\pi(\omega)}+d_2(i)_{\pi(k)}} + j^{-d_0-d_1(i+\tau)_{\pi(\omega)}-d_2(i+\tau)_{\pi(k)}}\big]$$

$$\sum_{i=0}^{2^n-1-\tau}\big(j^{A(i)-a_2(i+\tau)} + j^{a_2(i)-A(i+\tau)} + j^{B(i)-b_2(i+\tau)} + j^{b_2(i)-B(i+\tau)}\big)$$
$$= \sum_{i=0}^{2^n-1-\tau} j^{A(i)-A(i+\tau)}\big[1 + (-1)^{i_{\pi(1)}-(i+\tau)_{\pi(1)}}\big]$$
$$\big[j^{d'_0+d'_1(i)_{\pi(\omega)}+d'_2(i)_{\pi(k)}} + j^{-d'_0-d'_1(i+\tau)_{\pi(\omega)}-d'_2(i+\tau)_{\pi(k)}}\big]$$

$$\sum_{i=0}^{2^n-1-\tau}\big(j^{a_1(i)-a_2(i+\tau)} + j^{a_2(i)-a_1(i+\tau)} + j^{b_2(i)-b_1(i+\tau)} + j^{b_1(i)-b_2(i+\tau)}\big)$$
$$+ \sum_{i=0}^{2^n-1-\tau}\big(j^{A(i)-a_3(i+\tau)} + j^{a_3(i)-A(i+\tau)} + j^{B(i)-b_3(i+\tau)} + j^{b_3(i)-B(i+\tau)}\big)$$
$$= \sum_{i=0}^{2^n-1-\tau} j^{A(i)-A(i+\tau)}\big[1 + (-1)^{i_{\pi(1)}-(i+\tau)_{\pi(1)}}\big]\times$$
$$\big[j^{d'_0+d'_1(i)_{\pi(\omega)}+d'_2(i)_{\pi(k)}-d_0-d_1(i+\tau)_{\pi(\omega)}-d_2(i+\tau)_{\pi(k)}}$$
$$+j^{d_0+d_1(i)_{\pi(\omega)}+d_2(i)_{\pi(k)}-d'_0-d'_1(i+\tau)_{\pi(\omega)}-d'_2(i+\tau)_{\pi(k)}}\big]$$
$$+ \sum_{i=0}^{2^n-1-\tau} j^{A(i)-A(i+\tau)}\big[1 + (-1)^{i_{\pi(1)}-(i+\tau)_{\pi(1)}}\big]\times$$
$$\big[j^{d^*_0+d^*_1(i)_{\pi(\omega)}+d^*_2(i)_{\pi(k)}} + j^{-d^*_0-d^*_1(i+\tau)_{\pi(\omega)}-d^*_2(i+\tau)_{\pi(k)}}\big]$$

$$\sum_{i=0}^{2^n-1-\tau}\big(j^{a_1(i)-a_3(i+\tau)} + j^{a_3(i)-a_1(i+\tau)} + j^{b_1(i)-b_3(i+\tau)} + j^{b_3(i)-b_1(i+\tau)}\big)$$
$$= \sum_{i=0}^{2^n-1-\tau} j^{A(i)-A(i+\tau)}\big[1 + (-1)^{i_{\pi(1)}-(i+\tau)_{\pi(1)}}\big]\times$$
$$\big[j^{d^*_0+d^*_1(i)_{\pi(\omega)}+d^*_2(i)_{\pi(k)}-d_0-d_1(i+\tau)_{\pi(\omega)}-d_2(i+\tau)_{\pi(k)}}$$
$$+j^{d_0+d_1(i)_{\pi(\omega)}+d_2(i)_{\pi(k)}-d^*_0-d^*_1(i+\tau)_{\pi(\omega)}-d^*_2(i+\tau)_{\pi(k)}}\big]$$

$$\sum_{i=0}^{2^n-1-\tau}(j^{a_2(i)-a_3(i+\tau)}+j^{a_3(i)-a_2(i+\tau)}+j^{b_2(i)-b_3(i+\tau)}+j^{b_3(i)-b_2(i+\tau)})$$
$$=\sum_{i=0}^{2^n-1-\tau} j^{A(i)-A(i+\tau)}[1+(-1)^{i_{\pi(1)}-(i+\tau)_{\pi(1)}}]\times$$
$$[j^{d_0^*+d_1^*(i)_{\pi(\omega)}+d_2^*(i)_{\pi(k)}-d_0'-d_1'(i+\tau)_{\pi(\omega)}-d_2'(i+\tau)_{\pi(k)}}$$
$$+j^{d_0'+d_1'(i)_{\pi(\omega)}+d_2'(i)_{\pi(k)}-d_0^*-d_1^*(i+\tau)_{\pi(\omega)}-d_2^*(i+\tau)_{\pi(k)}}]$$

then similar to the case 4, $c(x)$ and $d(x)$ are Golay complementary pair if the following conditions hold.

$$\begin{aligned}
&16(j^{d_0+d_2y}+j^{-d_0-d_2z})+8(j^{d_0'+d_2'y}+j^{-d_0'-d_2'z})\\
&\quad+4(j^{d_0'+d_2'y-d_0-d_2z}+j^{d_0+d_2y-d_0'-d_2'z})+4(j^{d_0^*+d_2^*y}+j^{-d_0^*-d_2^*z})\\
&\quad+2(j^{d_0^*+d_2^*y-d_0-d_2z}+j^{d_0+d_2y-d_0^*-d_2^*z})\\
&\quad+(j^{d_0^*+d_2^*y-d_0'-d_2'z}+j^{d_0'+d_2'y-d_0^*-d_2^*z})\\
&=16(j^{d_0+d_1+d_2y}+j^{-d_0-d_1-d_2z})+8(j^{d_0'+d_1'+d_2'y}+j^{-d_0'-d_1'-d_2'z}) \qquad (10)\\
&\quad+4(j^{d_0'+d_1'+d_2'y-d_0-d_1-d_2z}+j^{d_0+d_1+d_2y-d_0'-d_1'-d_2'z})+4(j^{d_0^*+d_1^*+d_2^*y}+j^{-d_0^*-d_1^*-d_2^*z})\\
&\quad+2(j^{d_0^*+d_1^*+d_2^*y-d_0-d_1-d_2z}+j^{d_0+d_1+d_2y-d_0^*-d_1^*-d_2^*z})\\
&\quad+(j^{d_0^*+d_1^*+d_2^*y-d_0'-d_1'-d_2'z}+j^{d_0'+d_1'+d_2'y-d_0^*-d_1^*-d_2^*z})
\end{aligned}$$

$$\begin{aligned}
&16(j^{d_0+d_1y}+j^{-d_0-d_2z})+8(j^{d_0'+d_1'y}+j^{-d_0'-d_1'z})\\
&\quad+4(j^{d_0'+d_1'y-d_0-d_1z}+j^{d_0+d_1y-d_0'-d_1'z})+4(j^{d_0^*+d_1^*y}+j^{-d_0^*-d_1^*z})\\
&\quad+2(j^{d_0^*+d_1^*y-d_0-d_1z}+j^{d_0+d_1y-d_0^*-d_1^*z})\\
&\quad+(j^{d_0^*+d_1^*y-d_0'-d_1'z}+j^{d_0'+d_1'y-d_0^*-d_1^*z})\\
&=16(j^{d_0+d_1y+d_2}+j^{-d_0-d_1z-d_2})+8(j^{d_0'+d_1'y+d_2'}+j^{-d_0'-d_1'z-d_2'}) \qquad (11)\\
&\quad+4(j^{d_0'+d_1'y+d_2'-d_0-d_1z-d_2}+j^{d_0+d_1y+d_2-d_0'-d_1'z-d_2'})+4(j^{d_0^*+d_1^*y+d_2^*}+j^{-d_0^*-d_1^*z-d_2^*})\\
&\quad+2(j^{d_0^*+d_1^*y+d_2^*-d_0-d_1z-d_2}+j^{d_0+d_1y+d_2-d_0^*-d_1^*z-d_2^*})\\
&\quad+(j^{d_0^*+d_1^*y+d_2^*-d_0'-d_1'z-d_2'}+j^{d_0'+d_1'y+d_2'-d_0^*-d_1^*z-d_2^*})
\end{aligned}$$

Where $y, z \in \{0, 1\}$.

By exhaust search, there are 260 sets of $(d_0, d_1, d_2, d'_0, d'_1, d'_2)$ satisfying the above equation (10) and equation (11) , please see appendix 4.

6 Conlusion

We propose a new method to judge whether the sequences over the QAM constellation constructed using new offset pairs are Golay complementary sequences. Based on this method, we prove the conjectures[16] and find some new Golay complementary sequences over 64-QAM constellation. We propose a new offset pairs, based on the pairs, we construct new Golay complementary sequences over 64-QAM constellation. 260 new Golay complementary sequences over 128-QAM constellation can also be found. One can find many Golay complementary sequences over QAM and $Q-PAM$ constellation based on the method proposed in this paper.

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Appendix 1

	d_0	d_1	d_0'	d_1'
1	0	0	1	0
2	0	0	1	2
3	0	0	2	0
4	0	0	3	0
5	0	0	3	2
6	0	1	2	3
7	0	3	2	1
8	1	0	0	0
9	1	0	1	0
10	1	0	2	0
11	1	0	3	0
12	1	2	0	0
13	1	2	1	2
14	1	2	2	0
15	1	2	3	2
16	1	3	1	1
17	2	0	0	0
18	2	0	1	0
19	2	0	1	2
20	2	0	2	0
21	2	0	3	0
22	2	0	3	2
23	3	0	0	0
24	3	0	1	0
25	3	0	2	0
26	3	0	3	0
27	3	1	3	3
28	3	2	0	0
29	3	2	1	2
30	3	2	2	0
31	3	2	3	2
32	0	0	0	0

Appendix 2

	d_0	d_1	d_2	d_0'	d_1'	d_2'
1	0	0	0	1	0	0
2	0	0	0	1	0	2
3	0	0	0	1	2	0
4	0	0	0	2	0	0
5	0	0	0	3	0	0
6	0	0	0	3	0	2
7	0	0	0	3	2	0
8	0	0	1	2	0	3
9	0	0	3	2	0	1
10	0	1	0	2	3	0
11	0	1	3	2	3	1
12	0	3	0	2	1	0
13	0	3	1	2	1	3
14	1	0	0	0	0	0
15	1	0	0	1	0	0
16	1	0	0	2	0	0
17	1	0	0	3	0	0
18	1	0	2	0	0	0
19	1	0	2	1	0	2
20	1	0	2	2	0	0
21	1	0	2	3	0	2
22	1	0	3	1	0	1
23	1	2	0	0	0	0
24	1	2	0	1	2	0
25	1	2	0	2	0	0
26	1	2	0	3	2	0
27	1	3	0	1	1	0
28	1	3	3	1	1	1
29	2	0	0	0	0	0
30	2	0	0	1	0	0
31	2	0	0	1	0	2
32	2	0	0	1	2	0
33	2	0	0	2	0	0
34	2	0	0	3	0	0
35	2	0	0	3	0	2
36	2	0	0	3	2	0
37	3	0	0	0	0	0
38	3	0	0	1	0	0
39	3	0	0	2	0	0
40	3	0	0	3	0	0
41	3	0	1	3	0	3
42	3	0	2	0	0	0
43	3	0	2	1	0	2
44	3	0	2	2	0	0
45	3	0	2	3	0	2
46	3	1	0	3	3	0
47	3	1	1	3	3	3
48	3	2	0	0	0	0
49	3	2	0	1	2	0
50	3	2	0	2	0	0
51	3	2	0	3	2	0
52	0	0	0	0	0	0

Appendix 3

	d_0	d_1	d_2	d_3	d_0'	d_1'	d_2'	d_3'
1	0	0	0	0	1	0	0	0
2	0	0	0	0	1	0	0	2
3	0	0	0	0	1	0	2	0
4	0	0	0	0	1	2	0	0
5	0	0	0	0	2	0	0	0
6	0	0	0	0	3	0	0	0
7	0	0	0	0	3	0	0	2
8	0	0	0	0	3	0	2	0
9	0	0	0	0	3	2	0	0
10	0	0	0	1	2	0	0	3
11	0	0	0	3	2	0	0	1
12	0	0	1	0	2	0	3	0
13	0	0	1	3	2	0	3	1
14	0	0	3	0	2	0	1	0
15	0	0	3	1	2	0	1	3
16	0	1	0	0	2	3	0	0
17	0	1	0	3	2	3	0	1
18	0	1	3	0	2	3	1	0
19	0	3	0	0	2	1	0	0
20	0	3	0	1	2	1	0	3
21	0	3	1	0	2	1	3	0
22	1	0	0	0	0	0	0	0
23	1	0	0	0	1	0	0	0
24	1	0	0	0	2	0	0	0
25	1	0	0	0	3	0	0	0
26	1	0	0	2	0	0	0	0
27	1	0	0	2	1	0	0	2
28	1	0	0	2	2	0	0	0
29	1	0	0	2	3	0	0	2
30	1	0	0	3	1	0	0	1
31	1	0	2	0	0	0	0	0
32	1	0	2	0	1	0	2	0
33	1	0	2	0	2	0	0	0
34	1	0	2	0	3	0	2	0
35	1	0	3	0	1	0	1	0
36	1	0	3	3	1	0	1	1
37	1	2	0	0	0	0	0	0
38	1	2	0	0	1	2	0	0
39	1	2	0	0	2	0	0	0
40	1	2	0	0	3	2	0	0
41	1	3	0	0	1	1	0	0
42	1	3	0	3	1	1	0	1
43	1	3	3	0	1	1	1	0
44	2	0	0	0	0	0	0	0
45	2	0	0	0	1	0	0	0
46	2	0	0	0	1	0	0	2
47	2	0	0	0	1	0	2	0
48	2	0	0	0	1	2	0	0
49	2	0	0	0	2	0	0	0
50	2	0	0	0	3	0	0	0
51	2	0	0	0	3	0	0	2
52	2	0	0	0	3	0	2	0
53	2	0	0	0	3	2	0	0
54	3	0	0	0	0	0	0	0
55	3	0	0	0	1	0	0	0
56	3	0	0	0	2	0	0	0
57	3	0	0	0	3	0	0	0
58	3	0	0	1	3	0	0	3
59	3	0	0	2	0	0	0	0
60	3	0	0	2	1	0	0	2
61	3	0	0	2	2	0	0	0
62	3	0	0	2	3	0	0	2
63	3	0	1	0	3	0	3	0
64	3	0	1	1	3	0	3	3
65	3	0	2	0	0	0	0	0
66	3	0	2	0	1	0	2	0
67	3	0	2	0	2	0	0	0
68	3	0	2	0	3	0	2	0
69	3	1	0	0	3	3	0	0
70	3	1	0	1	3	3	0	3
71	3	1	1	0	3	3	3	0
72	3	2	0	0	0	0	0	0
73	3	2	0	0	1	2	0	0
74	3	2	0	0	2	0	0	0
75	3	2	0	0	3	2	0	0
76	0	0	0	0	0	0	0	0

Appendix 4

	d_0	d_1	d_2	$d_0^{'}$	$d_1^{'}$	$d_2^{'}$	d_0^{*}	d_1^{*}	d_2^{*}
1	0	0	0	0	0	0	1	0	0
2	0	0	0	0	0	0	1	0	2
3	0	0	0	0	0	0	1	2	0
4	0	0	0	0	0	0	2	0	0
5	0	0	0	0	0	0	3	0	0
6	0	0	0	0	0	0	3	0	2
7	0	0	0	0	0	0	3	2	0
8	0	0	0	1	0	0	0	0	0
9	0	0	0	1	0	0	1	0	0
10	0	0	0	1	0	0	2	0	0
11	0	0	0	1	0	0	3	0	0
12	0	0	0	1	0	2	0	0	0
13	0	0	0	1	0	2	1	0	2
14	0	0	0	1	0	2	2	0	0
15	0	0	0	1	0	2	3	0	2
16	0	0	0	1	2	0	0	0	0
17	0	0	0	1	2	0	1	2	0
18	0	0	0	1	2	0	2	0	0
19	0	0	0	1	2	0	3	2	0
20	0	0	0	2	0	0	0	0	0
21	0	0	0	2	0	0	1	0	0
22	0	0	0	2	0	0	1	0	2
23	0	0	0	2	0	0	1	2	0
24	0	0	0	2	0	0	2	0	0
25	0	0	0	2	0	0	3	0	0
26	0	0	0	2	0	0	3	0	2
27	0	0	0	2	0	0	3	2	0
28	0	0	0	3	0	0	0	0	0
29	0	0	0	3	0	0	1	0	0
30	0	0	0	3	0	0	2	0	0
31	0	0	0	3	0	0	3	0	0
32	0	0	0	3	0	2	0	0	0
33	0	0	0	3	0	2	1	0	2
34	0	0	0	3	0	2	2	0	0
35	0	0	0	3	0	2	3	0	2
36	0	0	0	3	2	0	0	0	0
37	0	0	0	3	2	0	1	2	0
38	0	0	0	3	2	0	2	0	0
39	0	0	0	3	2	0	3	2	0
40	0	0	1	1	0	3	2	0	3
41	0	0	1	3	0	1	2	0	3
42	0	0	3	1	0	3	2	0	1
43	0	0	3	3	0	1	2	0	1
44	0	1	0	1	3	0	2	3	0
45	0	1	0	3	1	0	2	3	0
46	0	1	3	1	3	3	2	3	1
47	0	1	3	3	1	1	2	3	1
48	0	3	0	1	3	0	2	1	0
49	0	3	0	3	1	0	2	1	0
50	0	3	1	1	3	3	2	1	3
51	0	3	1	3	1	1	2	1	3
52	1	0	0	0	0	0	0	0	0
53	1	0	0	0	0	0	1	0	0
54	1	0	0	0	0	0	2	0	0
55	1	0	0	0	0	0	3	0	0
56	1	0	0	1	0	0	0	0	0
57	1	0	0	1	0	0	1	0	0
58	1	0	0	1	0	0	2	0	0
59	1	0	0	1	0	0	3	0	0
60	1	0	0	1	0	2	2	0	2
61	1	0	0	1	2	0	2	2	0
62	1	0	0	2	0	0	0	0	0
63	1	0	0	2	0	0	1	0	0
64	1	0	0	2	0	0	2	0	0
65	1	0	0	2	0	0	3	0	0
66	1	0	0	2	0	1	0	0	3
67	1	0	0	2	1	0	0	3	0
68	1	0	0	3	0	0	0	0	0
69	1	0	0	3	0	0	1	0	0
70	1	0	0	3	0	0	2	0	0
71	1	0	0	3	0	0	3	0	0
72	1	0	0	3	0	2	0	0	2
73	1	0	0	3	0	3	3	0	1
74	1	0	0	3	2	0	0	2	0
75	1	0	0	3	3	0	3	1	0
76	1	0	2	0	0	0	0	0	0
77	1	0	2	0	0	0	1	0	2
78	1	0	2	0	0	0	2	0	0
79	1	0	2	0	0	0	3	0	2
80	1	0	2	1	0	0	2	0	2
81	1	0	2	1	0	2	0	0	0
82	1	0	2	1	0	2	1	0	2

83	1	0	2	1	0	2	2	0	0
84	1	0	2	1	0	2	3	0	2
85	1	0	2	1	2	0	2	2	2
86	1	0	2	2	0	0	0	0	0
87	1	0	2	2	0	0	1	0	2
88	1	0	2	2	0	0	2	0	0
89	1	0	2	2	0	0	3	0	2
90	1	0	2	2	0	3	0	0	1
91	1	0	2	2	1	3	0	3	1
92	1	0	2	3	0	0	0	0	2
93	1	0	2	3	0	2	0	0	0
94	1	0	2	3	0	2	1	0	2
95	1	0	2	3	0	2	2	0	0
96	1	0	2	3	0	2	3	0	2
97	1	0	2	3	0	3	3	0	1
98	1	0	2	3	2	0	0	2	2
99	1	0	2	3	3	3	3	1	1
100	1	0	3	0	0	1	1	0	1
101	1	0	3	0	0	3	1	0	1
102	1	2	0	0	0	0	0	0	0
103	1	2	0	0	0	0	1	2	0
104	1	2	0	0	0	0	2	0	0
105	1	2	0	0	0	0	3	2	0
106	1	2	0	1	0	0	2	2	0
107	1	2	0	1	0	2	2	2	2
108	1	2	0	1	2	0	0	0	0
109	1	2	0	1	2	0	1	2	0
110	1	2	0	1	2	0	2	0	0
111	1	2	0	1	2	0	3	2	0
112	1	2	0	2	0	0	0	0	0
113	1	2	0	2	0	0	1	2	0
114	1	2	0	2	0	0	2	0	0
115	1	2	0	2	0	0	3	2	0
116	1	2	0	2	3	0	0	1	0
117	1	2	0	2	3	1	0	1	3
118	1	2	0	3	0	0	0	2	0
119	1	2	0	3	0	2	0	2	2
120	1	2	0	3	2	0	0	0	0
121	1	2	0	3	2	0	1	2	0
122	1	2	0	3	2	0	2	0	0
123	1	2	0	3	2	0	3	2	0
124	1	2	0	3	3	0	3	1	0
125	1	2	0	3	3	3	3	1	1

126	1	3	0	0	1	0	1	1	0
127	1	3	0	0	3	0	1	1	0
128	1	3	3	0	1	3	1	1	1
129	1	3	3	0	3	1	1	1	1
130	2	0	0	0	0	0	0	0	0
131	2	0	0	0	0	0	1	0	0
132	2	0	0	0	0	0	1	0	2
133	2	0	0	0	0	0	1	2	0
134	2	0	0	0	0	0	2	0	0
135	2	0	0	0	0	0	3	0	0
136	2	0	0	0	0	0	3	0 -	2
137	2	0	0	0	0	0	3	2	0
138	2	0	0	0	0	1	2	0	3
139	2	0	0	0	0	3	2	0	1
140	2	0	0	0	1	0	2	3	0
141	2	0	0	0	1	3	2	3	1
142	2	0	0	0	3	0	2	1	0
143	2	0	0	0	3	1	2	1	3
144	2	0	0	1	0	0	0	0	0
145	2	0	0	1	0	0	1	0	0
146	2	0	0	1	0	0	2	0	0
147	2	0	0	1	0	0	3	0	0
148	2	0	0	1	0	2	0	0	0
149	2	0	0	1	0	2	1	0	2
150	2	0	0	1	0	2	2	0	0
151	2	0	0	1	0	2	3	0	2
152	2	0	0	1	0	3	1	0	1
153	2	0	0	1	2	0	0	0	0
154	2	0	0	1	2	0	1	2	0
155	2	0	0	1	2	0	2	0	0
156	2	0	0	1	2	0	3	2	0
157	2	0	0	1	3	0	1	1	0
158	2	0	0	1	3	3	1	1	1
159	2	0	0	2	0	0	0	0	0
160	2	0	0	2	0	0	1	0	0
161	2	0	0	2	0	0	1	0	2
162	2	0	0	2	0	0	1	2	0
163	2	0	0	2	0	0	2	0	0
164	2	0	0	2	0	0	3	0	0
165	2	0	0	2	0	0	3	0	2
166	2	0	0	2	0	0	3	2	0
167	2	0	0	3	0	0	0	0	0

168	2	0	0	3	0	0	1	0	0
169	2	0	0	3	0	0	2	0	0
170	2	0	0	3	0	0	3	0	0
171	2	0	0	3	0	1	3	0	2
172	2	0	0	3	0	2	0	0	0
173	2	0	0	3	0	2	1	0	2
174	2	0	0	3	0	2	2	0	0
175	2	0	0	3	0	2	3	0	2
176	2	0	0	3	1	0	3	3	0
177	2	0	0	3	1	1	3	3	3
178	2	0	0	3	2	0	0	0	0
179	2	0	0	3	2	0	1	2	0
180	2	0	0	3	2	0	2	0	0
181	2	0	0	3	2	0	3	2	0
182	3	0	0	0	0	0	0	0	0
183	3	0	0	0	0	0	1	0	0
184	3	0	0	0	0	0	2	0	0
185	3	0	0	0	0	0	3	0	0
186	3	0	0	1	0	0	0	0	0
187	3	0	0	1	0	0	1	0	0
188	3	0	0	1	0	0	2	0	0
189	3	0	0	1	0	0	3	0	0
190	3	0	0	1	0	1	1	0	3
191	3	0	0	1	0	2	0	0	2
192	3	0	0	1	1	0	1	3	0
193	3	0	0	1	2	0	0	2	0
194	3	0	0	2	0	0	0	0	0
195	3	0	0	2	0	0	1	0	0
196	3	0	0	2	0	0	2	0	0
197	3	0	0	2	0	0	3	0	0
198	3	0	0	2	0	3	0	0	1
199	3	0	0	2	3	0	0	1	0
200	3	0	0	3	0	0	0	0	0
201	3	0	0	3	0	0	1	0	0
202	3	0	0	3	0	0	2	0	0
203	3	0	0	3	0	0	3	0	0
204	3	0	0	3	0	2	2	0	2
205	3	0	0	3	2	0	2	2	0
206	3	0	1	0	0	1	3	0	3
207	3	0	1	0	0	3	3	0	3
208	3	0	2	0	0	0	0	0	0
209	3	0	2	0	0	0	1	0	2
210	3	0	2	0	0	0	2	0	0
211	3	0	2	0	0	0	3	0	2
212	3	0	2	1	0	0	0	0	2
213	3	0	2	1	0	1	1	0	3
214	3	0	2	1	0	2	0	0	0
215	3	0	2	1	0	2	1	0	2
216	3	0	2	1	0	2	2	0	0
217	3	0	2	1	0	2	3	0	2
218	3	0	2	1	1	1	1	3	3
219	3	0	2	1	2	0	0	2	2
220	3	0	2	2	0	0	0	0	0
221	3	0	2	2	0	0	1	0	2
222	3	0	2	2	0	0	2	0	0
223	3	0	2	2	0	0	3	0	2
224	3	0	2	2	0	1	0	0	3
225	3	0	2	2	3	1	0	1	3
226	3	0	2	3	0	0	2	0	2
227	3	0	2	3	0	2	0	0	0
228	3	0	2	3	0	2	1	0	2
229	3	0	2	3	0	2	2	0	0
230	3	0	2	3	0	2	3	0	2
231	3	0	2	3	2	0	2	2	2
232	3	1	0	0	1	0	3	3	0
233	3	1	1	0	3	0	3	3	0
234	3	1	1	0	1	3	3	3	3
235	3	2	0	0	3	1	3	3	3
236	3	2	0	0	0	0	0	0	0
237	3	2	0	0	0	0	1	2	0
238	3	2	0	0	0	0	2	0	0
239	3	2	0	0	0	0	3	2	0
240	3	2	0	1	0	0	0	2	0
241	3	2	0	1	0	2	0	2	2
242	3	2	0	1	1	0	1	3	0
243	3	2	0	1	1	1	1	3	3
244	3	2	0	1	2	0	0	0	0
245	3	2	0	1	2	0	1	2	0
246	3	2	0	1	2	0	2	0	0
247	3	2	0	1	2	0	3	2	0
248	3	2	0	2	0	0	0	0	0
249	3	2	0	2	0	0	1	2	0
250	3	2	0	2	0	0	2	0	0
251	3	2	0	2	0	0	3	2	0
252	3	2	0	2	1	0	0	3	0
253	3	2	0	2	1	3	0	3	1

254	3	2	0	3	0	0	2	2	0
255	3	2	0	3	0	2	2	2	2
256	3	2	0	3	2	0	0	0	0
257	3	2	0	3	2	0	1	2	0
258	3	2	0	3	2	0	2	0	0
259	3	2	0	3	2	0	3	2	0
260	0	0	0	0	0	0	0	0	0