

# FUNSHADE: Function Secret Sharing for Two-Party Secure Thresholded Distance Evaluation

Alberto Ibarrondo  
Copper.co\*  
Sophia Antipolis, France  
ibarrond@eurecom.fr

Hervé Chabanne  
Idemia & Telecom Paris  
Paris, France

Melek Önen  
EURECOM  
Sophia Antipolis, France

## ABSTRACT

We propose a novel privacy-preserving, two-party computation of various distance metrics (e.g., Hamming distance, Scalar Product) followed by a comparison with a fixed threshold, which is known as one of the most useful and popular building blocks for many different applications including machine learning, biometric matching, etc. Our solution builds upon recent advances in function secret sharing and makes use of an optimized version of arithmetic secret sharing. Thanks to this combination, our new solution named FUNSHADE is the first to require only one round of communication and two ring elements of communication in the online phase, outperforming all prior state-of-the-art schemes while relying on lightweight cryptographic primitives. Lastly, we implement our solution from scratch in portable C and expose it in Python, testifying its high performance by running secure biometric identification against a database of 1 million records in  $\sim 10$  seconds with full correctness and 32-bit precision, without parallelization.

## KEYWORDS

Function Secret Sharing, Secure Two Party Computation, Scalar Product, Hamming Distance

## 1 INTRODUCTION

The computation of privacy-preserving distance metrics  $f_{dist}(\mathbf{x}, \mathbf{y})$  between two vectors  $\mathbf{x}, \mathbf{y}$  followed by a comparison with a threshold  $\theta$  is a very popular building block in many applications in need of privacy protection, including machine learning (e.g., k-nearest neighbors [68], linear regression [35]), biometrics (e.g., biometric authentication [46, 57], biometric identification [32]) etc.

The literature counts many solutions based on various cryptographic techniques that allow computation over sensitive data while preserving its privacy: *Secure Multiparty Computation* (MPC) (garbled circuits [65], secret sharing (SS) [39, 60]) to split the distance computation across multiple entities [19, 29, 33], *Fully Homomorphic Encryption* (FHE) [23, 34, 36] supporting addition and multiplication between ciphertexts [4, 7, 22], and *Functional Encryption* (FE) [2, 9] as a public-key encryption scheme that supports evaluation of scalar products when decrypting the ciphertexts [3, 10].

However, not all operations are born equal. While linear operations are widely covered by all the privacy-preserving techniques, the protection of non-linear operations including the comparison to

a threshold  $\theta$  is much harder to attain. Computing this non-linear operation with most MPC primitives is often communication intensive (e.g., [29, 64]) both in terms of communication size and in number of rounds; FHE-based techniques must resort to computation-intensive algorithms [24, 42]; and FE-based techniques are limited to linear function evaluations. Luckily, recent solutions [11, 14, 58] show a considerable improvement to securely realize the comparison to  $\theta$  by resorting to Function Secret Sharing (FSS) primitives.

In [15], the authors study the computation of distance metrics. They propose GSHADE, a decomposition of each metric into a combination of local single-input functions and a cross-product, and preserve the privacy of these operations via Oblivious Transfer [54].

We draw inspiration from the family of distance metrics covered in GSHADE. Integrating FSS-based threshold comparison primitives from [11] with an optimized version of Secret Sharing [53] in a two-party computation (2PC) protocol to perform privacy-preserving distance metric computations with a subsequent comparison to  $\theta$ . To summarize our contributions, our solution:

- requires just one round of communication in the online phase, lowering the communication costs with respect to the two-round state-of-the-art solutions from AriaNN [58] and Boyle et. al. [11] by merging the communication required for the scalar product with that of the comparison to  $\theta$ ,
- sends two ring elements only in the online phase, reducing the communication size of previous solutions by a factor of  $2l$  (for input vectors with  $l$  elements),
- features 100% correctness in the comparison result, as opposed to [58],
- is implemented and open-sourced in a portable C module with lightweight wrapping to high-level languages such as Python, Rust or Golang,
- is extensively tested, showcasing its high efficiency and correctness in a variety of scenarios (LAN/WAN, Authentication/ Identification), requiring less than 300ms for realistic biometric identification against 5000 identities with full 32-bit precision, and  $\sim 10$ s against 1 million identities.

The paper is outlined as follows. Section 2 describes the preliminaries, the distance metrics we consider in this work and some applications. Section 3 details the proposed solution, including its security analysis. Section 4 tackles the implementation and experiments. Section 5 addresses previous work and positions our contribution, wrapping up with the conclusions in Section 6.

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## 2 PRELIMINARIES

### Notation

We use bold letters to denote vectors (e.g.,  $\mathbf{x}$ ,  $\mathbf{y}$ ) and plain letters (e.g.,  $a$ ,  $k$ ) for scalars.  $\mathbf{x}^{(i)}$  denotes the  $i$ th element of vector  $\mathbf{x}$ . For convenience we omit the  $(i)$  superscripts in element-wise additions of the form  $\Sigma[\mathbf{a}^{(i)} + \mathbf{b}^{(i)} + \dots]$ . We write  $\mathbf{a} \cdot \mathbf{b} = \mathbf{c}$  to denote the element-wise multiplication of two vectors where  $\mathbf{c}^{(i)} = \mathbf{a}^{(i)}\mathbf{b}^{(i)}$ , and  $\mathbf{a}^T \mathbf{b}$  to denote the inner (scalar) product between two vectors.

$P_0, P_1$  denote the two computing parties in the 2PC paradigm. We generalize behavior common to these two computing parties by resorting to  $P_j$ , where  $j \in \{0, 1\}$ . Similarly, we reserve  $R_{descr}$  to indicate an entity in our scenario playing a role with a certain description, e.g.,  $R_{setup}$  for the entity in charge of the setup,  $R_{in_x}$  for the entity holding the input vector  $\mathbf{x}$ . We write  $P_j \supseteq R_{descr}$  to denote that party  $P_j$  takes on a role  $R_{descr}$ . We use  $q \leftarrow 4$  to set a local variable  $q$  to 4, and  $P_a : \text{SEND } q \Rightarrow P_b$  for party  $a$  sending value  $q$  to party  $b$ . We note  $\mathcal{U}_{[S]}$  as the uniform random distribution in the set  $S$ , and write  $r \sim \mathcal{U}_{[S]}$  to indicate sampling that distribution and assigning the sample to  $r$ . We employ  $1_{x \in A}$  to denote the indicator function (e.g.,  $1_{x > 5} = 1 \iff x > 5$ ):

$$1_{x \in A} \equiv 1_A(x) \triangleq \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A, \end{cases}$$

As a special case of indicator function, the unit step function is defined as  $1_{x \in \mathbb{R}_+^*} = 1_{x \geq 0}$ . We implicitly consider a two's complement encoding to map between signed and unsigned  $n$ -bit integers, a bijective mapping between  $[-2^{n-1}, 2^{n-1} - 1]$  and  $[0, 2^n - 1]$  by applying  $\text{mod } 2^n$ , where the interval of negative values  $[-2^{n-1}, -1]$  is mapped to the upper half of the unsigned interval  $[2^{n-1}, 2^n - 1]$ . As such, the unit step function for  $n$ -bit integers corresponds to  $1_{x \in \mathbb{Z}_n^+} = 1_{0 \leq x \leq 2^{n-1} - 1}$ .

We write  $\langle x \rangle$  to indicate that value  $x$  is arithmetically secret shared into shares  $(x_0, x_1)$  among computing parties ( $P_0, P_1$ ) such that  $P_0$  holds the share  $x_0$  and  $P_1$  holds the share  $x_1$ . Likewise, we write  $\langle\langle x \rangle\rangle$  to indicate that value  $x$  is  $\Pi$ -secret shared (Section 2.1.2) into three shares  $(\Delta_x, \delta_{x0}, \delta_{x1})$ , where both parties ( $P_0, P_1$ ) hold  $\Delta_x$  and each party  $P_j$  holds  $\delta_{xj}$ .

### 2.1 Multi-Party Computation

Secure multi-party computation (or MPC) [6, 21, 39, 65] allows two or more parties to compute any mathematical function on private inputs without revealing anything but the output of the function. Typically, MPC is instantiated in the preprocessing model, where specially crafted randomness is generated in an offline input-independent phase from either a trusted dealer or via an offline interaction, and then this randomness is used in the online phase to compute the function, once the inputs are known. This two-phase approach yields considerable performance benefits. Some examples of this correlated randomness include Beaver multiplication triples [5] and garbled circuit preprocessing [29, 65].

When used to evaluate circuits based on only binary or only arithmetic interactions, MPC protocols present very fast online execution. However, applications such as biometrics or machine learning require a combination of linear operations (additions and multiplications over a large ring) and non-linear operations such as integer comparison or truncation. The cost of blindly implementing

these two types of operations with only one MPC circuit type can be prohibitively high. To address this, many works have tackled mixed-mode MPC to provide efficient conversions between arithmetic and binary domains, supporting both linear and non-linear operations [19, 29, 50, 53]. Yet, these conversions often entail a hefty communication overhead in the online phase.

In line with the TinyTable protocol [27] to secret share truth tables in a succinct manner, Boyle et al. propose a very promising approach [11, 14] based on Function Secret Sharing (FSS) [12, 13]. Offering the same online communication and round complexity for non-linear function evaluations as for pure arithmetic computations in arithmetic-only circuits, FSS relies on fast symmetric cryptography primitives to also yield fast online evaluation.

The present work will benefit from standard arithmetic secret sharing techniques [5], more evolved secret sharing techniques emanating from research in mixed-mode operations [53] and modern FSS approaches [11] to achieve a lightweight and highly communication efficient biometric matching protocol. As such, we now delve into the details of these techniques.

**2.1.1 Additive Secret Sharing.** Secret sharing is a cryptographic primitive that allows a secret  $x$  to be shared among  $n$  parties, such that any  $t$  of them can reconstruct the secret. The secret sharing scheme is defined by  $k$ , the number of parties, and threshold  $t$ , minimum number of parties required to reconstruct the secret. In the domain of two-party computation (2PC), the number of parties is  $n = 2$  and the threshold is  $t = 2$ . This work focuses on 2PC arithmetic secret sharing in rings (shortened to SS for convenience), where a secret value  $x$  is split into two random shares  $x_0$  and  $x_1$  such that  $x = x_0 + x_1 \text{ mod } N$ , with  $N$  being the ring size. The shares are distributed to the two computing parties such that party  $P_j$  receives the share  $x_j$ . With this sharing scheme, the two parties can perform local addition/subtraction of two secret shared values. Additionally, parties can resort to Beaver's multiplication triples [5] to perform multiplication at the cost of one round of communication:

$$\begin{aligned} \text{SS.add: } \text{Online}(P_0, P_1): \quad & \langle x \pm y \rangle = \langle x \rangle \pm \langle y \rangle \\ \text{SS.mult: } \text{Offline}(R_{setup}): \quad & \langle a \rangle, \langle b \rangle \sim \mathcal{U}_{[\mathbb{Z}_N^{2 \times 2}]} \\ & \langle c \rangle \leftarrow \langle a \cdot b \rangle \\ & \text{SEND}(a_j, b_j, c_j) \Rightarrow P_j \\ \text{Online}(P_0, P_1): \quad & \text{SEND}(x_j - a_j, y_j - b_j) \Rightarrow P_{1-j} \\ & \langle x \cdot y \rangle = \langle b \rangle (x - a) + \langle a \rangle (y - b) + \langle c \rangle \\ & \quad + (x - a)(y - b) \end{aligned} \tag{1}$$

At the end of the computation, the resulting secret shared value can be reconstructed by sending both shares to a chosen party  $P_{res}$ , to add the two shares together and reconstruct the result. We work with  $N = 2^n$  for values of  $n \in \{8, 16, 32, 64\}$  to benefit from a considerable speedup when dealing with  $n$ -bit modular arithmetic thanks to native integer types present in modern computers.

Of special interest for this work, computing a scalar product  $\mathbf{x}^T \mathbf{y} = \sum_{i=1}^l \mathbf{x}^{(i)} \cdot \mathbf{y}^{(i)}$  with SS requires sending 2 terms per multiplication, for a total of  $2l$  values sent.

**2.1.2  $\Pi$ -Secret Sharing.** Originally inspired by ASTRA [20] in the 3PC scenario, ABY2.0 [53] introduced a novel way to perform

additive secret sharing in 2PC<sup>1</sup>, where a value  $x$  is split into three random shares  $(\Delta_x, \delta_{x0}, \delta_{x1})$  such that  $\Delta_x = x + \delta_{x0} + \delta_{x1} \pmod N$ . The  $\delta$ -shares  $\delta_{xj}$  are distributed to each computing party  $P_j$  forming an arithmetic secret sharing  $\langle \delta_x \rangle$  of  $\delta_x = \delta_{x0} + \delta_{x1}$ , while the  $\Delta$ -share  $\Delta_x$  is held by both parties at once. We name this sharing scheme as  $\Pi$ -secret sharing, due to the "horizontally" mutual  $\Delta$ -share and the two "vertically" separated  $\delta$ -shares, and denote the  $\Pi$ -sharing of value  $x$  as  $\langle\langle x \rangle\rangle$ . The  $\Pi$ -sharing scheme allows local addition/subtraction, and multiplication at the cost of one round of communication. The essential difference with respect to the SS scheme is that the  $\delta$ -shares can be precomputed (leaving only the  $\Delta$ -share to be determined in the online phase), and thus carry extra correlation that was not possible with standard SS. The main arithmetic operations in PISS are defined as follows:

$$\begin{aligned}
\text{PISS.add: } & \text{Online}(P_0, P_1): \langle\langle x \pm y \rangle\rangle = \langle\langle x \rangle\rangle \pm \langle\langle y \rangle\rangle \\
\text{PISS.mult: } & \text{Offline}(R_{\text{setup}}): \langle\delta_x \rangle, \langle\delta_y \rangle, \langle\delta_z \rangle \sim \mathcal{U}_{\mathbb{Z}_N^{3 \times 2}} \\
& \langle\delta_{xy} \rangle \leftarrow \langle\delta_x \cdot \delta_y \rangle \\
& \text{SEND } (\delta_{xj}, \delta_{yj}, \delta_{xyj}) \Rightarrow P_j \\
& \text{Online}(P_0, P_1): \langle x \cdot y \rangle \equiv \langle z \rangle \leftarrow j \cdot \Delta_x \Delta_y - \Delta_x \langle\delta_y \rangle \\
& \quad - \Delta_y \langle\delta_x \rangle + \langle\delta_{xy} \rangle \\
& \langle\Delta_z \rangle \equiv \langle z \rangle + \langle\delta_z \rangle \\
& \text{SEND } (\Delta_{zj}) \Rightarrow P_{1-j} \\
& \langle\langle z \rangle\rangle \equiv (\Delta_{z0} + \Delta_{z1}, \langle\delta_z \rangle)
\end{aligned} \tag{2}$$

Crucially, the online phase of the  $\Pi$ -sharing multiplication first computes a local arithmetic sharing of the result, and then uses one round of communication to convert the result back into  $\Pi$ -shares. As promptly explained in [53], this moves the communication from the multiplication inputs to the multiplication outputs, which yields sizeable advantages in terms of communication size for operations such as the scalar product: computing a scalar product  $\mathbf{x}^T \mathbf{y} = \sum_{i=1}^l \mathbf{x}^{(i)} \cdot \mathbf{y}^{(i)}$  with PISS requires sending 2 values only for the entire operation, thus reducing the communication size by a factor of  $l$  with respect to SS.

**2.1.3 Function Secret Sharing.** A 2PC Functional Secret Sharing (FSS) scheme [12, 13] for a function family  $\mathcal{F}$  splits a function  $f \in \mathcal{F}$  into two additive shares  $(f_0, f_1)$ , such that each  $f_j$  hides  $f$  and  $f_0(x) + f_1(x) = f(x)$  for every input  $x$ . Beyond trivial solutions such as secret-sharing the truth-table of  $f$ , FSS schemes seek succinct descriptions of  $f_j$  (function keys  $\mathbf{k}_0, \mathbf{k}_1$ ) with efficient online execution. Since both function shares must evaluate on the same value  $x$ , this value must be made public to both computing parties  $P_j$ . To maintain input data privacy, a random mask  $r$  is added to the secret input  $x$ , so that the opened value  $\hat{x} = x + r$  completely hides  $x$  before using it as input to the FSS evaluation. In order to obtain full correctness on the function evaluation with respect to  $f(x)$ , the class of functions  $\mathcal{F}$  is restricted to  $f_r(x) = f(x + r)$ , where the mask is known by the dealer and used for the key generation.

For addition and multiplication gates over a ring  $\mathbb{Z}_{2^n}$ , the FSS gates correspond to Beaver's protocol [5]. A much more interesting case arises in [11, 14], where non-linear operations including zero-test, integer comparison or bit decomposition are efficiently constructed using a small number of invocations of FSS primitives. Luckily, these FSS gates make a black-box use of any secure pseudorandom generator (PRG), yielding short keys and fast implementations based on AES.

Grounded on the MPC preprocessing model, a FSS gate is composed of two algorithms:

- $\text{Gen}(1^\lambda, f) \rightarrow (\mathbf{k}_0, \mathbf{k}_1)$  is a PPT key generation algorithm that, given the security parameter  $\lambda$  and the description of a function  $f : \mathbb{G}_{in} \mapsto \mathbb{G}_{out}$ , outputs a pair of function keys  $(\mathbf{k}_0, \mathbf{k}_1)$  containing the descriptions for  $f_0, f_1$  and the input mask shares  $r_0, r_1$  respectively.
- $\text{Eval}(j, \mathbf{k}_j, \hat{x}) \rightarrow f(x)$  is a polynomial-time deterministic algorithm that, given the party index  $j$ , the function key  $\mathbf{k}_j$  and the masked input  $\hat{x}$  outputs an additive share  $f_j(\hat{x})$ , such that  $f_0(\hat{x}) + f_1(\hat{x}) = f(x)$ .

As central building block of many FSS gates, we recall the concept of Distributed Comparison Function (DCF) (Section 3 of [11]) to be a comparison function  $f_{\alpha, \beta}^<$  outputting  $\beta$  if  $x > \alpha$  and zero otherwise. Built on top of two evaluations of DCF, [11] later proposes a FSS gate for Interval Containment (IC) computing  $f_{p,q}(x) = 1_{x \in [p,q]}$  (Section 4.1 of [11]). To compute the unit step function of a  $n$ -bit signed integer, it suffices to employ their construction (detailed in Figure 3 of [11]) setting  $p = 0$  and  $q = 2^{n-1} - 1$ , obtaining  $1_{p \leq x \leq q}$ . For convenience, we detail this FSS gate instantiation in Protocols 1 (key generation) and 2 (evaluation), keeping the DCF calls to the original protocol in [11].

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**Algorithm 1**  $\text{FSS.Gen}^{IC}(\lambda, n, r) \rightarrow \mathbf{k}_0^{IC}, \mathbf{k}_1^{IC}$

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**Inputs:**  $\lambda$ : computational security parameter.  
 $r$ : Mask for the input to the function.  
**Output:**  $\mathbf{k}_0, \mathbf{k}_1$ : preprocessing keys, to send to  $P_0, P_1$  respectively.  
 $\langle\delta_x \rangle, \langle\delta_y \rangle$ :  $\delta$ -shares of input vectors, to send to  $P_{in_x}, P_{in_y}$  (input owners) resp.

**Note:** All arithmetic operations  $(+, -, \cdot)$  are defined in  $\mathbb{Z}_{2^n}$ , thus their results are susceptible to "overflow" due to modular reduction.

Define the interval  $[p, q]$  for sign extraction:

$$1: p \leftarrow 0; \quad q \leftarrow 2^{n-1} - 1$$

Generate a DCF for  $\gamma$ , an arbitrary value above the two interval limits:

$$2: \gamma \leftarrow (2^n - 1)$$

$$3: (\mathbf{k}_{\gamma 0}, \mathbf{k}_{\gamma 1}) \leftarrow \text{FSS.Gen}_n^<(1^\lambda, \gamma + r, 1, \mathcal{U}_{\mathbb{Z}_{2^n}})$$

Generate the correction terms<sup>2</sup> to fix overflows:

$$4: c \leftarrow 1_{p+r > q+r} + 1_{q+r+1 > q+1} - 1_{p+r > p} + 1_{p+r=2^n-1}$$

$$5: c_0 \sim \mathcal{U}_{\mathbb{Z}_{2^n}}; \quad c_1 \leftarrow c - c_0$$

Compose keys:

$$6: \mathbf{k}_0^{IC} \leftarrow (\mathbf{k}_{\gamma 0}, c_0); \quad \mathbf{k}_1^{IC} \leftarrow (\mathbf{k}_{\gamma 1}, c_1)$$

$$7: \text{return } \mathbf{k}_0^{IC}, \mathbf{k}_1^{IC}$$


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<sup>1</sup>Note that ABY2.0 [53] refers to arithmetic secret sharing as  $[\cdot]$ -sharing and  $\Pi$ -secret sharing as  $\langle \cdot \rangle$ -sharing.

<sup>2</sup>The correction terms test three standard overflow cases and one corner case. The standard case terms test if  $q+r$  overflows ( $1_{p+r > q+r}$ ), if  $q+r+1$  overflows ( $1_{q+r+1 > q+1}$ ),

**Table 1: Reformulation of the distance metrics into a composition of local evaluations of  $f_{local}$  and the cross product  $f_{cp} \cdot \mathbf{x}^T \mathbf{y}$** 

Distance Metric	Formula	$f_{local}(\mathbf{x}) + f_{local}(\mathbf{y}) + f_{cp} \cdot \mathbf{x}^T \mathbf{y}$	$f_{local}(\mathbf{v})$	$f_{cp}$
Scalar/Inner Product	$\sum \mathbf{x}^{(i)} \cdot \mathbf{y}^{(i)}$	$0 + 0 + 1 \sum (\mathbf{x}^{(i)} \cdot \mathbf{y}^{(i)})$	$0$	$1$
Hamming Distance	$\sum \mathbf{x}^{(i)} \oplus \mathbf{y}^{(i)}$	$\sum (\mathbf{x}^{(i)})^2 + \sum (\mathbf{y}^{(i)})^2 - 2 \sum (\mathbf{x}^{(i)} \cdot \mathbf{y}^{(i)})$	$\sum (\mathbf{v})^2$	$-2$
Squared Euclidean	$\sum (\mathbf{x}^{(i)} - \mathbf{y}^{(i)})^2$	$\sum (\mathbf{x}^{(i)})^2 + \sum (\mathbf{y}^{(i)})^2 - 2 \sum (\mathbf{x}^{(i)} \cdot \mathbf{y}^{(i)})$	$\sum (\mathbf{v})^2$	$-2$
Squared Mahalanobis	$(\mathbf{x} - \mathbf{y})^T \mathbf{M} (\mathbf{x} - \mathbf{y})$	$\mathbf{x}^T \mathbf{M} \mathbf{x} + \mathbf{y}^T \mathbf{M} \mathbf{y} - 2 (\mathbf{x}^T \mathbf{M}) \cdot \mathbf{y}$	$(\mathbf{v}^T \mathbf{M} \mathbf{v})$	$-2$

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**Algorithm 2** FSS.Eval<sup>IC</sup>( $j, \mathbf{k}_j, \hat{x}$ )  $\rightarrow o_0, o_1$ 


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**Inputs:**  $j$ : The party number,  $j \in \{0, 1\}$ .

 $\mathbf{k}_j$ : The key for  $P_j$ , composed of a DCF key for  $\gamma$  and a correction share  $c_j$ .

 $\hat{x}$ : Masked public input, result of reconstructing  $x + r$ .

**Output:**  $o_0, o_1$ : Additive secret shares of  $1_{x \in [0, 2^{n-1}-1]}$ .

 Define the interval  $[p, q]$  for sign extraction:

 1:  $p \leftarrow 0$ ;  $q \leftarrow 2^{n-1} - 1$ 

 Deserialize key and obtain local overflow term  $\eta$ :

 2:  $(\mathbf{k}_{\gamma j}, c_j) \leftarrow \mathbf{k}_j$ 

 3:  $\eta \leftarrow 1_{\hat{x} > p} - 1_{\hat{x} > q+1}$ 

Evaluate the DCF with two inputs and compute result:

 4:  $o_j^L \leftarrow \text{FSS.Eval}_n^<(j, \mathbf{k}_{\gamma j}, 1, \hat{x} - 1)$ 

 5:  $o_j^R \leftarrow \text{FSS.Eval}_n^<(j, \mathbf{k}_{\gamma j}, 1, \hat{x} - q - 2)$ 

 6: **return**  $o_j \leftarrow j \cdot \eta - o_j^L + o_j^R + c_j$ 


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**2.1.4 On security guarantees.** This work focuses on 2PC with security against a semi-honest adversary non-adaptively corrupting at most one computing party. Also referred to as *Honest-but-Curious*, the computing parties  $P_j$  are to follow the protocol faithfully, while a party corrupted by the adversary will try to extract as much information as possible from his computation.

Employing simulation based security proofs [17, 38], previous works have proven SS and IISS to be perfectly information theoretic secure against computationally unbounded semi-honest adversaries [29, 53]. In contrast, FSS schemes rely on the security of the underlying PRG to be proven computationally secure against time bounded adversaries [11].

## 2.2 Thresholded Distance Metrics and Applications

Inspired by GSHADE [15], we introduce the thresholded distance metrics that we seek to protect in this work alongside motivating real-world applications:

- **Scalar Product:**  $f_{SP}(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \mathbf{y} = \sum_{i=1}^n \mathbf{x}^{(i)} \mathbf{y}^{(i)}$  is a common distance metric in face recognition where  $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$  are two vectors of the same dimension.
- **Hamming Distance:**  $f_{HD}(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^n (\mathbf{x}^{(i)} \oplus \mathbf{y}^{(i)})$  is a distance metric frequently used in information theory and

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and if  $p + r$  does not overflow ( $1_{p+r > p}$ , which is always 1 in our instantiation since  $p = 0$  and  $r < 2^n - 1$ ). The corner case term tests whether  $p + r = 2^n - 1$  ( $1_{p+r=2^n-1}$ , yielding zero except if  $r = 2^n - 1$  in our case). Proofs of the need of these correctness terms are given in [11].

computer science to measure the distance between two bit-strings. Besides its interest in iris and fingerprint recognition, it is the base of the perceptual hashing technique [49] used in image comparison, with applications ranging from image watermarking [31] to detection of Child Sexual Abuse Material (CSAM)[25].

- **Squared Euclidean Distance:**  $f_{SED}(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^n (\mathbf{x}^{(i)} - \mathbf{y}^{(i)})^2$  is a distance metric used in many machine learning applications, such as clustering [48]. It is also used in the context of face recognition [37].
- **Squared Mahalanobis Distance:**  $f_{MD}(\mathbf{x}, \mathbf{y}) = (\mathbf{x} - \mathbf{y})^T \mathbf{M} (\mathbf{x} - \mathbf{y})$  is a distance metric used in many machine learning applications, such as clustering [48] and recognition of hand shape/keystrokes/signatures [15].

## 3 OUR SOLUTION

We now describe our solution for a lightweight and efficient 2PC distance metric with comparison, requiring a single round of communication and two ring elements in the online phase.

### 3.1 Distance Metrics

We start off by writing the generic function we wish to protect:

$$f(f_{dist}, \theta, \mathbf{x}, \mathbf{y}) = 1_{f_{dist}(\mathbf{x}, \mathbf{y}) \geq \theta} = \begin{cases} 1 & \text{if } f_{dist}(\mathbf{x}, \mathbf{y}) \geq \theta, \\ 0 & \text{if } f_{dist}(\mathbf{x}, \mathbf{y}) < \theta, \end{cases} \quad (3)$$

To adapt to 2PC, we reformulate the distance metrics  $f_{dist}$  from Section 2.2 as

$$z = f_{dist}(\mathbf{x}, \mathbf{y}) = f_{local}(\mathbf{x}) + f_{local}(\mathbf{y}) + f_{cp} \cdot \sum (\mathbf{x}^{(i)} \cdot \mathbf{y}^{(i)}) \quad (4)$$

where  $f_{local}$  is a function that can be computed locally by each input data holder, and  $f_{cp}$  is the "cross product" constant factor that applies to the scalar product evaluation present in all the metrics. Using this blueprint, we rewrite all the distance metrics in Table 1.

We remark that the Hamming Distance can be reformulated as the Squared Euclidean Distance as long as the input vectors are composed of binary values  $\mathbf{x}^{(i)}, \mathbf{y}^{(i)} \in \{0, 1\} \forall i$ , since the boolean XOR operation between two binary values can be rewritten in the arithmetic domain as  $\mathbf{x}^{(i)} \oplus \mathbf{y}^{(i)} = (\mathbf{x}^{(i)} - \mathbf{y}^{(i)})^2$ , the square of its difference.

### 3.2 The Two-Party Computation Scenario

This work is set in a context of two party computation (2PC), where two non-colluding parties ( $P_0, P_1$ ) are in charge of performing the secure computation. We argue the security of our protocols assuming these two parties behave in a semi-honest manner, following the protocol steps faithfully while attempting to extract as much

information as possible from the process. In this context, Funshade guarantees privacy of all the input data against a semi-honest adversary corrupting either of the two parties (see Section 3.6 for a detailed security analysis).

Our work is set in the *MPC with preprocessing* model, leveraging off a setup/offline phase in order to optimize the cost of the online (input-dependent) phase. Much like for other 2PC scenarios [19, 29, 53, 55], our protocols also require several additional *roles* to be filled in a complete solution:

- $R_{setup}$ : The setup covers the generation of preprocessing material during the offline phase, and its distribution to the computing parties involved in the online phase.
- $R_{in_x}, R_{in_y}$ : The input data holders, with access to the input vectors  $\mathbf{x}$  and  $\mathbf{y}$  respectively. These vectors are to be shared with the computing parties either during the offline phase if available beforehand, or at the beginning of the online phase. By convention, and following Equation 3, we employ  $\mathbf{x}$  for the live template and  $\mathbf{y}/Y$  for the reference template(s).
- $R_{res}$ : Receives the shares of the secure computation and reconstructs the result.

A role can be performed by more than one party at the same time, e.g.,  $P_0, P_1$  can jointly perform the role  $R_{setup}$  (more in Section 3.5). A party can carry out multiple roles as well.

### 3.3 Sketching the Solution

With the different parties and roles in place, we are now ready to sketch our solution. In a nutshell, we combine  $\Pi$ -sharing to locally compute a scalar product, with the FSS gate for interval containment from [11] with full correctness.

The key insight driving our design stems from the intermediate SS state in the  $\Pi$ -sharing multiplication (Equation 2). By providing  $\Pi$ -shared input vectors to the computing parties  $P_j$ , we can locally obtain the SS shares of the element-wise multiplication, and perform local cumulative addition to obtain shares of the scalar product result. Compared to the pure SS approach, we no longer need a round of communication to reconstruct the intermediate values  $x-a$  and  $y-b$  masked by Beaver triples (Mult. in Equation 1). As pointed out in ABY2.0 [53], the communication in a IISS multiplication gate happens at the output wires, as opposed to SS multiplication gates where the round of communication is tied to the input wires.

The subsequent FSS gate for interval containment requires a publicly reconstructed input held by both parties, which, to preserve the input data privacy, must be masked prior to its reconstruction (in line with previous FSS-based works [11, 14, 58]). Crucially, the masking of the private input via local shares addition followed by its reconstruction (at the cost of one round of communication) happens at the input wire of the FSS gate.

All we have left is to put together the two pieces of the puzzle. We can skip the  $\Pi$ -sharing reconstruction and instead add the input mask directly to the scalar product output, and then reconstruct this masked value to serve as public input for the FSS interval containment gate. Figure 1 depicts our idea applied to the scalar product metric.

To obtain the other metrics we would have each input data holder  $R_{in_x}, R_{in_y}$  run  $f_{local}$  on its input and secret share this result with the computing parties to add it to the output of the scalar product. In

addition to that, both parties would multiply the shares of the scalar product result with the corresponding public value  $f_{cp}$ , resulting in the correct distance metric evaluation  $z = f_{dist}(\mathbf{x}, \mathbf{y})$ .

To keep the threshold  $\theta$  hidden from the computing parties (and known only by  $R_{setup}$ ), we subtract the value of  $\theta$  from the additive random mask  $r$  during the offline/setup phase to get  $r_\theta$ , and then compute  $z_\theta = z - \theta$ .

### 3.4 Protocol Specification

Embracing this combination of IISS for the locally computed scalar product and FSS for the comparison to  $\theta$ , we can now outline each of the protocols that compose the full solution:

- (1) FUNSHADE.Setup (Protocol 3): generation of the correlated randomness required for the scalar product multiplications, as well as the keys for the interval containment, and distribution of all the preprocessing material.

---

**Protocol 3** FUNSHADE.Setup( $l, n, \lambda, \theta$ )  $\rightarrow \mathbf{k}_0, \mathbf{k}_1, \langle \delta_{\mathbf{x}} \rangle, \langle \delta_{\mathbf{y}} \rangle$

---

**Players:**  $R_{setup}$

**Input:**  $l$ : length of the input vectors.

$n$ : number of bits for the secret sharing ring  $\mathbb{Z}_{2^n}$ .

$\lambda$ : security parameter.

$\theta$ : threshold for the comparison  $\in \mathbb{Z}_{2^n}$ .

**Output:**  $\mathbf{k}_0, \mathbf{k}_1$ : preprocessing keys, sent to  $P_0, P_1$  respectively.  
 $\langle \delta_{\mathbf{x}} \rangle, \langle \delta_{\mathbf{y}} \rangle$ :  $\delta$ -shares of input vectors, sent to  $P_{in_x}, P_{in_y}$  (input owners) resp.

**Note:** All arithmetic operations ( $+, -$ ) are defined in  $\mathbb{Z}_{2^n}$ .

*Beaver Triples for  $\Pi$ -sharing scalar product:*

$$1: \langle \delta_{\mathbf{x}} \rangle, \langle \delta_{\mathbf{y}} \rangle \equiv ((\delta_{x_0}, \delta_{x_1}), (\delta_{y_0}, \delta_{y_1})) \sim \mathcal{U}_{[\mathbb{Z}_{2^n}^{l \times 4}]}$$

$$2: \delta_{xy_0} \sim \mathcal{U}_{[\mathbb{Z}_{2^n}^l]}$$

$$\delta_{xy_1} \leftarrow (\delta_{x_0} + \delta_{x_1}) \cdot (\delta_{y_0} + \delta_{y_1}) - \delta_{xy_0}$$

$$\langle \delta_{xy} \rangle \equiv (\delta_{xy_0}, \delta_{xy_1})$$

$$3: \langle r \rangle \equiv (r_0, r_1) \sim \mathcal{U}_{[\mathbb{Z}_{2^n}^2]} \quad r \leftarrow r_0 + r_1$$

$$\langle r_\theta \rangle \equiv (r_{\theta 0}, r_{\theta 1}) \leftarrow (r_0, r_1 - \theta)$$

*FSS interval containment:*

$$4: \mathbf{k}_0^{IC}, \mathbf{k}_1^{IC} \leftarrow \text{FSS.Gen}^{IC}(\lambda, n, r)$$

$$5: \mathbf{k}_j \equiv (\delta_{x_j}, \delta_{y_j}, \delta_{xy_j}, r_{\theta j}, \mathbf{k}_j^{IC}), j \in \{0, 1\}$$

*Dealing the preprocessing material:*

$$6: \text{SEND } \mathbf{k}_0 \Rightarrow P_0, \quad (\delta_{x_0} + \delta_{x_1}) \Rightarrow R_{in_x}$$

$$\mathbf{k}_1 \Rightarrow P_1, \quad (\delta_{y_0} + \delta_{y_1}) \Rightarrow R_{in_y}$$


---

- (2) FUNSHADE.Share (Protocol 4):  $R_{in_x}$  and  $R_{in_y}$  prepare the  $\Pi$ -shares of their corresponding inputs using the correlated randomness and then send these shares to  $P_0, P_1$ .
- (3) FUNSHADE.Eval (Protocol 5):  $P_0, P_1$  engage in an online protocol upon acquiring the  $\Pi$ -shares of both inputs, using local multiplication and addition to compute the scalar product, and then evaluate the interval containment FSS scheme to determine whether the result is below the threshold  $\theta$ .
- (4) FUNSHADE.Result (Protocol 6):  $P_0, P_1$  send the arithmetic shares of the result to  $R_{res}$  for its reconstruction.

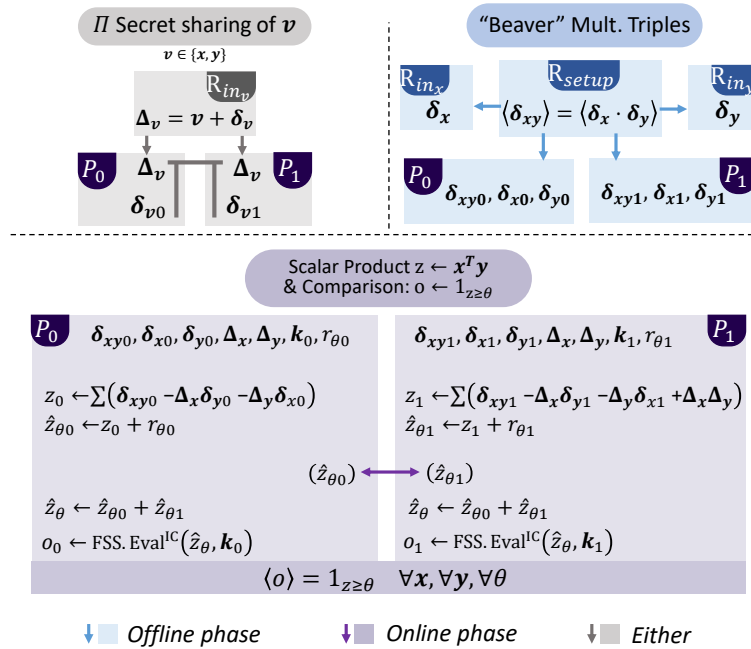


Figure 1: Overview of Funshade primitives

---

**Protocol 4** FUNSHADE.Share( $v, \delta_v$ )  $\rightarrow \Delta_v, \langle d_v \rangle$ 


---

**Players:**  $R_{in_v}$ , holding the input vector  $v$  (where  $v \in \{x, y\}$ ).

**Input:**  $v$ : input vector  $\in \mathbb{Z}_{2^n}^l$  held by  $P_{in_v}$ .

 $\delta_v$ : Precomputed sum of  $\delta$ -shares  $\in \mathbb{Z}_{2^n}^l$ .

**Output:**  $\Delta_v$ :  $\Delta$ -shares of vector  $v$  distributed to both  $P_0$  &  $P_1$ .

 $d_{v_j}$ : Arithmetic shares of the local computation  $f_{local}(v)$ .

- 1:  $\Delta_v \leftarrow (v + \delta_v)$
  - 2:  $d_v \leftarrow f_{local}(v)$ ;  $\langle d_v \rangle \equiv (d_{v0}, d_{v1}) \leftarrow (\sim \mathcal{U}_{[\mathbb{Z}_{2^n}]}, d_v - d_{v0})$
  - 3: SEND  $(\Delta_v, d_{v0}) \Rightarrow P_0$ ,  $(\Delta_v, d_{v1}) \Rightarrow P_1$
- 

### 3.5 Applications and Practical Considerations

We can employ our solution in a variety of use-cases, from biometric verification to CSAM detection. In this section we apply FUNSHADE to a scenario of biometric authentication for access control.

There is a **Gate** allowing access to a flight/train/concert with a small computer and a camera, and a **Biometric Provider (BP)** operating a server that holds the database of registered/enrolled users.  $y$  corresponds to a biometric template belonging to an enrolled user, and  $x$  to a freshly captured biometric template. This setting naturally includes two computing parties  $\{Gate, BP\} \equiv \{P_0, P_1\}$ , each of them playing specific roles, with  $Gate \supseteq R_{in_x}, R_{res}$  and  $BP \supseteq R_{in_y}$ . The solution is split into two phases:

*Enrollment (offline):*

- ①  $R_{setup}$  carries out the generation of preprocessing material and distributes it to BP and Gate.
- ② BP ( $R_{in_y}$ ) enrolls a user by collecting its biometric template  $y$ , and then secret shares it with Gate ( $P_0$ ).

---

**Protocol 5** FUNSHADE.Eval( $j, \Delta_x, \Delta_y, \langle d_x \rangle, \langle d_y \rangle, k_j$ )  $\rightarrow \langle o \rangle$ 


---

**Players:**  $P_j$ ,  $j \in \{0, 1\}$  computing parties.

**Input:**  $\Delta_x, \Delta_y$ :  $\Delta$ -shares of  $\langle x \rangle, \langle y \rangle$  ( $\Pi$ -shared inputs  $x, y$ ) held by both  $P_0$  and  $P_1$ .

 $\langle d_x \rangle, \langle d_y \rangle$ : Arithmetic shares of locally computed single-input terms  $f_{local}(x), f_{local}(y)$  of  $f_{dist}(x, y)$ .

 $k_j$ : preprocessing keys from FUNSHADE.Setup containing:

- $\delta_{x_j}, \delta_{y_j}$ :  $\delta$ -shares of  $\Pi$ -shared input vectors  $x, y$ ,
- $\delta_{xy_j}$ : arith. shares of Beaver triple s.t.  $\langle \delta_x \rangle \langle \delta_y \rangle = \langle \delta_{xy} \rangle$ ,
- $r_{\theta_j}$ : arith. shares of FSS input mask  $r$  minus threshold  $\theta$ ,
- $k_j^{IC}$ : FSS key for the IC gate of [11].

**Output:**  $\langle o \rangle$ : arithmetic shares of the result  $o = 1_{f_{dist}(x, y) \geq \theta}$ .

**Note:** All steps apply to both computing parties  $P_j$ ,  $j \in \{0, 1\}$ . All arithmetic operations  $(+, -, \cdot)$  are defined in  $\mathbb{Z}_{2^n}$ .

*$\Pi$ -sharing based scalar product:*

- 1:  $\hat{z}_{\theta_j} \leftarrow r_{\theta_j} + d_{x_j} + d_{y_j} + f_{cpf} \cdot \sum^l [j \Delta_x \Delta_y - \Delta_x \delta_{y_j} - \Delta_y \delta_{x_j} + \delta_{xy_j}]$

*Reconstruction of masked input to FSS gate:*

- 2:  $P_j$ : SEND  $\hat{z}_{\theta_j} \Rightarrow P_{1-j}$ ;  $\hat{z}_{\theta} \leftarrow \hat{z}_{\theta 0} + \hat{z}_{\theta 1}$

*Interval Containment for sign extraction:*

- 3:  $o_j \leftarrow \text{FSS.Eval}^{IC}(j, k_j^{IC}, \hat{z}_{\theta})$

- 4: **return**  $o_j$
- 

*Identification (Online):*

- ③ A user<sup>3</sup> attempts to cross the Gate. Gate ( $R_{in_x}$ ) gets and secret shares his live template with BP ( $P_1$ ).

<sup>3</sup>Note that the users are not participating in the protocol themselves.

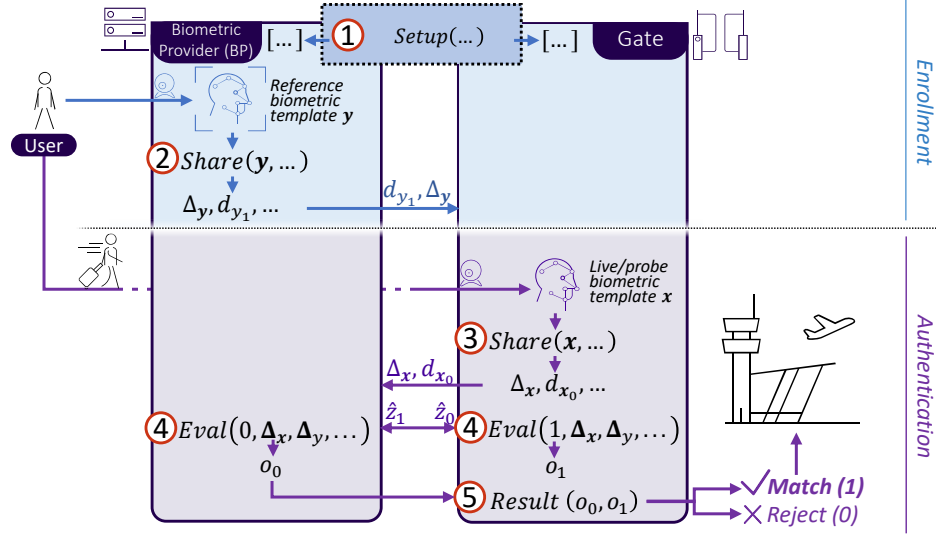


Figure 2: Diagram of a privacy-preserving biometric authentication solution for flight boarding.

**Protocol 6**  $\text{FUNSHADE.Result}(\langle o \rangle) \rightarrow o$

**Players:**  $P_j, j \in \{0, 1\}$  computing parties,  $R_{res}$  result holder.

**Input:**  $\langle o \rangle$ : secret shares  $o_0, o_1 \in \mathbb{Z}_2^n$  of the result  $o$  held by  $P_0, P_1$ .

**Output:**  $o$ : Output value.

1:  $P_j$ : SEND  $o_j \Rightarrow R_{res}$ .

2:  $R_{res}$ :  $o \leftarrow (o_0 + o_1)$

- ④ Gate ( $P_0$ ) and BP ( $P_1$ ) jointly perform Protocol 5, incurring in one round of communication where each party sends its share of the distance evaluation to the other.
- ⑤ The final result is sent to Gate ( $R_{res}$ ), who accepts or rejects the user accordingly.

This instantiation can be parallelized for different reference inputs  $\mathbf{y}^{(k)}$  in cases where the reference database contains more than one record, e.g., biometric identification against a database of multiple subjects, CSAM detection against a large database of image hashes. In these cases, the individual secret shared outputs  $o_j^{(k)}$  can be locally summed up to yield a single value as output. Additionally, these use-cases normally gather their reference databases ahead of time, allowing for early deployment of the  $\Pi$ -shares of  $\mathbf{y}^{(k)}$ .

*Realizing the setup  $R_{setup}$ .* In line with previous work on semi-honest MPC [8, 26, 45, 56, 64], fresh randomness is generated for each 1:1 verification in the offline phase and used only once in the online phase, as the security of our protocols would weaken if we were to reuse randomness/masks. Protocols in this work are presented in the preprocessing model, where  $P_0, P_1$  receive correlated randomness from a trusted dealer taking the role of  $R_{setup}$ . Protocols within this model can be converted to protocols in the standard model by:

- (a) Resorting to *trusted hardware* [52]. The role  $R_{setup}$  can be emulated within a trusted execution environment inside one of the computing parties, such as Intel SGX or ARM TrustZone [51].

- (b) A semi-honest 3PC setting with honest-majority [63]: A third party  $P_2$  can be added to the protocol to enact  $R_{setup}$  during the offline phase and remain dormant in the online phase.
- (c) Pure 2PC [11]:  $R_{setup}$  can be jointly emulated by the two parties  $P_0, P_1$  via a small-scale two-party secure protocol.

For the biometric authentication solution illustrated in Figure 2 we favor the 2PC approach, resorting to generic 2PC techniques for the FSS gate key generation (Appendix A.2 of [11]), and either Oblivious Transfer (OT) or Homomorphic Encryption (HE) for the preprocessing of the  $\Pi$ SS scalar product (Section 3.1.3 of [53]).

### 3.6 Security Analysis

We consider security against a Honest-but-Curious adversary  $\mathcal{A}$  that corrupts up to one of the two computing parties  $P_j$ . We consider a static corruption model where the adversary must choose which participant to corrupt before the execution of the computations. This is a standard security model in previous MPC frameworks [11, 19, 29, 50, 53, 58]. Under this threat model, we define and later prove the security and correctness of our constructions.

We employ the standard real world - ideal world paradigm, providing the simulation for the case of a corrupt  $P_j$ . The ideal world simulation contains an additional trusted party that receives all the inputs from  $P_0, P_1$ , computes the ideal functionality correctly and sends the corresponding results back to  $P_0, P_1$ . Conversely, the real world simulation executes the protocol as described in the FUNSHADE algorithms in the presence of  $\mathcal{A}$ .

Our security proof works in the  $\mathcal{F}_{\text{FUNSHADE.setup}}$ -hybrid model where  $\mathcal{F}_{\text{FUNSHADE.setup}}$  represents the ideal functionality corresponding to protocol FUNSHADE.setup.

**DEFINITION 1 (SECURITY OF FUNSHADE).** *For each  $j \in \{0, 1\}$ , there is a PPT algorithm  $\mathcal{S}$  (simulator) such that  $\forall \theta \in \mathbb{Z}_n^*, \forall \mathbf{x}, \mathbf{y} \in \mathbb{Z}_n^l$  and every function  $f_{\text{dist}}(\mathbf{x}, \mathbf{y}) : \mathbb{Z}_n^l \rightarrow \mathbb{Z}_n$  from Table 1,  $\mathcal{S}$  realizes the ideal functionality  $\mathcal{F}_{\text{th-dist}}$ , such that its behavior is computationally*



indistinguishable from a real world execution of protocols 4-5-6 in the presence of a semi-honest adversary  $\mathcal{A}$ .

**Ideal Functionality  $\mathcal{F}_{th-dist}$**

$\mathcal{F}_{th-dist}$  interacts with the parties  $P_0, P_1$  and the adversary  $\mathcal{S}$  and is parametrized by a publicly know function  $f_{dist}(x, y)$  and a threshold  $\theta$ .

- **Inputs:**  $\mathcal{F}_{th-dist}$  receives the inputs  $\Delta_x, \Delta_y, \delta_{x_j}, \delta_{y_j}$  from the computing parties  $P_0, P_1$ .
- **Computation:**  $\mathcal{F}_{th-dist}$  reconstructs  $x = \Delta_x - (\delta_{x_0} + \delta_{x_1})$  and  $y = \Delta_y - (\delta_{y_0} + \delta_{y_1})$ , computes  $z = f_{dist}(x, y)$  and  $o = 1_{z \geq \theta}$ .
- **Output:** Sends  $o_j$  to  $P_{res}$ .

**THEOREM 1.** *In the  $\mathcal{F}_{FUNSHADE.setup}$ -hybrid model, protocols 4-5-6 (online phase) securely realize the functionality  $\mathcal{F}_{th-dist}$ .*

**PROOF.** The semi-honest adversary corrupts  $P_j$  during the sequential execution of protocols 4-5-6. For this case,  $\mathcal{S}$  executes the setup phase honestly on behalf of  $P_{1-j}$  (in case of interactive setup), and will simulate the entire circuit evaluation, assuming the circuit-inputs of  $P_{1-j}$  to be 0. In the FUNSHADE.Result protocol,  $\mathcal{S}$  adjusts the shares of  $\langle o \rangle$  on behalf of  $P_{1-j}$  so that  $\mathcal{A}$  sees the same transcript as in the real-world protocol.

- **FUNSHADE.Setup:** For the offline phase, we consider it as an ideal functionality  $\mathcal{F}_{FUNSHADE.setup}$ , which generates the required FSS preprocessing keys and  $\delta$ -shares. Since we make only black-box access to FUNSHADE.setup, its simulation follows from the security of the underlying primitive used to instantiate it (OT or HE for the PISSE preprocessing material stemming from setupMULT of [53], generic 2PC for the FSS keys following Appendix A.2 of [11]). Alternatively, the Setup can be delegated into a third party, emulated with trusted hardware inside  $P_0|P_1$  or with an independent semi-honest party (as explained in Section 3.5).
- **FUNSHADE.Share:** We generalize the behavior of  $\mathcal{S}$  for both inputs  $v \in \{x, y\}$ . For the instances where  $P_j$  is the owner of the values (e.g.,  $P_j \supseteq R_{in_x}$ ),  $\mathcal{S}$  has to do nothing since  $\mathcal{A}$  is not receiving any messages.  $\mathcal{S}$  receives  $\Delta_v$  from  $\mathcal{A}$  on behalf of  $P_{1-j}$ . For the instances where  $P_{1-j}$  is the owner,  $\mathcal{S}$  sets  $v = 0$  and performs the protocol steps honestly.
- **FUNSHADE.Eval:** During the online phase,  $\mathcal{S}$  follows the protocol steps honestly using the data obtained from the setup phase. The scalar product requires  $l$  local additions (non-interactive and thus they don't need to be simulated) and a subsequent reconstruction of  $\langle \hat{z}_\theta \rangle$  as  $\hat{z}_\theta = \hat{z}_{\theta_0} + \hat{z}_{\theta_1}$  that behaves just like FUNSHADE.Result and serves as input to the FSS IC gate. For the FSS IC gate, we resort to the Simulation-based security of [11] (Definition 2) to argue computational indistinguishability of the ideal and real world executions, hiding the information of  $r$  contained in  $k_0$  and  $k_1$  from  $\mathcal{A}$ .
- **FUNSHADE.Result:** To reconstruct a value  $\langle o \rangle$ ,  $\mathcal{S}$  is given the output  $o$ , which is the output of  $\mathcal{A}$ . Using  $o$  and the share  $o_{1-j}$  corresponding to  $P_{1-j}$ ,  $\mathcal{S}$  computes  $o_j = o - o_{1-j}$  and sends this to  $\mathcal{A}$  on behalf of  $P_{1-j}$ .  $\mathcal{S}$  receives  $o_j$  from  $\mathcal{A}$  on behalf of  $P_{1-j}$ .  $\square$

**DEFINITION 2 (CORRECTNESS OF FUNSHADE).** *For every threshold  $\theta \in \mathbb{Z}_{n^+}^*$ , every pair of input vectors  $x, y \in \mathbb{Z}_n^l$  and every function  $f_{dist}(x, y) : \mathbb{Z}_n^l \rightarrow \mathbb{Z}_n$  from Table 1,*

$$\begin{aligned}
 & \text{if } (k_0, k_1, \langle \delta_x \rangle, \langle \delta_y \rangle) \leftarrow \text{FUNSHADE.Gen}(l, n, \lambda, \theta) \\
 & \text{and } (\Delta_x, \langle d_x \rangle) \leftarrow \text{FUNSHADE.Share}(x, \langle \delta_x \rangle), \\
 & \Delta_y, \langle d_y \rangle \leftarrow \text{FUNSHADE.Share}(y, \langle \delta_y \rangle) \\
 & \text{then } \Pr[\text{FUNSHADE.Eval}(0, \Delta_x, \Delta_y, d_{x_0}, d_{y_0}, k_0) \\
 & \quad + \text{FUNSHADE.Eval}(1, \Delta_x, \Delta_y, d_{x_1}, d_{y_1}, k_1) \\
 & \quad = 1_{f_{dist}(x, y) \geq \theta}] = 1.
 \end{aligned} \tag{5}$$

**THEOREM 2.** *Jointly, protocol 3 (offline), and protocols 4-5-6 (online), realize the function  $f(f_{dist}, \theta, x, y) = 1_{f_{dist}(x, y) \geq \theta}$  correctly.*

**PROOF.** We first decompose the  $\Pi$ -sharing based scalar product (step 1 of Protocol 5) for the joint result of the two computing parties  $\hat{z}_\theta$  in Equation 6,

$$\begin{aligned}
 \hat{z}_\theta &= \hat{z}_{\theta_0} + \hat{z}_{\theta_1} = (r_{\theta_0} + r_{\theta_1}) + (d_{x_0} + d_{x_1}) + (d_{y_0} + d_{y_1}) + f_{cp} \cdot \Sigma^l \\
 & \quad [\Delta_x \Delta_y - (\Delta_x \delta_{y_0} + \Delta_x \delta_{y_1}) - (\Delta_y \delta_{x_0} + \Delta_y \delta_{x_1}) + (\delta_{x_0} \delta_{y_0} + \delta_{x_1} \delta_{y_1})] \\
 &= r_\theta + d_x + d_y + f_{cp} \cdot \Sigma^l [\Delta_x \Delta_y - \Delta_x \delta_y - \Delta_y \delta_x + \delta_x \delta_y] \\
 &= r - \theta + d_x + d_y + f_{cp} \cdot \Sigma^l [\Delta_x \Delta_y - \Delta_x \delta_y - \Delta_y \delta_x + \delta_x \delta_y] \\
 &= r - \theta + d_x + d_y + f_{cp} \cdot \Sigma^l (\Delta_x - \delta_x) \cdot (\Delta_y - \delta_y) \\
 &= r - \theta + f_{local}(x) + f_{local}(y) + f_{cp} \cdot \Sigma^l (x^{(i)} \cdot y^{(i)}) \\
 &= r - \theta + f_{dist}(x, y) = z_\theta + r
 \end{aligned} \tag{6}$$

where we group all the SS shares and reconstruct their original values, replace  $r_\theta$  and  $\delta_{xy}$  by the corresponding values (from definitions in protocol 1), group the  $\Pi$ -shares of  $x$  and  $y$  to later reconstruct their values, and finally make use of Equation 4.

With the public input  $\hat{z}$  sorted out, we analyze the Interval Containment evaluation with output reconstruction in Equation 7,

$$\begin{aligned}
 o &= o_1 + o_2 = \text{FSS.Eval}^{IC}(0, k_0^{IC}, \hat{z}_\theta) + \text{FSS.Eval}^{IC}(1, k_1^{IC}, \hat{z}_\theta) \\
 &= \text{FSS.Eval}^{IC}(0, \text{FSS.Gen}^{IC}(\lambda, n, r)^{(0)}, z_\theta + r) \\
 & \quad + \text{FSS.Eval}^{IC}(1, \text{FSS.Gen}^{IC}(\lambda, n, r)^{(1)}, z_\theta + r) \\
 &= 1_{z_\theta \in \mathbb{Z}_{n^+}^*} = 1_{0 \leq z - \theta} = 1_{f_{dist}(x, y) \geq \theta}
 \end{aligned} \tag{7}$$

where we resort to Theorem 3 of [11] to argue that the two protocols ( $\text{FSS.Gen}^{IC}(\lambda, n, r)$ ,  $\text{FSS.Eval}^{IC}(j, k_j^{IC}, \hat{z}_\theta)$ ) constitute an FSS gate<sup>4</sup> correctly realizing  $f(z_\theta) = 1_{p \leq z_\theta \leq q}$ . Then, following Definition 2 (Correctness) of [11], we can argue that

$$\begin{aligned}
 & \Pr[\text{FSS.Eval}^{IC}(0, k_0^{IC}, \hat{z}_\theta) + \text{FSS.Eval}^{IC}(1, k_1^{IC}, \hat{z}_\theta) = \\
 & \quad 1_{0 \leq z_\theta \leq 2^{n-1}-1}] = 1 \\
 & \text{equating the FSS gate output to } 1_{z_\theta \in \mathbb{Z}_{n^+}^*}, \text{ the unit step function. } \square
 \end{aligned}$$

<sup>4</sup>There are several notation elements to adapt in order to align with [11]. Our mask  $r$  is written as  $r^{in}$  in Figure 3 of [11] depicting the FSS IC gate. We set the parameters  $p = 0$  and  $q = 2^{n-1} - 1$  to define the interval containing all positive integers.  $1_{p \leq z_\theta \leq q} = g_{IC, n, p, q}(z_\theta)$  is a function that belongs (per definition of IC gate in Section 4 of [11]) to the family of functions  $\mathcal{G}_{n, p, q}^{IC}$  referenced in Theorem 3 of [11].



**Table 2: Communication size, computation time and latency (in both LAN and WAN settings) for the different steps of FUNSHADE applied to biometrics, with  $n = 32$  and  $\lambda = l = 128$ , in two scenarios: authentication ( $K = 1$ ) and identification ( $K = 5000$ ).**

Phase/Step	Authentication (K=1)					Identification (K=5000)							
	Comm.	Computation	Latency LAN WAN		Total	Comm.	Computation	Latency LAN WAN		Total			
Offline	① Setup <sup>10</sup>	5.7 KB	33.63 $\mu$ s	10.46 ms   70.46 ms		10.49 ms   70.49 ms		28.5 MB	133 ms	2.29 s   2.35 s		2.42 s   2.48 s	
	② Share( $\{y\}$ )	512 B	0.038 $\mu$ s	10.04 ms   70.04 ms		10.04 ms   70.04 ms		2.56 MB	0.54 ms	215 ms   275 ms		215 ms   275 ms	
	Total	6.2 KB	33.67 $\mu$ s	20.50 ms   140.50 ms		20.53 ms   140.53 ms		31.06 MB	133.54 ms	2.50 s   2.62 s		2.63 s   2.90 s	
Online	③ Share(x)	512 B	0.038 $\mu$ s	10.04 ms   70.04 ms		10.04 ms   70.04 ms		512 B	0.038 $\mu$ s	10.04 ms   70.04 ms		10.04 ms   70.04 ms	
	④ Eval (SP)	8 B	0.19 $\mu$ s	10.00 ms   70.00 ms		10.00 ms   70.00 ms		40 KB	1.22 ms	10.40 ms   70.40 ms		11.62 ms   71.62 ms	
	④ Eval (IC)	-	9.2 $\mu$ s	-		9.2 $\mu$ s		-	46.65 ms	-		46.65 ms	
	⑤ Result	4 B	-	10.00 ms   70.00 ms		10.00 ms   70.00 ms		4 B	-	10.00 ms   70.00 ms		10.00 ms   70.00 ms	
	Total	524 B	9.43 $\mu$ s	30.04 ms   210.04 ms		30.05 ms   210.05 ms		40.5 KB	47.87 ms	30.44 ms   210.44 ms		78.31 ms   258.31 ms	

## 4 EXPERIMENTS

### 4.1 Implementation and Environment

We implement FUNSHADE in a compact<sup>5</sup>, portable<sup>6</sup> and standalone<sup>7</sup> C module, and we open source it<sup>8</sup>. We also provide a lightweight wrapper to Python by virtue of Cython. We instantiate the PRG function  $G$  (employed in the DCF gate) with a Miyaguchi-Preneel one-way compression function over an AES block cipher, an extended variant of Matyas-Meyer-Oseas function used in previous works [58]. We concatenate several fixed key block ciphers to achieve the desired output length.

### 4.2 Results

To assess the efficiency of our constructions, we test the execution of the full set of protocols from Figure 2 on a single core with an Intel(R) Core(TM) i7-7800X CPU, limiting the RAM consumption to up to 8GB, averaging measurements over at least 10 runs. For the intra and inter-protocol communication we employ a secure channel with capacity of 100Mbps and consider two scenarios:

- A LAN setting with 10ms of transmission latency.
- A WAN setting with 70ms of transmission latency.

We time the execution of the FSS primitives, summarizing the results in Table 3. We remark a  $\times 5$  speedup in the PRG function  $G$  by resorting to AES-NI CPU instructions, and therefore employ it for all the FSS primitives. As expected, the cost of both Gen and Eval algorithms increase linearly with  $n$ , the bit size of the ring  $\mathbb{Z}_{2^n}$  where the operations take place.

Subsequently, we assess the communication costs for each step of our solution using the Scalar Product (SP) as metric<sup>9</sup>, and we list them in Table 4. The size of the offline phase’s output can be interpreted as the total preprocessing. Overall, we highlight the extremely low communication size in the online phase, of just  $2Kn$  bits (2 ring elements for each of the  $K$  distance evaluations).

<sup>5</sup>Around 1000 Lines of Code for the core implementation.

<sup>6</sup>We employ only generic integer types, C arrays and plain C89 instructions (supported by every C compiler) to integrate smoothly with higher-level languages such as Python, Rust or Golang.

<sup>7</sup>The only optional dependency is libsodium for secure randomness generation.

<sup>8</sup>Our code is available at <https://<anonymous>/funshade>.

<sup>9</sup>Note that the evaluation of other distance metrics would require adding just  $n$  bits (one ring element, corresponding to a share of  $d_\sigma$ ) to the sharing of  $x$  and  $y$ .

<sup>10</sup>Depends on the instantiation of FUNSHADE.Setup (Section 3.5). Trusted hardware or a third party would yield the results from Table 2 while requiring only one round of communication; a 2PC instantiation would require more rounds and more computation.

**Table 3: Timings (ns) for the execution of FSS primitives over  $n$ -bit inputs, for  $\lambda = 128$ . The PRG function  $G_{\text{tiny}}$  implements a standalone AES block, whereas  $G_{\text{ni}}$  does so with faster AES-NI CPU instructions. All the FSS primitives use  $G_{\text{ni}}$  internally.**

Algorithm	Input Size			
	8-bit	16-bit	32-bit	64-bit
$G_{\text{ni}}$	162	162	162	202
$G_{\text{tiny}}$	871	871	871	1149
$\text{FSS.Gen}_n^<$	4306	6818	11521	26720
$\text{FSS.Eval}_n^<$	1207	2372	4700	11974
$\text{FSS.Gen}_n^{\text{C}}$	4974	7387	12287	28461
$\text{FSS.Eval}_n^{\text{C}}$	2409	4727	9296	23872

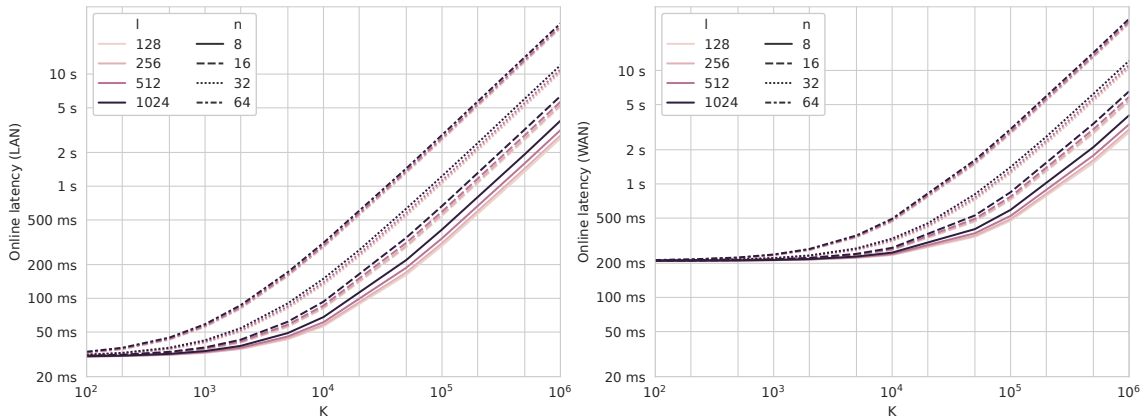
**Table 4: Communication costs for each of the FUNSHADE protocols.  $K$  is the number of reference vectors ( $K = 1$  for biometric authentication,  $K > 1$  for identification),  $l$  is the vector length,  $n$  is the bit size of the vector elements, and  $\lambda$  is the security parameter.**

Phase / Step	# rounds	Comm. size (bits)
Offline	① Setup	$(*)^{10}$ $Kn(2\lambda+8l+4n+10)+2K\lambda$
	② Share( $\{y_1 \dots y_K\}$ )	$Kn$
	Total	$(*)^{10}$ $Kn(2\lambda+9l+4n+10)+2K\lambda$
Online	③ Share(x)	$nl$
	④ Eval (SP)	$2Kn$
	④ Eval (IC)	-
	⑤ Result	$2n$
	Total	$n(l+2K+2)$

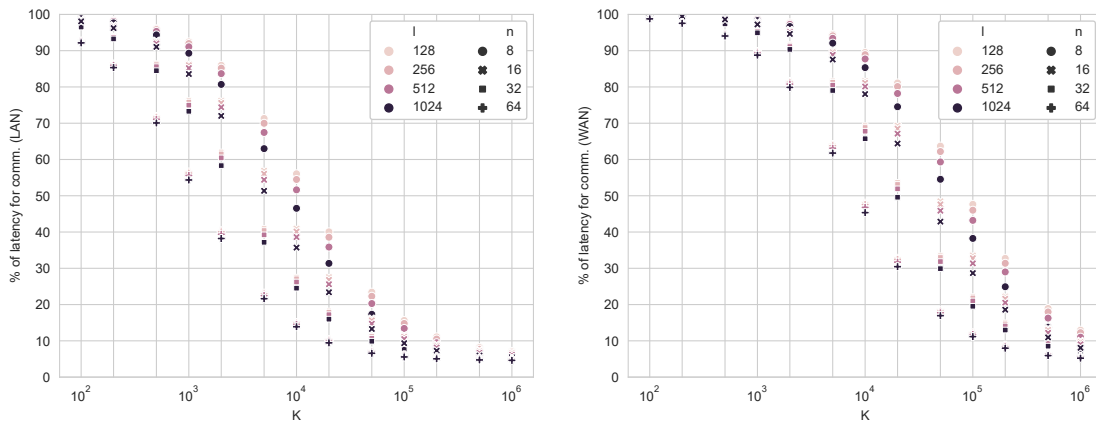
Focusing on the biometrics use-case, we resort to the Labelled Faces in the Wild (LFW) dataset [40] (13233 faces from more than 5000 identities) and extract biometric templates of length  $l = 128$  with an ArcFace-based [30] feature extractor<sup>11</sup>. We set  $n = 32$  to ensure that the protocols perfectly mimic the plaintext operations and thus obtain the exact same results, without any drop in accuracy. We report in Table 2 the results for two scenarios:

- *Authentication*, a 1:1 verification of the live template with a single reference.
- *Identification*, a 1:K verification of the live template with a set of  $K$  references, employing a subset ( $K = 5000$ ) of the identities in the LFW dataset, enough to be of use for the

<sup>11</sup>Feature extractors with similar characteristics can be obtained from <https://github.com/deepinsight/insightface/wiki/Model-Zoo>



**Figure 3: Online latency timings for the evaluation of Funshade with varying  $K$  (number of reference vectors),  $l$  (vector length) and  $n$  (size of vector elements in bits).**



**Figure 4: Ratio (%) between communication latency and total latency (communication + computation) for the online execution of Funshade with varying  $K$  (number of reference vectors),  $l$  (vector length) and  $n$  (size of vector elements in bits).**

applications mentioned beforehand (e.g., access control for a flight/train).

As shown in Table 2, the computation costs of our solution for authentication are negligible with respect to the communication latency. The total online latency for the identification scenario is more balanced in the LAN setting, whereas communication still dominates in the WAN setting. In any case, the total online latency amounts to less than 300ms including input sharing and output reconstruction, as well as its low-interaction lightweight communication makes FUNSHADE an ideal solution for privacy-preserving biometric verification in real time due to its low interaction.

Last but not least, we test our protocols with randomized input vectors of varying length  $l$  corresponding to typical template sizes of modern biometric feature extractors [30], employing bit-sizes ( $n$ ) corresponding to common CPU integer types to benefit from cheap

modular operations. We record the total online latency of all these experiments in the identification scenario for increasing references  $K$ . Figure 3 displays these results and shows how the vector size  $l$  has a much lower impact in the latency than  $n$ , hinting that applications that require lower numerical precision (e.g.,  $n = 16$ ) will be noticeably faster. Figure 4 shows the ratio of the communication latency to the total online latency. We observe that communication is the main bottleneck up until  $K \approx 1000$  in the LAN setting, and  $K \approx 10000$  in WAN. While this suggests that applications with higher requirements of  $K$  (e.g., nation-wide identification) should consider additional optimizations for the local computation (e.g., using multiple cores for parallelization or even resorting to a GPU), our non-parallelized solution requires around 10s to identify an individual against a database of 1 million records for  $n = 32$ .

To conclude, we verify the 100% correctness of our constructions in these experiments as long as natural overflows ( $-2^{n-1} \geq z \geq 2^{n-1} - 1$  for signed integers,  $0 \geq z \geq 2^n - 1$  for unsigned integers) are avoided.

## 5 PREVIOUS WORK

Distance metric evaluations, specially for Hamming Distance and Scalar Products, range among the most typical applications of privacy-preserving computation techniques. Consequently, a wide variety of previous work in MPC, FHE and FE have dealt with some form of it.

The Multi Party Computation field includes a plethora of works covering distance metric evaluations. All the frameworks for privacy preserving neural networks cover scalar-product-based matrix multiplications often followed by ReLU activations [8, 26, 45, 56, 64], covering a mixture of Garbled Circuits, Secret Sharing and their conversions. Secure hamming distance evaluation has motivated work such as [16] based on Oblivious Transfer, with its generalization to multiple metrics in [15]. Mixed-mode protocols have also tackled distance evaluations [29, 50, 53]. However, the majority of these solutions incur in a considerable communication cost to perform comparison. More recently, solutions based on FSS [11, 14, 58] have shown promising results, leading to this work.

In the field of Homomorphic Encryption, the biometrics use-case has led to a variety of approaches, including [4, 47] for hamming distance or [66] for scalar product. However, these approaches do not include comparison to a threshold, and often rely on costly cryptographic primitives that make them slow.

Since the advent of Functional Encryption [9], scalar product and hamming distance have been the most suitable candidates to study. Inner Product Encryption (IPE) started off with selective security in [1], already envisioning biometric use-cases, and reaching full security with [28] and [61]. [44] applied FE to biometric authentication with hamming distance and to nearest-neighbor search on encrypted data; [46] employs IPE for hamming-weight based matchings of real-world iris templates. [43] and [41] are the latest iterations of privacy-preserving scalar product techniques based on FE, demonstrating performances in the order of hundreds of *ms* for vectors of 128 values. While FE does not require an extra operation after the "evaluation" to retrieve the result, these schemes scale polynomially with the input vector length (thus are unsuitable for very large vectors), and their computation does not include comparison to a threshold. To include it, one must resort to techniques such as Threshold Predicate Encryption [67].

There also exist techniques in the literature not resorting to these three main fields, such as [68] with a custom scheme, or [59] with Identity Based Encryption.

To position FUNSHADE in the literature, we compare the costs of the online phase of our solution with that of selected previous works in Table 5. FUNSHADE is the first work in the 2PC setting requiring one single round of communication to evaluate  $1_{\mathbf{x}^T \mathbf{y} > \theta}$  while also presenting the lowest communication size of 2 ring elements. An additional side-by-side comparison with AriaNN [58] is provided in Appendix A.

*On the importance of the threshold comparison in privacy-preserving distance metrics.* The security provided by our construction, and

that of all privacy-preserving techniques in general (MPC, FHE, FE), does not prevent the reconstructed outputs  $o = f(\mathbf{x}, \mathbf{y})$  from revealing information about the inputs  $\mathbf{x}, \mathbf{y}$ . Indeed,  $P_{res}$  can leverage on his knowledge about the function being computed and attempt to extract information about the inputs from the outputs by inverting the function being computed  $Leak(\mathbf{x}, \mathbf{y}) \leftarrow Leak(f^{-1}(o))$ . Labeled as "input leakage" in previous works [41], this leakage affects the practical privacy of real-world deployments of privacy-preserving solutions. Applications using distance metric calculations as one of many building blocks (e.g., Machine Learning) might be more naturally protected thanks to the complexity of the function (beware! black-box model extraction attacks are real [62]), yet applications requiring only one distance metric evaluation (e.g., biometric matching, CSAM detection) are much more sensitive to this leakage, since these distance metrics are linear functions and thus easily invertible.

While solutions exist to add controlled noise to the input (e.g., Differential Privacy in [18]), the most straightforward method to reduce this leakage is to output the least information possible. For applications like biometric matching and CSAM detection, one-bit outputs suffice to determine whether there is a match or not, and hence performing the comparison in a privacy-preserving manner reduces considerably the input leakage of the construction. As such, FHE and FE-based solutions without privacy-preserving threshold comparison are more risky to apply in real-world scenarios than threshold-enabled solutions that MPC (ours included) offers out of the shelf.

## 6 CONCLUSIONS

In this work we presented FUNSHADE, a novel 2PC privacy preserving solution of various distance metrics (e.g., Hamming distance, Scalar Product) followed by threshold comparison. We build our protocols upon PISS, a version of arithmetic secret sharing optimized for the secure evaluation of scalar products, and function secret sharing with 100% correctness for comparison. Thanks to this, FUNSHADE proposes the first solution in the 2PC literature requiring one single round of communication in the online phase while outperforming all previous works in online communication size (two ring elements), all while relying on lightweight cryptographic primitives. We implement our solution from scratch in a portable C module, and showcase its extreme efficiency by achieving secure biometric identification against 5000 records in less than 300ms with 32-bit precision, and against 1 million records in  $\sim 10s$ .

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**Table 5: Benchmark of theoretical costs on evaluating a scalar product and comparison to threshold between two vectors with  $l$  integers of  $n$ -bits each. We exclude the costs of input sharing (1 round of communication) and output reconstruction (1 round of communication), which are equivalent for all protocols.**

Work	Type	#Rounds of communication	#ring elements in communication	Correctness	Online Computation Blocks
AriaNN [58]	2PC SS: Arith., FSS	2 (1+1)	$4l + 4$	N	SS scalar product, FSS Comparison (1 DCF)
Boyle et. al. [11]	2PC SS: Arith., FSS	2 (1+1)	$4l + 4$	Y	SS scalar product, FSS IC gate (2 DCF)
ABY [29]	2PC SS: Boolean&Arith, GC	3 (1+2+0)	$\gg 6l$	Y	SS scalar product, Arith. to Yao conversion, GC evaluation
ABY2.0 [53]	2PC PISS: Boolean&Arith.	5 (1+1+3)	$\gg 2$	Y	PISS scalar product, Arith. to Boolean conversion, BitExtraction
GSHADE [15] (only scalar prod.)	2PC OT	2	$> 2l$	Y	correlated OTs.
CryptFlow2 [55]	2PC SS: Arith., OT	5	$> (128 + 14)l$	Y	Linear layer (1-dim weights), dReLU
Falcon [64]	3PC Replicated SS: Arith.	8 (1+7)	$> 6$	Y	MatMult with 1-dim matrices, Private Compare
<b>FUNSHADE (OURS)</b>	<b>2PC</b> <b>PISS: Arith., FSS</b>	<b>1</b>	<b>2</b>	<b>Y</b>	<b>PISS scalar product,</b> <b>FSS IC gate (2 DCF)</b>

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## A SIDE-BY-SIDE COMPARISON WITH ARIANN

In terms of theoretical gains, we display in Figure 5 the main differences between our approach and that of AriaNN [58], strengthening the comparison provided in Table 5.

In terms of performance, AriaNN is expected to run the local computation of  $FSS.cmp$  in roughly half the time required for our primitive  $FSS.Eval^I C$  (since their construction requires just one call to a DCF instead of two), at the expense of non-negligible error rates. While these errors are shown in [58] to be inconsequential for Machine Learning inference use-cases, their consequences to use-cases such as biometric verification would be much more severe (e.g., yielding high rates of false positives that would break the purpose of the verification). Even then, thanks to the reduction in intermediate rounds of communication from 2 to 1, FUNSHADE is faster than AriaNN when the distributed comparison to  $\theta$  takes less than the transmission latency (10ms for LAN, 70 ms for WAN).

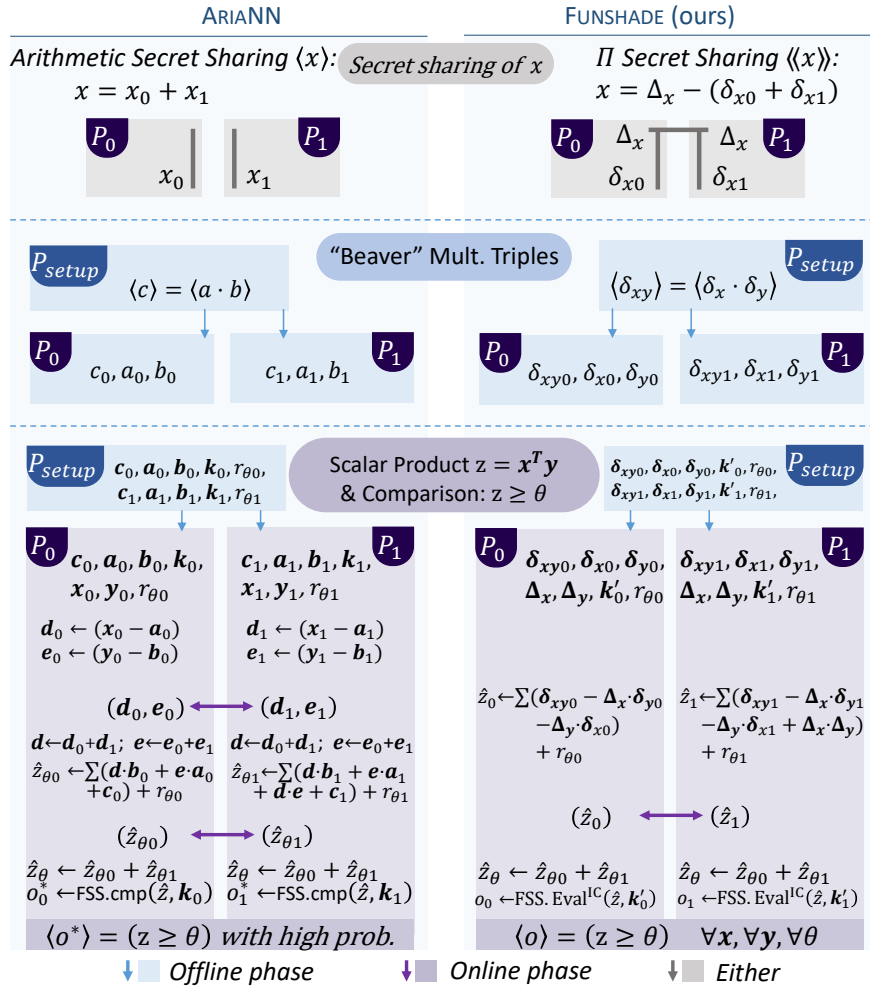


Figure 5: Side-by-side comparison between AriaNN and Funshade (ours)