Short Signatures From Diffie-Hellman: Realizing Short Public Key*

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Abstract. Efficient signature scheme whose security is relying on reliable assumptions is important. There are few schemes based on the standard assumptions such as the Diffie-Hellman (DH) in the standard model. We present a new approach for (hash-and-sign) DH-based signature scheme in the standard model. First, we combine two known techniques, programmable hashes and a tag-based signature scheme so that we obtain a short signature scheme with somewhat short public key of $\Theta(\frac{\lambda}{\log \lambda})$ group elements. Then, we developed a new technique for *asymmetric trade* between the public key and random tags, which are part of signatures. Roughly speaking, we can dramatically reduce the public key size by adding one field element in each signature. More precisely, our proposal produces public key of $\Theta(\sqrt{\frac{\lambda}{\log \lambda}})$ group elements, where λ is the security parameter. The signature size is still short, requiring two elements in a group of order p and two integers in $\mathbb{Z}_p$.

In our approach, we can guarantee the security against adversaries that make an a-priori bounded number of queries to signing oracle (we call *bounded CMA*). i.e., the maximum number q of allowable signing queries is prescribed at the parameter generating time. Note that for polynomial q, we limit ourselves to dealing with only polynomial-time reductions in all security proofs.

1 Introduction

Digital signature scheme is one of fundamental primitives of modern cryptography and is used in many cryptographic protocols as an important building block. From both a practical and theoretical standpoint, it is important to design efficient signature schemes whose security is proven under reliable assumptions. Most of efficient signature schemes follow a hash-and-sign paradigm (rather than a tree-based approach [17, 13]) to obtain efficient signature scheme, in particular short signatures; collision-resistant hash functions mapping from an arbitrarily long message to a short bit-string are first used, then the hashed message is signed onto. Each of them requires strong assumptions (e.g., random oracles [19, 36, 34, 2, 7, 22, 21], strong RSA assumptions [20, 15, 18], q-strong type assumptions [6, 35, 24, 26], or LRSW assumption [9]), inefficient signing/verification processes [28, 24], long public parameters [38], or that the signer must keep state [27].

Our Results. In this paper, we propose a new approach to practical signature scheme from standard assumptions in the standard model. To this end, we first introduce a new adversarial model which we call a *bounded chosen-message-attack (bounded CMA)*. In such an adversarial model, the maximum number q of allowable queries to the signing oracle is prescribed at the parameter generating time. The concept of bounded queries is already used in Cramer *et al.*'s *bounded CCA2* security notion for encryption schemes, in which the adversary is restricted to making an a-priori bounded number of queries to the decryption oracle [14]. In the *chosen-message-attack (CMA)* security notion, the adversary can select any polynomial number of messages and receive the corresponding signatures so that the bounded CMA security is a weaker notion than the standard CMA security. Even if there is a standard model stateless signature scheme under the standard assumption (e.g., Waters signature scheme from the Diffie-Hellman (DH) assumption) that is secure against the chosen message attack, there is a simple reason why a weaker security notion is important in practice. The Waters signature scheme is the sole construction in the category of the standard model DH-based stateless signature schemes, but it suffers from the large public key size. (In this paper, we only focus on schemes based on prime-order groups

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since it usually allows shorter signatures in practice.) We aim at designing a practical signature scheme under DH assumption in reasonable adversarial model.

In our new adversarial model, we propose the first (hash-and-sign) stateless signature scheme that has sublinear public key of $\Theta(\sqrt{\frac{\lambda}{\log \lambda}})$ group elements and secure under the Diffie-Hellman (DH) assumption in the standard model. In our scheme, a signature is comprised of two elements in a group of order p and two integers in $\mathbb{Z}_p$. This is roughly double size of the Waters signatures, which was the sole construction in a category of hash-and-sign stateless signatures under the DH-based assumption in the standard model.[1] From a practical standpoint, the public key size of the proposed scheme is much shorter than that of the Waters signatures for practical parameters, and the signature size of our scheme is still short. (We can reduce signature size to two group elements and one field element by applying tag compression technique and will explain the detail later.) For example, in our scheme, public key consists of at most 16 group elements when $\lambda \in [80, 256]$ and $q \in \{2^{30}, 2^{40}\}$, where q is the maximum of the number of signing queries issued by adversary and the reduction loss of the proposed scheme is comparable with that of Waters signature scheme in [38]. For the same setting, the public key of the Waters signature scheme consists of 164 to 516 group elements (that is, $2\lambda + 4$).

The signer of the proposed signature scheme does not need to maintain certain states [27] so that our construction is a stateless signature scheme. Naccache [32] proposed a variant of Waters signatures that offers a trade-off between the public key size and the concrete security level, but the asymptotic behavior is the same as the Waters signatures. Note that the variant, in which Naccache's trade-off is applied, requires a super-polynomial time reduction to obtain (asymptotically) the same public key size as ours. In this paper, for polynomial q, we limit ourselves to dealing with only polynomial-time reductions in the security proof.

Our Strategy for Sub-linear Public Key. We first apply a well-known technique for a generic transformation from weakly-secure signatures to fully-secure signatures using a chameleon hash [29]. In contrast to the full security model, the adversary should send all messages for signing queries before receiving the public key in the weak security model. There is an efficient chameleon hash that is secure under the Discrete Logarithm (DL) assumption [29]. Since the description of the DL-based chameleon hashes consists of two group elements and the DL assumption is weaker than the DH assumption, the generic transformation using chameleon hashes does not inflict a loss in the security and the asymptotic efficiency of the proposed signature scheme.

Next, we explore two different techniques of obtaining short signatures. One technique is to use so-called programmable hash functions [26] so that one can obtain weakly secure (stateless) DH-based signatures with a large public key. The other technique is to use tags and so in the security proof the simulator can restrict the adversaries to forging in the polynomial number of tags by maintaining states so that one can obtain stateful DH-based signatures with both short public key and signatures [27]. Since our aim is a stateless signature scheme, we should modify this technique so that the signer does not maintain the current index but randomly chooses it from some fixed set for each signature. The naive combination of these two different techniques allows us to obtain short and stateless DH-based signatures with *somewhat* short public key of $\Theta(\frac{\lambda}{\log \lambda})$ group elements.

To achieve sublinear public key of $\Theta(\sqrt{\frac{\lambda}{\log \lambda}})$ group elements, we develop a new technique for *asymmetric trade* between the public key and random tags, which are part of signatures. A trivial proof strategy for the resulting signature scheme would end up in an inefficient (super-polynomial time) reduction. We introduce a new proof technique for an efficient (polynomial time) reduction. The proposed security proof is based on the lemma, which is given in the section 3.2, about a generalization of the 'generalized birthday bound', which was essentially used for designing short signatures based on the RSA and the q-DH assumptions in [24, 26]. In other words, we can consider Hofheinz et al.'s generalized birthday bound as a special case of our lemma. Our proof technique is of independent interest.

Related Works (Hash-and-Sign Signatures). By relying on the random oracle heuristic, many constructions for efficient signature schemes in several settings have been proposed (e.g., DL setting [19, 36, 34, 7, 22], RSA setting [2], and Lattice setting [21]). However, there have been some studies showing the limitations of the random oracles [10, 16, 30].

[1] There are some variants of the Waters signatures [32, 28, 25]. However, the basic framework of construction is essentially same as the Waters signatures.

In the DL setting, Boneh and Boyen [6] proposed the first signature scheme in the standard model where signature size is comparable with that of BLS signature scheme [7] in the random oracle model. Okamoto [35] proposed a signature scheme which is more effective in many applications such as blind signatures, group signatures, and anonymous credentials. Recently, Hofheinz, Jager, and Kiltz [24, 26] proposed short signatures using programmable hash functions where the signature size is a bit shorter than previous schemes. However, the security of all these signature schemes are proven under non-static q-type assumptions. The size of the problem instance of q-type assumptions is (linearly) increased according to the number of signing queries. There are analyses for the q-type assumptions [8, 12]. Camenisch and Lysyanskaya [9] proposed a signature scheme which can be used to construct efficient group signatures, identity escrow schemes, and anonymous credential systems. They proved security of their signature scheme under interactive assumption, called the LRSW assumption [31]. Interactive assumptions are non-falsifiable and there is a criticism for non-falsifiable assumptions [33].

To the best of our knowledge, there were only two signature scheme based on the standard DH assumption in the standard model [38, 27]. However, the signer of the signature scheme in [27] needs to maintain certain states, i.e., stateful signature scheme. Therefore, there was only one construction for stateless signatures [38] that is proven secure under the standard DH assumption in the standard model. However, the signature scheme in [38] has a large public key $\Theta(\lambda)$ as compared with all aforementioned signature schemes based on the strong assumptions.

In the RSA setting, the first standard model construction was developed by Gennaro, Halevi, and Rabin [20]. Subsequently, Cramer and Shoup [15] and Fischlin [18] proposed more efficient signature scheme. However, these schemes were proven secure under the strong RSA assumption. Hohenberger and Waters proposed the first hash-and-sign signatures [27] from the RSA assumption. However, their scheme requires the signer to maintain states, i.e., stateful signature scheme. In the same year they proposed first stateless RSA-based signatures [28]. Subsequently, Hofheinz, Jager, and Kiltz [24] and Yamada, Hanaoka, and Kunihiro [39] improved its efficiency. However, all these signature schemes based on the RSA assumption require a large number of primality tests at signing and verifying.

Independent Work. Independently of our work, Böhl *et al.* [4] propose essentially the same DH-based signature scheme but different security analysis. They propose a new strategy (call confined guessing) for constructing signature schemes and proving the security of schemes, and the DH-based signature scheme is one instantiation of their strategy. Their analysis leads better security statement and asymptotic efficiency than ours. More precisely, in contrast to our analysis theirs do not need to restrict the adversarial model, and they achieve shorter public key of size $O(\log \lambda)$ group elements, which is asymptotically shorter than ours $O(\sqrt{\frac{\lambda}{\log \lambda}})$. Note that the short public key yielded from their analysis is attained at the price of a worse security reduction (though it is still a polynomial time reduction). For practical security parameters (and realistic bound of allowable signing queries), therefore, our analysis provides more efficient parameters such as group size than Böhl *et al.*'s analysis.

Outline. In the next section, we give preliminaries to explain our results. In Section 3, we explain our intuition behind our construction, a new proof strategy, and the proposed signature scheme. Section 4 analyze the security and efficiency of the proposed scheme. In Section 5, we give several extensions for shorter signatures using a pseudorandom function and using asymmetric pairings.

2 Preliminary and Definitions

Notation. We use $[a, b]$ to denote a set of integers between two integers, a and b. For a set S, $s \xleftarrow{\$} S$ denotes that the element s is uniformly chosen from S. For an algorithm Alg, $Alg(x) \to a$ means that Alg outputs a on input x. If the input of Alg is clear from the context, we sometimes omit it and simply write $Alg \to a$.

Signature Scheme. A signature scheme consists of three algorithms, KeyGen, Sign, and Verify.

KeyGen(λ): It takes the security parameter λ and outputs a keypair (PK,SK).
Sign(PK,M,SK): It takes the public key PK, the secret key SK, and a message M and outputs a signature σ.
Verify(PK,M,σ): It takes the public key PK, a message M, and a signature σ and returns 1 if the signature is valid; otherwise, 0.

q-bounded Chosen Message Attack. We define two new security notions for signatures, called *existential unforgeability with respect to q-bounded chosen-message-attacks* (EU-q-CMA) and *existential unforgeability with respect to q-bounded weak chosen-message-attacks* (EU-q-wCMA). These security notions are similar as well-known security notions EU-CMA formalized by Goldwasser, Micali, and Rivest [23] and its weaker version, EU-wCMA [27], respectively, but our definitions are weaker notions than previous ones, respectively since adversaries are allowed to queries to the signing oracle at most prefixed q times. The adversary in both new security models is given the public key and access to a signing oracle, and wins if he can produce a valid pair of a signature and a message on which the adversary did not query to the signing oracle. In the EU-q-CMA security model, the adversary is allowed to query any time (at most q times) before he outputs a forgery. However, the adversary in the EU-q-wCMA model should send the challenger an entire list of messages (of size at most q) he wants to query before receiving the public key. We provide the formal definition of EU-q-CMA and EU-q-wCMA below. Let $\mathsf{SIG} = (\mathsf{KeyGen}, \mathsf{Sign}, \mathsf{Verify})$ be a signature scheme. We consider two following experiments.

$\mathbf{Exp}^{\text{EU-}q\text{-CMA}}_{\mathsf{SIG},\mathcal{A}}(\lambda)$
$(PK, SK) \leftarrow \mathsf{KeyGen}(\lambda)$;
$(M, \sigma) \leftarrow \mathcal{A}^{\mathsf{Sign}(\cdot)}(PK)$;
Let $List$ be the set of messages queried by the adversary such that $|List| \leq q$;
If $M \notin List$ and $\mathsf{Verify}(PK, M, \sigma) = 1$ return 1;
Otherwise, return 0.

$\mathbf{Exp}^{\text{EU-}q\text{-wCMA}}_{\mathsf{SIG},\mathcal{A}}(\lambda)$
$(M_1, \ldots, M_{q'}, st) \leftarrow \mathcal{A}(st)$ s.t. $q' \leq q$;
$(PK, SK) \leftarrow \mathsf{KeyGen}(\lambda)$;
For $\forall i \in [1, q']$, $\sigma_i \leftarrow \mathsf{Sign}(PK, M_i, SK)$;
$(M, \sigma) \leftarrow \mathcal{A}(PK, \sigma_1, \ldots, \sigma_{q'}, st)$;
If for $\forall i \in [1, q']$, $M \neq M_i$ and $\mathsf{Verify}(PK, M, \sigma) = 1$ return 1; Otherwise, return 0.

We define

$$\mathbf{Adv}^{\text{EU-}q\text{-CMA}}_{\mathsf{SIG},\mathcal{A}}(\lambda) = \Pr\left[\mathbf{Exp}^{\text{EU-}q\text{-CMA}}_{\mathsf{SIG},\mathcal{A}}(\lambda) = 1\right] \quad \text{and} \quad \mathbf{Adv}^{\text{EU-}q\text{-wCMA}}_{\mathsf{SIG},\mathcal{A}}(\lambda) = \Pr\left[\mathbf{Exp}^{\text{EU-}q\text{-wCMA}}_{\mathsf{SIG},\mathcal{A}}(\lambda) = 1\right].$$

Definition 1 *Let SIG be a signature scheme. If for fixed polynomial $q(\lambda)$ and any probabilistic polynomial-time adversary $\mathcal{A}$, $\mathbf{Adv}^{EU\text{-}q\text{-}CMA}_{\mathsf{SIG},\mathcal{A}}(\lambda)$ ($\mathbf{Adv}^{EU\text{-}q\text{-}wCMA}_{\mathsf{SIG},\mathcal{A}}(\lambda)$, respectively) is a negligible function in λ, we say that the signature scheme SIG is EU-q-CMA secure (EU-q-wCMA secure, respectively). More precisely, if for any algorithm $\mathcal{A}$ with issuing at most (polynomial) q signing queries and running time T, $\mathbf{Adv}^{EU\text{-}q\text{-}CMA}_{\mathsf{SIG},\mathcal{A}}(\lambda) < \epsilon$ ($\mathbf{Adv}^{EU\text{-}q\text{-}wCMA}_{\mathsf{SIG},\mathcal{A}}(\lambda) < \epsilon$, respectively), then we say that SIG is (q, ϵ, T)-EU-CMA secure ((q, ϵ, T)-EU-wCMA secure, respectively).*

Chameleon Hash Functions and Generic Transformation. Krawczyk and Rabin [29] formalized the notion of chameleon hash function and provided a simple construction based on the DL assumption in the standard model. A chameleon hash function H takes two inputs m (message) and r (randomness) and outputs a hash value $H(m; r)$. It satisfies three properties, collision-resistance, trapdoor collisions, and uniformity. The collision-resistance property states that it is infeasible (for the polynomial-time adversary) to find two distinct messages m and m' and randomness r and r' such that $H(m; r) = H(m'; r')$. The uniformity means that for each message m, $H(m; r)$ has the same probability distribution where r is chosen uniformly at random. The trapdoor collisions property states that, given some trapdoor information, any pair m, r, and any additional message m', it is possible to efficiently find a randomness r' such that $H(m; r) = H(m'; r')$.

We review the chameleon hashes based on the DL assumption [29].[2] Let $\mathcal{G}_{ch}$ is a group generator that takes security parameter λ as input and outputs a cyclic group of prime order p' of 2λ-bits (e.g., an elliptic curve group generator).

$\mathsf{CHSetup}(\lambda)$: $\mathcal{G}_{ch}(\lambda) \to \mathbb{G}_{ch}$.
Choose $g_{ch} \xleftarrow{\$} \mathbb{G}_{ch}$ and $\beta \xleftarrow{\$} \mathbb{Z}_{p'}$ and compute $h_{ch} = g_{ch}^{\beta}$.
Output $\{g_{ch}, h_{ch}\}$, as the description of $H(\cdot;\cdot)$, and trapdoor $tr = \{\beta\}$,
where $H(\cdot;\cdot) : \mathbb{Z}_{p'} \times \mathbb{Z}_{p'} \to \mathbb{G}_{ch}$ is defined by $(x, r) \mapsto g_{ch}^{x} h_{ch}^{r}$.
$\mathsf{Trapdoor\ collision}(tr, x, r, x')$: Solve the equation $x + \beta r = x' + \beta r$ with a variable r'.
Output r'

[2] In [29], the chameleon hash function is constructed over a multiplicative subgroup of a finite field. We can easily generalize it to the chameleon hash function over any cyclic groups, in which the DL assumption holds.

We can easily check that the above scheme satisfies three properties of chameleon hashes. In particular, the collision resistance of the above scheme tightly comes from the DL assumption on $\mathbb{G}_{ch}$; that is, if there exists an adversary finding collisions of the above chameleon hashes with ϵ_{ch} probability in time T_{ch}, then we can construct an algorithm solving the DL problem with ϵ_{ch} probability in time T'_{ch} such that $T'_{ch} \approx T_{ch}$.

The generic transformation from EU-wCMA secure signatures to EU-CMA secure signatures was used in many previously proposed signature schemes (e.g., [29, 37, 6, 27, 28]). Similarly, we can construct a generic transformation from EU-q-wCMA secure signatures to EU-q-CMA secure signatures. Suppose that $(\mathsf{G}, \mathsf{S}, \mathsf{V})$ is a EU-q-wCMA secure signature scheme for arbitrary length messages and $\mathsf{CHSetup}$ is a generator for description of a chameleon hash based on the DL assumption. Then, the following scheme is a EU-q-CMA secure signature scheme for fixed length messages.[3]

$\mathsf{KeyGen}(\lambda)$: Run $\mathsf{CHSetup}(\lambda) \to (H(\cdot;\cdot), tr)$ and $\mathsf{G}(\lambda) \to (\mathsf{pk}, \mathsf{sk})$, publish $PK = (pk, H)$, and then keep $SK = \{sk\}$.
$\mathsf{Sign}(PK, M, SK)$: Pick a random $r \in \mathbb{Z}_p$, compute $y = H(M; r)$, run $\mathsf{S}(pk, y, sk) \to \sigma'$, and then output the signature $\sigma = (\sigma', r)$.
$\mathsf{Verify}(PK, M, \sigma)$: Parse σ as (σ', r), compute $y = H(M; r)$, and then output $\mathsf{V}(pk, y, \sigma')$.

Lemma 1 *If $(\mathsf{G}, \mathsf{S}, \mathsf{V})$ is (q, ϵ, T)-EU-wCMA secure and $\mathsf{CHSetup}$ is a generator for secure chameleon hashes, then the above scheme is $(q, 2(\epsilon+\epsilon_{ch}), T')$-EU-CMA secure signature scheme, where the chameleon hash satisfies (ϵ_{ch}, T_{ch})-collision resistance and $T' \approx T \approx T_{ch}$.*

Proof. Proof is essentially same as that for the generic transformation from EU-wCMA secure signatures to EU-CMA secure signatures, given in [28].[4]

Bilinear Groups. We define bilinear groups of prime order. The proposed signature scheme essentially uses a bilinear map.

Definition 2 *We say that $\mathcal{G}$ is a bilinear group generator, if on inputting the security parameter λ, it outputs a tuple $(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_t, e)$, where p is a $(2\lambda+1)$-bit prime, $\mathbb{G}_1$, $\mathbb{G}_2$, and $\mathbb{G}_t$ are finite abelian groups of order p, and $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_t$ is a non-degenerate bilinear map, that is, (bilinearity) for all $a, b \in \mathbb{Z}_p$ and $g \in \mathbb{G}_1$, $g' \in \mathbb{G}_2$, $e(g^a, g'^b) = e(g, g')^{ab}$ and (non-degeneracy) for generators $g \in \mathbb{G}_1$ and $g' \in \mathbb{G}_2$, $e(g, g') \neq 1$.*

If $\mathbb{G}_1 = \mathbb{G}_2$, then we use a notation $\mathbb{G}$ to denote $\mathbb{G}_1 = \mathbb{G}_2$ and we say that e is a type-1 pairing. If $\mathbb{G}_1 \neq \mathbb{G}_2$ but there is an efficiently computable homomorphism $\phi : \mathbb{G}_2 \to \mathbb{G}_1$, then we say that e is a type-2 pairing. Otherwise (that is, $\mathbb{G}_1 \neq \mathbb{G}_2$ and there are no efficiently computable homomorphisms between $\mathbb{G}_1$ and $\mathbb{G}_2$), we say that e is a type-3 pairing.

(Computational) Diffie-Hellman Assumption. We define the (computational) DH assumption in the bilinear group setting.

Definition 3 *Let $\mathcal{G}$ be a bilinear group generator. We say that $\mathcal{G}$ satisfies the (ϵ_{dh}, T_{dh})-DH assumption if for any T_{dh}-time probabilistic algorithm $\mathcal{B}$ the following advantage $\mathbf{Adv}^{\mathbf{DH}}_{\mathcal{B}}$ is less than ϵ_{dh}:*

$$\mathbf{Adv}^{\mathbf{DH}}_{\mathcal{G},\mathcal{B}} = \Pr\left[\mathcal{A}(p, \mathbb{G}, \mathbb{G}_t, e, g, g^a, g^b) \to g^{ab} \middle| \mathcal{G}(\lambda) \to (p, \mathbb{G}, \mathbb{G}_t, e), a, b \xleftarrow{\$} \mathbb{Z}_p, g \xleftarrow{\$} \mathbb{G}\right].$$

3 Short Signatures with Short Public Key

In this section, we first propose a EU-q-wCMA secure signature scheme with somewhat short public key by combining two techniques in [24, 27], and then propose a EU-q-wCMA secure signature scheme with short

[3] For signing arbitrary length messages, the signer can first apply collision-resistance hash functions.
[4] In [28], a generic transformation is given for EU-wCMA secure signature scheme for fixed length messages. Note that we can easily generalize it and prove an analogous lemma for arbitrary length messages. In fact, if the message size of EU-q-wCMA secure signature scheme is larger than or equal to the size of the representation of the chameleon hash values, then the generic transformation derives EU-q-CMA secure signature scheme.

$\mathsf{KeyGen}(\lambda)$: Run $\mathcal{G} \to (p, \mathbb{G}, \mathbb{G}_t, e)$ and choose $v, u_1, \ldots, u_q, g \xleftarrow{\$} \mathbb{G}$ and $\alpha \xleftarrow{\$} \mathbb{Z}_p$.
Output $PK = \{v, u_1, \ldots, u_q, g, g^\alpha\}$ and $SK = \{\alpha\}$.

$\mathsf{Sign}(PK, M, SK)$: Compute $\sigma = (v \prod_{i=1}^{q} u_i^{M^i})^\alpha$, and output σ.

$\mathsf{Verify}(PK, M, \sigma)$: Check whether $e(\sigma, g) \stackrel{?}{=} e(g^\alpha, v \prod_{i=1}^{q} u_i^{M^i})$
: Output 1 if the equation holds; otherwise, 0.

Fig. 1. Short signatures with large public key

$\mathsf{KeyGen}(\lambda)$: Run $\mathcal{G} \to (p, \mathbb{G}, \mathbb{G}_t, e)$ and choose $v, u_1, \ldots, u_m, g_1, h, g \xleftarrow{\$} \mathbb{G}, \alpha \xleftarrow{\$} \mathbb{Z}_p$.
Output $PK = \{v, u_1, \ldots, u_m, g_1, h, g, g^\alpha\}$ and $SK = \{\alpha\}$.

$\mathsf{Sign}(PK, M, SK)$: Choose $t \xleftarrow{\$} \mathbb{Z}_p$ and $s_1 \xleftarrow{\$} [1, Q]$.
Compute $\sigma_1 = (v \prod_{i=1}^{m} u_i^{M^i})^\alpha (h g_1^{s_1})^t$ and $\sigma_2 = g^{-t}$.
Output $\sigma = (\sigma_1, \sigma_2, s_1)$.

$\mathsf{Verify}(PK, M, \sigma)$: Parse σ to $(\sigma_1, \sigma_2, s_1)$.
Check whether $s_1 \in [1, Q]$. If not, abort and output 0.
Check whether $e(\sigma_1, g) e(\sigma_2, h g_1^{s_1}) \stackrel{?}{=} e(g^\alpha, v \prod_{i=1}^{m} u_i^{M^i})$.
Output 1 if the equation holds; otherwise, 0.

Fig. 2. Construction for somewhat short public key

public key. The proposed signature scheme is for fixed length messages, but we note that we can easily modify it for arbitrary length messages by using collision resistant hash functions; first, compute a hash value of a long message, and then use it as a message for the signature scheme.

We use λ to denote the security parameter and q to denote the maximum number of signing queries. Since we restrict the adversary to be computationally bounded (that is, we only consider the probabilistic polynomial time adversary), q is a polynomial in λ.

3.1 Combining Two Techniques: 'Somewhat' Short Public Key

We begin with exploring two techniques for obtaining short signatures in the standard model. In the simulation of the EU-q-wCMA model, the simulator should give a set of signatures on messages queried by the adversary, but the simulator should not be able to create signatures on all messages other than those queried by the adversary. If the simulator can create signatures on all messages, then the simulator does not need help from the adversary to obtain the forgery since the simulator can sign on all messages himself; hence, we cannot extract the solution of the DH problem from the output of the adversary. We can use programmable hash functions [26] to allow the simulator to produce only signatures on messages queried by the adversary. In particular, we use weak programmable hash functions [24] to construct EU-q-wCMA secure short signatures. We describe the short signature scheme with a polynomial size public key in Figure 1. We assume that the public key contains the bilinear group description. For a 2λ-bit message M, we consider M as an element of $\mathbb{Z}_p$. $(v \prod_{i=1}^{q} u_i^{M^i})$ is a weak programmable hash function on input M that, in the EU-q-wCMA model, allows the simulator to sign on at most q messages, which are given by the adversary before generating the public key.[5] Furthermore, we can construct a simulator that extracts the solution of g^{ab} from the forgery by imbedding g^a in v and u_i and setting g and g^α by g and g^b, respectively.

There is the other technique that obtains short signatures with short public key by maintaining the index counter in the signer side [27]. The idea of this technique is first to restrict the adversary to attack one of the polynomially many indexes and then uses the technique for selectively-secure signatures such as that used in the Boneh-Boyen signature scheme [5]. We can combine this technique with programmable hash functions. Since our aim is a stateless signature scheme, we should modify this technique so that the signer does not maintain the current index but randomly chooses it from some fixed set for each signature. Then, we obtain a short signature

[5] M^i is not the i-th bit of M, but the i times product of M.

$\mathsf{KeyGen}(\lambda)$: Run $\mathcal{G} \to (p, \mathbb{G}, \mathbb{G}_t, e)$.
Choose $v, u_1, \ldots, u_m, g_1, \ldots, g_k, h, g \xleftarrow{\$} \mathbb{G}$, $\alpha \xleftarrow{\$} \mathbb{Z}_p$.
Output $PK = \{v, u_1, \ldots, u_m, g_1, \ldots, g_k, h, g, g^\alpha\}$ and $SK = \{\alpha\}$.

$\mathsf{Sign}(PK, M, SK)$: Uniformly choose $t \xleftarrow{\$} \mathbb{Z}_p$ and $s_1, \ldots, s_k \xleftarrow{\$} [1, Q]$.
Compute $\sigma_1 = (v \prod_{i=1}^m u_i^{M^i})^\alpha (h \prod_{i=1}^k g_i^{s_i})^t$ and $\sigma_2 = g^{-t}$.
Output $\sigma = (\sigma_1, \sigma_2, \overrightarrow{s})$, where $\overrightarrow{s} = (s_1, \ldots, s_k) \in [1, Q]^k$.

$\mathsf{Verify}(PK, M, \sigma)$: Parse σ to $(\sigma_1, \sigma_2, \overrightarrow{s})$.
Check whether $\overrightarrow{s} \in [1, Q]^k$. If not, abort and output 0.
Check whether $e(\sigma_1, g) e(\sigma_2, h \prod_{i=1}^k g_i^{s_i}) \stackrel{?}{=} e(g^\alpha, v \prod_{i=1}^m u_i^{M^i})$.
Output 1 if the above equation holds; otherwise, 0.

Fig. 3. Main construction for short public key

with somewhat short public key and give the description of the scheme in Figure 2. In Figure 2, s_1 plays the role of index in [27] and we call s_1 tag. In Figure 2, we set Q to be polynomial in λ. The strategy of the simulation in the EU-q-wCMA model is as follows: The simulator guesses s_1^*, the tag of the forgery, (with non-negligible $\frac{1}{Q}$ probability) and uses the technique for the selectively-secure signature scheme of the Boneh-Boyen signatures. For each signature, the tag is randomly chosen so that there may exist several signatures containing the same tag as s_1^* among the resulting signatures of singing queries. Under normal circumstances, the simulator cannot produce signatures with tag s_1^* (since we use technique for selectively-secure scheme). We can resolve this by using the weak programmable hash functions. If we uniformly choose a tag from $[1, Q]$ at most q times for polynomial $Q \geq q$, there are at most $\Theta(\frac{\lambda}{\log \lambda})$ same tags as the tag of the forgery with overwhelming probability. Therefore, we can set $m = \Theta(\frac{\lambda}{\log \lambda})$ and the simulator can create m signatures, which has the same tag as that of the forgery. We omit detailed analysis since the analysis for the generalized scheme, which is our main construction, is given in Section 4.

Remark. We used the combination of two techniques in this section for signature schemes based on the DH assumption. There is similar approach for signature schemes based on the RSA assumption and q-DH assumption [24]. Note that our original contribution is explained in Section 3.2.

3.2 Asymmetric Trade: Realizing Short Public Key

Let Q, m, and k be functions in λ. For readers who want to see the specific parameters a little early, we give an example parameter below. We will explain about selecting parameters in Section 4.3.

Example Parameter 1. $Q = 2^3 q$, $m = \left\lceil \sqrt{\frac{\lambda}{\log \lambda}} \right\rceil$, and $k = \left\lceil \sqrt{\frac{\lambda}{\log \lambda}} \right\rceil$.

We describe our main signature scheme in Figure 3.

For each signature $\sigma = (\sigma_1, \sigma_2, \overrightarrow{s})$, we call $\overrightarrow{s}$ tag vector. In contrast to the scheme discussed in the previous section, we use a vector $\overrightarrow{s}$ instead of an integer s_1 in signatures. Roughly speaking, our analysis shows that the signature scheme in Figure 3 satisfies weak unforgeability (against bounded CMA) when $mk = \Omega(\frac{\lambda}{\log \lambda})$ (this result contains the signatures with somewhat short public key in Figure 2). In addition (roughly speaking again), since the public key size is $\Theta(m + k)$ group elements, we can attain the minimal public key size when m and k are nearly equal. On the other hand, the size of signatures will increase when the parameter k increases. However, each s_i is a $\log Q$-bit integer, and so $\overrightarrow{s}$ is asymptotically much shorter than $\Theta(\lambda)$-bit (if we set Q as a polynomial in λ). This is an *asymmetric trade* between the public key and tag vectors. When we apply the example parameter 1, the signature size will be bounded by two group and a field element, that is, the signature size is $\Theta(\lambda)$ bits. We give precise analysis of the efficiency of the proposed signature scheme in Section 4.3.

Our construction of the short signatures with short public key in Figure 3 is a simple generalization of the short signatures with somewhat short public key in Figure 2. However, the analysis of the security in the EU-q-wCMA model is more challenging than the construct itself. The basic strategy of the simulator in the

EU-q-wCMA model of the signature scheme in Figure 2 is guessing the tag s_1^* of the forgery and then using the programmability of the weak programmable hash function ($v \prod_{i=1}^{m} u_i^{M^i}$) to sign for the signature with the same tag. We cannot naively apply this proof strategy to the generalized construction. To obtain short public key, we should set k sufficiently large (but not too much). However, if k is large, then the simulator cannot guess the tag vector of the forgery, $\overrightarrow{s}^* \in [1, Q]^k$, with non-negligible probability. That is, we would fail to construct a polynomial-time reduction. We developed a proof technique to resolve this problem.

Our proof strategy. We now explain our proof strategy for polynomial-time reduction from solving the DH problem to the breaking the weak unforgeability of the proposed signature scheme. In particular, we explain the method to guess the tag vector $\overrightarrow{s}^*$ of the forgery with non-negligible probability. In fact, we cannot guess all the bits of $\overrightarrow{s}^*$, but only part of $\overrightarrow{s}^*$ with non-negligible probability. This is sufficient for our proof strategy.

We begin with defining notations for efficient explanation. Let S and S^i be sets $[1, Q]$ and $[1, Q]^i$ (i times canonical product set), respectively. For $j \in [1, q]$, let $\overrightarrow{s}_j \in S^k$ be the tag vector (randomly chosen by the simulator) of the signature on the jth message (queried by the adversary). Let $\overrightarrow{s}^* = (s_1^*, \ldots, s_k^*) \in S^k$ be the tag vector of the forgery output by the adversary. For $\overrightarrow{s} \in S^k$ and $i \leq k$, let $\overrightarrow{s}^{(i)} \in S^i$ be the first i entries of $\overrightarrow{s}$ (e.g., $\overrightarrow{s} = (s_1, \ldots, s_k)$ and $\overrightarrow{s}^{(i)} = (s_1, \ldots, s_i)$). We separate the adversaries into several types according to the relations between $\overrightarrow{s}^*$ and $\{\overrightarrow{s}_i\}_{i \in [1,q]}$. To this end, for fixed $\{\overrightarrow{s}_i\}_{i \in [1,q]}$, we first define the set S_i as

$$\{\hat{s} \in S^i \mid \exists \text{ at least } (m+1) \text{ distinct } j_1, \ldots, j_{m+1} \in [1, q] \text{ such that } \hat{s} = \overrightarrow{s}_{j_1}^{(i)} = \ldots = \overrightarrow{s}_{j_{m+1}}^{(i)}\}.$$

Let us consider an example to help the readers understand the definition of S_i.

Example. Suppose that

$$\overrightarrow{s}_1^{(i)} = \ldots = \overrightarrow{s}_{m+2}^{(i)} \neq \overrightarrow{s}_j^{(i)} \text{ for } j \in [m+3, q],$$
$$\overrightarrow{s}_1^{(i+1)} = \ldots = \overrightarrow{s}_{m+1}^{(i+1)} \neq \overrightarrow{s}_j^{(i+1)} \text{ for } j \in [m+2, q],$$

and $\overrightarrow{s}_{m+3}^{(i)}, \ldots, \overrightarrow{s}_q^{(i)}$ are distinct. Then,

$$\begin{cases} \overrightarrow{s}_j^{(i)} \in S_i \text{ for } j \in [1, m+2] \\ \overrightarrow{s}_j^{(i)} \notin S_i \text{ for } j \in [m+3, q], \end{cases}, \quad \begin{cases} \overrightarrow{s}_j^{(i+1)} \in S_{i+1} \text{ for } j \in [1, m+1] \\ \overrightarrow{s}_j^{(i+1)} \notin S_{i+1} \text{ for } j \in [m+2, q], \end{cases}$$

and $|S_i| = |S_{i+1}| = 1$. □

We can easily see that $|S_{i+1}| \leq |S_i|$. Let n be the largest integer in $[1, k]$ such that $S_n \neq \emptyset$. If we choose m, k, and Q appropriately, we then obtain the following two properties with overwhelming probability, where the probability is taken over the choice of $\{\overrightarrow{s}_i\}_{i \in [1,q]}$.

1. $|S_1| < \lambda$
2. $n < k$ (equivalently $S_k = \emptyset$, that is, $|S_k| < 1$)

When $Q \geq q$, the following lemma implies the above two properties. (e.g., we obtain the above properties when we apply the example parameter 1 to Lemma 2.)

Lemma 2 $\Pr_{\overrightarrow{s}_1, \ldots, \overrightarrow{s}_q \xleftarrow{\$} S^k}[|S_i| \geq j] < (\frac{q^{m+1}}{(m+1)!Q^{im}})^j$.

We prove Lemma 2 in the next section.

For now, let us assume that we have m, k, and Q such that the above two properties hold. We separate the types of adversaries according to $\overrightarrow{s}^*$ as follows.

$$\begin{aligned} \text{Type-1} &: \overrightarrow{s}^{*(1)} \notin S_1. \\ \text{Type-2} &: \overrightarrow{s}^{*(1)} \in S_1, \text{ and } \overrightarrow{s}^{*(2)} \notin S_2. \\ &\vdots \\ \text{Type-}i &: \overrightarrow{s}^{*(i-1)} \in S_{i-1}, \text{ and } \overrightarrow{s}^{*(i)} \notin S_i. \\ &\vdots \\ \text{Type-}n &: \overrightarrow{s}^{*(n-1)} \in S_{n-1}, \text{ and } \overrightarrow{s}^{*(n)} \notin S_n. \\ \text{Type-}(n+1) &: \overrightarrow{s}^{*(n)} \in S_n. \end{aligned}$$

Here, $\overrightarrow{s}^{*(i-1)} \in S_{i-1}$ implies that $\overrightarrow{s}^{*(j)} \in S_j$ for all $j \in [1, i-1]$. Therefore, we can see that the above $n+1$ types of adversaries are pairwise disjoint and cover all possible adversaries. For the type-i adversary, the simulator can guess $\overrightarrow{s}^{*(i)}$ with probability $\frac{1}{|S_{i-1}|\cdot|S|}$; it guesses $\overrightarrow{s}^{*(i-1)}$ with $\frac{1}{|S_{i-1}|}$ and s_i^* with $\frac{1}{|S|}$ (we use the second property for the case $i = n+1$). Since the simulator can guess the type of the adversary with probability $\frac{1}{k}$, it can guess the tag vector of the forgery with at least probability $\frac{1}{k\lambda Q}$ (we use the first property for the inequality $|S_{i-1}| \leq |S_1| < \lambda$).

The other parts of the proof strategy are similar to the strategy for the short signatures with somewhat short public key in Figure 2 as we mentioned in Section 3.1; (1) guess $\overrightarrow{s}^{*(i)}$, (2) use the proof technique for the reduction from solving the DH problem to breaking the selectively secure signatures, and (3) generate for the signature with the same tag vector as $\overrightarrow{s}^{*(i)}$, using the programmability of the weak programmable hash functions. Since $\overrightarrow{s}^{*(i)} \notin S_i$ (for $i = n+1$, $\overrightarrow{s}^{*(i)} \notin S_i = \emptyset$) implies that there are at most m tag vectors same as $\overrightarrow{s}^{*(i)}$, the simulator can response m signatures with tag vector $\overrightarrow{s}^{*(i)}$ using the programmability of $(v \prod_{i=1}^{m} u_i^{M^i})$. If $k\lambda Q$ is bounded by a polynomial in λ, then we obtain the polynomial-time reduction.

By applying the above strategy, we give the following theorem.

Theorem 1 *The signature scheme in Figure 3 is (q, ϵ, T)-EU-wCMA secure assuming the (ϵ', T')-DH assumption holds such that*

$$\epsilon' = \frac{1}{k\lambda Q}(\epsilon - \frac{q^{m+1}}{(m+1)!Q^{km}} - \frac{q}{p} - (\frac{q^{m+1}}{(m+1)!Q^m})^\lambda) \quad \textit{and} \quad T \approx T'.$$

We prove Theorem 1 in the next section.

For a EU-q-CMA secure signature scheme, we can apply the generic transformation from a EU-q-wCMA secure scheme to a EU-q-CMA secure scheme given in Lemma 1.[6] From Theorem 1 and Lemma 1, we obtain the following corollary (by using a (ϵ_{ch}, T_{ch})-collision resistant hash function if need be).[7]

Corollary 1. *Let* SIG *be the signature scheme for fixed length messages described in Figure 3 and* SIG′ *be the signature scheme for arbitrary length messages obtained from* SIG *and a (ϵ_{ch}, T_{ch})-collision resistant hash function. Let* CHSetup *is a generator for the chameleon hashes based on the DL assumption. Let* SIG″ *be the signature scheme resulting from the generic transformation in Lemma 1 on* SIG′ *and* CHSetup*. If $\mathcal{G}$ satisfies $(\frac{1}{k\lambda Q}(\frac{\epsilon}{4} - \frac{q^{m+1}}{(m+1)!Q^{km}} - \frac{q}{p} - (\frac{q^{m+1}}{(m+1)!Q^m})^\lambda), T_{dh})$-DH assumption and $\mathcal{G}_{ch}$ satisfies the (ϵ_{dl}, T_{dl})-DL assumption, then* SIG *is $(q, \epsilon + 4\epsilon_{crh} + 2\epsilon_{dl}, T)$-EU-CMA secure, where $\mathcal{G}$ and $\mathcal{G}_{ch}$ are group generators used in* SIG *and* CHSetup*, respectively, q is the maximum number of signatures the adversary can obtain, and $T_{dh} \approx T_{dl} \approx T$.*

If the collision resistant hash function is not used (that is, the representation of the output of chameleon hashes needs less than or equal to 2λ-bit), and so SIG″ *is the signature scheme resulting from the generic transformation on* SIG *and* CHSetup*, then* SIG″ *is $(q, \frac{\epsilon}{2} + 2\epsilon_{dl}, T)$-EU-CMA secure.*

Proof. By Theorem 1, SIG is $(q, \frac{\epsilon}{4}, T)$-EU-wCMA secure. By the standard hybrid argument, we can show that SIG′ is $(q, 2(\frac{\epsilon}{4} + \epsilon_{crh}), T)$-EU-wCMA secure. Lastly, by Lemma 1, SIG″ is $(q, 2(2(\frac{\epsilon}{4} + \epsilon_{crh}) + \epsilon_{dl}), T)$-EU-CMA secure since the chameleon hash function based on the (ϵ_{dl}, T_{dl})-DL assumption has (ϵ_{dl}, T_{dl})-collision resistance. If the collision resistant hash function is not used, then Theorem 1 and Lemma 1 directly imply that SIG″ is $(q, 2(\frac{\epsilon}{4} + \epsilon_{dl}), T)$-EU-CMA secure. □

To derive a meaningful result about the asymptotic security from Theorem 1 and Corollary 1, we need the following three conditions.

[6] If the representation of a chameleon hash value needs more than 2λ-bit, then we first apply $(\epsilon_{crh}, T_{crh})$-collision resistant hash function for messages of the EU-q-wCMA secure signature scheme for arbitrary length messages since the chameleon hash values are messages of EU-q-wCMA secure signatures.

[7] If we use elliptic curve groups as the base group $\mathbb{G}_{ch}$, over which the chameleon hashes are defined, then the representation of the chameleon hash values are sufficiently short to be used as messages for EU-q-wCMA secure signatures without firstly applying collision-resistance hash functions. However, if we use multiplicative subgroups of finite fields as $\mathbb{G}_{ch}$, then we need collision-resistance hash functions to map the chameleon hash values into short messages for EU-q-wCMA secure signatures.

Condition 1. $k\lambda Q$ is polynomially bounded in λ.
Condition 2. $\frac{q^{m+1}}{(m+1)!Q^{km}}$ is a negligible function in λ.
Condition 3. $(\frac{q^{m+1}}{(m+1)!Q^{m}})^{\lambda}$ is a negligible function in λ.

We give asymptotic values of m, k, and Q for satisfying the above conditions and short public key in Section 4.3. For such parameters (e.g., example parameter 1), we obtain the following corollary.

Corollary 2. *Let* SIG'' *be a signature scheme in given in Corollary 1 with parameter* m*,* k*, and* Q *satisfying the above three conditions. Then,* SIG'' *is* $(q, \epsilon + 4\epsilon_{crh} + 2\epsilon_{dl}, T)$*-EU-CMA secure assuming* $(\frac{\epsilon}{4k\lambda Q} - neg(\lambda), T_{dh})$*-DH assumption holds, where* $neg(\lambda)$ *is a negligible function in* λ *and* $T \approx T_{dh}$*.*

If SIG'' *is the signature scheme resulting from the generic transformation on* SIG *and* $\mathsf{CHSetup}$ *(that is, the collision resistant hash function is not used), then* SIG'' *is* $(q, \frac{\epsilon}{2} + 2\epsilon_{dl}, T)$*-EU-CMA secure assuming* $(\frac{\epsilon}{4k\lambda Q} - neg(\lambda), T_{dh})$*-DH assumption holds, where* $neg(\lambda)$ *is a negligible function in* λ *and* $T \approx T_{dh}$*.*

4 Analysis

4.1 Proof of Lemma 2

In this subsection, we prove Lemma 2. Let F be the set of all functions from $[1, q]$ to S^i. For $\overrightarrow{y} \in S^i$ and $f \in F$, let $|f^{-1}(\overrightarrow{y})|$ be the number of the distinct pre-images of $\overrightarrow{y}$. Let T_f be the set of all $\overrightarrow{y} \in Im(f)$ such that $|f^{-1}(\overrightarrow{y})| \geq m+1$, where $Im(f)$ means the set of all images of f. Then, we can consider $\Pr_{\overrightarrow{s}_1,\ldots,\overrightarrow{s}_q \xleftarrow{\$} S^k}[|S_i| \geq j]$ as

$$\Pr_{f \xleftarrow{\$} F}[|T_f| \geq j].$$

To compute $\Pr_{f \xleftarrow{\$} F}[|T_f| \geq j]$, we count all functions f such that $|T_f| \geq j$, then divide the result by $|S^i|^q$ (the number of all elements in F). In fact, we count the number of f such that $|T_f| \geq j$, allowing duplications, so that we compute the upper bound of $\Pr_{f \xleftarrow{\$} F}[|T_f| \geq j]$. To define an f, we choose j distinct subsets $A_1, \ldots, A_j$ of size $m+1$ from $[1, q]$ and j distinct vectors $\overrightarrow{y}_1, \ldots, \overrightarrow{y}_j$ from S^i, and then set $f(a) = \overrightarrow{y}_t$ for all $a \in A_t$ and $t \in [1, j]$. For other integers $a \in [1, q] \setminus (A_1 \cup \ldots \cup A_j)$, we arbitrarily define $f(a)$. This way of defining a function covers all f such that $|T_f| \geq j$. We count all f that are defined as above. Then, the number of such f is bounded by

$$\Big(\prod_{t=0}^{j-1} \binom{q - t(m+1)}{m+1} \cdot (|S^i| - t)\Big) \cdot (|S^i|)^{(q-j(m+1))},$$

where the notation $\binom{\cdot}{\cdot}$ denotes the binomial coefficient.

Therefore, we can obtain the desired result as follows:

$$\begin{aligned}
\Pr_{\overrightarrow{s}_1,\ldots,\overrightarrow{s}_q \xleftarrow{\$} S^k}[|S_i| \geq j] &= \Pr_{f \xleftarrow{\$} F}[|T_f| \geq j] \\
&< \frac{\Big(\prod_{t=0}^{j-1} \binom{q-t(m+1)}{m+1} \cdot (Q^i - t)\Big) \cdot (Q^i)^{(q-j(m+1))}}{|S^i|^q} \\
&< \frac{\Big(\frac{q^{m+1}}{(m+1)!}\Big)^j Q^{ij + i(q-j(m+1))}}{Q^{iq}} \\
&= (\tfrac{q^{m+1}}{(m+1)!Q^{im}})^j.
\end{aligned}$$

Remark. The result in Lemma 2 is similar as the lemma given in [24] called 'generalized birthday bound'. Note that Lemma 2 is more general than 'generalized birthday bound'; e.g., if we set $i = 1$ and $j = 1$, then the result in Lemma 2 provides a more tighter upper bound than 'generalized birthday bound' given in [24].

4.2 Proof of Theorem 1

In this subsection, we prove Theorem 1. Suppose that there exists a probabilistic (polynomially bounded) adversary $\mathcal{A}$ which makes at most q signing queries and outputs a valid forgery with probability ϵ. We construct an algorithm $\mathcal{B}$ that uses $\mathcal{A}$ as an internal sub-algorithm and breaks the DH assumption.

Simulation Description. We now describe an algorithm $\mathcal{B}$ on input (g, g^a, g^b), the uniform instance of the DH problem. Let $A = g^a$ and $B = g^b$. We use the notations S, S^i, and S_i defined in 3.2.

KeyGen: Without loss of generality, we assume that $\mathcal{A}$ makes the maximum q queries for signing. First, $\mathcal{B}$ receives a list L of messages $M_1, \ldots, M_q$ on which $\mathcal{A}$ requests signatures. $\mathcal{B}$ uniformly generates random vectors $\overrightarrow{s}_1, \ldots, \overrightarrow{s}_q \xleftarrow{\$} S^k$ such that $\overrightarrow{s}_j$ will be used in the j-th signing query on M_j. $\mathcal{B}$ separately behaves according to whether $n = k$. If $n = k$, we say that the event $E1$ occurs. When $E1$ occurs, $\mathcal{B}$ acts as a real challenger by running the KeyGen algorithm. When $E1$ does not occur, $\mathcal{B}$ guesses the tag vector of the forgery; $\mathcal{B}$ uniformly chooses ℓ from $[1, n+1]$, $(s'_1, \ldots, s'_{\ell-1})$ from $S_{\ell-1}$ except for the case $\ell = 1$, and s'_ℓ from S such that $(s'_1, \ldots, s'_\ell) \notin S_\ell$. Since $|S_\ell| \leq \frac{Q}{m+1} < Q = |S|$, there always exists such a s'_ℓ. We use notation $\overrightarrow{s}'^{(\ell)}$ to denote $(s'_1, \ldots, s'_\ell)$.

Next, $\mathcal{B}$ generates a public key. Since $\overrightarrow{s}'^{(\ell)} \notin S_\ell$, there exist at most m distinct integers $j_1, \ldots, j_m \in [1, q]$ such that $\overrightarrow{s}'^{(\ell)} = \overrightarrow{s}^{(\ell)}_{j_1} = \ldots = \overrightarrow{s}^{(\ell)}_{j_m}$. Let J be a set of such integers, so that $|J| \leq m$. Let $f(x)$ be a polynomial defined as

$$f(x) = \begin{cases} \prod_{i \in J}(X - M_i) & \text{if } |J| \geq 1, \\ 1 & \text{otherwise.} \end{cases}$$

Let $x_0, \ldots, x_m$ be coefficients of $f(x)$ such that $f(x) = \sum_{i=0}^{m} x_i X^i$ (for $i > |J|$, set $x_i = 0$). $\mathcal{B}$ uniformly chooses integers $y_0, \ldots, y_m, z_0, \ldots, z_\ell, w_0, \ldots, w_k \xleftarrow{\$} \mathbb{Z}_p$. If for some $j \in [1, q] \setminus J$, $\sum_{i=1}^{\ell}(s_{ji} - s'_i)z_i = 0$, where $\overrightarrow{s}_j = (s_{j1}, \ldots, s_{jk})$, then we say that the event $E2$ occurs. If $E2$ occurs, $\mathcal{B}$ behaves like a real challenger running the KeyGen algorithm and using $\overrightarrow{s}_j$ in the j-th signing query. Otherwise ($E2$ does not occur), $\mathcal{B}$ generates a public key as follows.

$$\begin{aligned} v &:= A^{x_0} g^{y_0}, & u_i &:= A^{x_i} g^{y_i} \text{ for } i \in [1, m], \\ h &:= A^{-\sum_{i=1}^{\ell} s'_i z_i} g^{w_0}, & g_i &:= \begin{cases} A^{z_i} g^{w_i} & \text{for } i \in [1, \ell] \\ g^{w_i} & \text{for } i \in [\ell+1, k] \end{cases}, \\ g &:= g, & g^\alpha &:= B. \end{aligned}$$

The secret key is b, which is unknown to $\mathcal{B}$.

Sign: When $E1$ or $E2$ occurs, $\mathcal{B}$ behaves like a real challenger by running the Sign algorithm except for using $\overrightarrow{s}_1, \ldots, \overrightarrow{s}_q$ generated in the *KeyGen* phase.

Otherwise (both $E1$ and $E2$ do not occur), $\mathcal{B}$ generates signatures on $M_1, \ldots, M_q$ as follows. For the j-th signing query, $\mathcal{B}$ separately signs on M_j according to whether $j \in J$.

Case $j \in [1, q] \setminus J$: Since $E2$ does not occur, $\sum_{i=1}^{\ell}(s_{j1} - s'_i)z_i \neq 0$. Choose $t' \xleftarrow{\$} \mathbb{Z}_p$ and compute

$$\begin{aligned} \sigma_{j1} &= B^{(\sum_{i=0}^{m} y_i M_j^i) - (w_0 + \sum_{i=1}^{k} s_{ji} w_i) \frac{(\sum_{i=0}^{m} x_i M_j^i)}{(\sum_{i=1}^{\ell}(s_{j1} - s'_i)z_i)}} \cdot (A^{(\sum_{i=1}^{\ell}(s_{j1} - s'_i)z_i)} g^{w_0 + \sum_{i=1}^{k} s_{ji} w_i})^{t'} \\ \text{and } \sigma_{j2} &= B^{(\sum_{i=0}^{m} x_i M_j^i)/(\sum_{i=1}^{\ell}(s_{j1} - s'_i)z_i)} \cdot g^{-t'}. \end{aligned}$$

Case $j \in J$: Choose $t \xleftarrow{\$} \mathbb{Z}_p$ and compute

$$\sigma_{j1} = B^{(\sum_{i=0}^{m} y_i M_j^i)} (g^{w_0 + \sum_{i=1}^{k} s_{ji} w_i})^t \text{ and } \sigma_{j2} = g^{-t}.$$

For both cases, we define the j-th signature $\sigma_{(j)}$ on M_j as $(\sigma_{j1}, \sigma_{j2}, \overrightarrow{s}_j)$.

Response: $\mathcal{B}$ sends $\mathcal{A}$ the public key $PK = (v, u_1, \ldots, u_m, h, g_1, \ldots, g_k, g, g^\alpha)$ along with signatures $\sigma_{(1)}, \ldots, \sigma_{(q)}$.

Extract solution of DH problem from forgery: When $E1$ or $E2$ occurs, $\mathcal{B}$ outputs a random group element as the answer of the DH problem. Otherwise, do the following.

$\mathcal{B}$ receives a message M^* along with a forgery $\sigma^* = (\sigma_1^*, \sigma_2^*, \overrightarrow{s}^*)$ on M^* from $\mathcal{A}$ such that $M^* \notin L$. If $\mathsf{Verify}(PK, M^*, \sigma^*) = 0$, $\mathcal{B}$ aborts. If $\overrightarrow{s}^{*(\ell)} \neq \overrightarrow{s}^{\prime(\ell)}$, $\mathcal{B}$ aborts. Otherwise, $M^* \notin L$, so that $f(M^*) = \sum_{i=0}^{m} x_i (M^*)^i \neq 0$. $\mathcal{B}$ outputs $(\sigma_1^* \cdot B^{-\sum_{i=0}^{m} y_i (M^*)^i} \cdot (\sigma_2^*)^{w_0 + \sum_{i=1}^{k} s_i^* w_i})^{\frac{1}{f(M^*)}}$.

During an interaction between $\mathcal{A}$ and $\mathcal{B}$, $\mathcal{B}$ randomly chooses a group element from $\mathbb{G}$ and outputs it if $\mathcal{B}$ aborts or $\mathcal{A}$ quits the interaction.

Analysis (Distribution of Simulated Transcript). First, we argue that the distribution of the public key and signatures on $M_1, \ldots, M_q$ generated by $\mathcal{B}$ are identical to that of the real challenger. $\overrightarrow{s}_1, \ldots, \overrightarrow{s}_q$ are uniformly distributed. Even if $E1$ and $E2$ are determined by $\overrightarrow{s}_1, \ldots, \overrightarrow{s}_q$, the tag vectors $\overrightarrow{s}_1, \ldots, \overrightarrow{s}_q$ are used no matter whether $E1$ or $E2$ occurs. Therefore, in the view of $\mathcal{A}$, $\overrightarrow{s}_1, \ldots, \overrightarrow{s}_q$ are uniformly distributed, so we only focus on the other randomness used in the public key and signatures. When the events $E1$ or $E2$ occurs, $\mathcal{B}$ acts as the real challenger, so that the distributions generated by $\mathcal{B}$ and the real challenger are identical. Otherwise, $y_0, \ldots, y_m$, and $w_0, \ldots, w_k$ are uniformly chosen from $\mathbb{Z}_p$ and are independent from $E1$ and $E2$. Furthermore, the DH instance is also uniformly generated. Therefore, the distributions of the public key generated by $\mathcal{B}$ are identical to those of the output of the KeyGen algorithm.

Next, we consider the distribution of the j-th signature on M_j when $E1$ and $E2$ do not occur (that is, $\sum_{i=1}^{\ell}(s_{j1} - s'_i) z_i \neq 0$). If $j \in [1, q] \setminus J$, randomness t is distributed as if $t = -\frac{(\sum_{i=0}^{m} x_i M_j^i b)}{(\sum_{i=1}^{\ell}(s_{j1} - s'_i) z_i)} + t'$. More precisely, we can check the following equations from the equality $t = -\frac{(\sum_{i=0}^{m} x_i M_j^i b)}{(\sum_{i=1}^{\ell}(s_{j1} - s'_i) z_i)} + t'$.

$$
\begin{aligned}
\sigma_{j1} &= B^{(\sum_{i=0}^{m} y_i M_j^i) - (w_0 + \sum_{i=1}^{k} s_{ji} w_i) \frac{(\sum_{i=0}^{m} x_i M_j^i)}{(\sum_{i=1}^{\ell}(s_{ji} - s'_i) z_i)}} \cdot (A^{\sum_{i=1}^{\ell}(s_{j1} - s'_i) z_i} g^{w_0 + \sum_{i=1}^{k} s_{ji} w_i})^{t'} \\
&= B^{(\sum_{i=0}^{m} y_i M_j^i)} (g^{ab})^{\sum_{i=0}^{m} x_i M_j^i} \cdot (A^{\sum_{i=1}^{\ell}(s_{ji} - s'_i) z_i} g^{w_0 + \sum_{i=1}^{k} s_{ji} w_i})^{\frac{(-\sum_{i=0}^{m} x_i M_j^i b)}{(\sum_{i=1}^{\ell}(s_{ji} - s'_i) z_i)}} \cdot (A^{\sum_{i=1}^{\ell}(s_{ji} - s'_i) z_i} g^{w_0 + \sum_{i=1}^{k} s_{ji} w_i})^{t'} \\
&= (\prod_{i=0}^{m} (A^{x_i} g^{y_i})^{M_j^i})^b \cdot (A^{-\sum_{i=1}^{\ell} s'_i z_i} g^{w_0} \cdot \prod_{i=1}^{\ell} (A^{z_i} g^{w_i})^{s_{ji}} \cdot \prod_{i=\ell+1}^{k} (g^{w_i})^{s_{ji}})^t \\
&= (v \prod_{i=1}^{m} u_i^{M_j^i})^b (h \prod_{i=1}^{k} g_i^{s_{ji}})^t
\end{aligned}
$$

and

$$
\sigma_{j2} = B^{(\sum_{i=0}^{m} x_i M_j^i)/(\sum_{i=1}^{\ell}(s_{j1} - s'_i) z_i)} \cdot g^{-t'} \;=\; g^{-t}.
$$

Since t' is uniformly chosen from $\mathbb{Z}_p$ and independent of all other values in $\frac{(\sum_{i=0}^{m} x_i M_j^i b)}{(\sum_{i=1}^{\ell}(s_{j1} - s'_i) z_i)}$, t is also uniformly distributed. Therefore, the distribution of σ_{j1} and σ_{j2} is identical to that of the output of Sign algorithm.

If $j \in J$, t is uniformly distributed, that is,

$$
\begin{aligned}
\sigma_{j1} &= B^{(\sum_{i=0}^{m} y_i M_j^i)} (g^{w_0 + \sum_{i=1}^{k} s_{ji} w_i})^t \\
&= B^{(\sum_{i=0}^{m} y_i M_j^i)} (g^{ab})^{\sum_{i=0}^{m} x_i M_j^i} \cdot (A^{\sum_{i=1}^{\ell}(s_{ji} - s'_i) z_i} g^{w_0 + \sum_{i=1}^{k} s_{ji} w_i})^t \\
&= (\prod_{i=0}^{m} (A^{x_i} g^{y_i})^{M_j^i})^b \cdot (A^{-\sum_{i=1}^{\ell} s'_i z_i} g^{w_0} \prod_{i=1}^{\ell} (A^{z_i} g^{w_i})^{s_{ji}} \prod_{i=\ell+1}^{k} (g^{w_i})^{s_{ji}})^t \\
&= (v \prod_{i=1}^{m} u_i^{M_j^i})^b (h \prod_{i=1}^{k} g_i^{s_{ji}})^t
\end{aligned}
$$

By the definition of J and $f(x)$, $f(M_j) = (\sum_{i=0}^{m} x_i M_j^i) = 0$ and $s_{ji} = s'_i$ for all $j \in J$ and $i \in [1, \ell]$. This implies that the second equality holds. Since $\sigma_{j2} = g^{-t}$, σ_{j1} and σ_{j2} are distributed as the output of Sign algorithm.

Analysis (Success Probability). We now show that the success probability of $\mathcal{B}$ breaking the DH assumption. We separate the types of adversaries according to $\overrightarrow{s}^*$ as follows.

Type-1: $\overrightarrow{s}^{*(1)} \notin S_1$.
Type-2: $\overrightarrow{s}^{*(1)} \in S_1$, and $\overrightarrow{s}^{*(2)} \notin S_2$.
$\vdots$
Type-i: $\overrightarrow{s}^{*(i-1)} \in S_{i-1}$, and $\overrightarrow{s}^{*(i)} \notin S_i$.
$\vdots$

Type-n: $\overrightarrow{s}^{*(n-1)} \in S_{n-1}$, and $\overrightarrow{s}^{*(n)} \notin S_n$.
Type-$(n+1)$: $\overrightarrow{s}^{*(n)} \in S_n$.

As we mentioned in Section 3.2, we can see that the above $n+1$ types of the adversaries are pairwise disjoint and cover all possible adversaries.

We define several events. Let ℓ^* be the type of adversary and C be the event such that $\ell^* = \ell$ and $\overrightarrow{s}^{*(\ell)} = \overrightarrow{s}^{\prime(\ell)}$. Let D be the event such that $|S_1| \geq \lambda$, F be the event such that $\mathcal{A}$ successfully (weakly) forges a signature, and S be the event such that $\mathcal{B}$ outputs g^{ab}. From the definition of events, we derive the followings.

$$\begin{aligned}\Pr[S] &> \Pr[S \wedge C \wedge \neg E1 \wedge \neg E2 \wedge \neg D \wedge F] \\ &= \Pr[S \wedge C | \neg E1 \wedge \neg E2 \wedge \neg D \wedge F] \cdot \Pr[\neg E1 \wedge \neg E2 \wedge \neg D \wedge F] \\ &= \Pr[S | \neg E1 \wedge \neg E2 \wedge \neg D \wedge F \wedge C] \cdot \Pr[C | \neg E1 \wedge \neg E2 \wedge \neg D \wedge F] \cdot \Pr[\neg E1 \wedge \neg E2 \wedge \neg D \wedge F],\end{aligned}$$

where the probability is over all randomness used by $\mathcal{A}$ and $\mathcal{B}$ in the simulation.

We first show that the probability

$$\Pr[S | \neg E1 \wedge \neg E2 \wedge \neg D \wedge F \wedge C] = 1.$$

Suppose that the event $(\neg E1 \wedge \neg E2 \wedge \neg D \wedge F \wedge C)$ occurs. Since $\mathcal{A}$ outputs a valid forgery such that σ_1^* and σ_2^* satisfy the verification equation (event F), we can see that σ_1^* and σ_2^* are $(v \prod_{i=1}^m u_i^{M^{*i}})^\alpha (h \prod_{i=1}^k g_i^{s_i^*})^t$ and g^{-t} for some t, respectively. From the hypothesis $\overrightarrow{s}^{*(\ell)} = \overrightarrow{s}^{\prime(\ell)}$ (event C), we know

$$\sigma_1^* = (g^{ab})^{f(M^*)} (g^b)^{\sum_{i=0}^m y_i M^{*i}} (g^t)^{w_0 + \sum_{i=1}^k s_i^* w_i}.$$

We can show that the output of $\mathcal{B}$ is

$$(\sigma_1^* \cdot B^{-\sum_{i=0}^m y_i (M^*)^i} \cdot (\sigma_2^*)^{w_0 + \sum_{i=1}^k s_i^* w_i})^{\frac{1}{f(M^*)}} = g^{ab}.$$

That is, $\Pr[S | \neg E1 \wedge \neg E2 \wedge \neg D \wedge F \wedge C] = 1$.

Next, we show that

$$\Pr[C | \neg E1 \wedge \neg E2 \wedge \neg D \wedge F] \geq \frac{1}{k} \cdot \frac{1}{\lambda} \cdot \frac{1}{Q}.$$

Event E_2 is independent from the other events since it is determined by independent variables z_i's, which are perfectly hidden from adversarial view, so that $\Pr[C | \neg E1 \wedge \neg D \wedge F] = \Pr[C | \neg E1 \wedge \neg E2 \wedge \neg D \wedge F]$. Since ℓ^* should be in $[1, n+1]$ and ℓ is perfectly hidden from adversarial view, $\Pr[\ell = \ell^* | \neg E1 \wedge \neg D \wedge F] = \frac{1}{n+1} \geq \frac{1}{k}$ (ℓ is chosen independently from other values used in the simulation so that it is independent from the other events). Suppose that $\ell^* = \ell$. Then, $\overrightarrow{s}^{*(\ell-1)} \in S_{\ell-1}$. The event $\overrightarrow{s}^{*(\ell)} = \overrightarrow{s}^{\prime(\ell)}$ is equal to $(\overrightarrow{s}^{*(\ell-1)} = \overrightarrow{s}^{\prime(\ell-1)}) \wedge (s_\ell^* = s_\ell')$ (except for the case $\ell = 1$). Since $\overrightarrow{s}^{\prime\ell-1}$ and s_ℓ' are uniformly chosen from $S_{\ell-1}$ and S, respectively and $\overrightarrow{s}^{\prime\ell}$ is perfectly hidden from adversarial view,

$$\Pr[\overrightarrow{s}^{*(\ell)} = \overrightarrow{s}^{\prime(\ell)} | (\ell = \ell^*) \wedge \neg E1 \wedge \neg D \wedge F] = \begin{cases} \frac{1}{Q} & \text{if } \ell = 1 \\ \frac{1}{|S_{\ell-1}|} \cdot \frac{1}{Q} & \text{otherwise.} \end{cases}$$

From the condition $\neg D$, we know that $|S_{\ell-1}| \leq |S_1| < \lambda$. Therefore, $\Pr[\overrightarrow{s}^{*(\ell)} = \overrightarrow{s}^{\prime(\ell)} | (\ell = \ell^*) \wedge \neg E1 \wedge \neg D \wedge F] > \frac{1}{\lambda Q}$; thus,

$$\begin{aligned}\Pr[C | \neg E1 \wedge \neg E2 \wedge \neg D \wedge F] &= \Pr[C | \neg E1 \wedge \neg D \wedge F] \\ &= \Pr[\overrightarrow{s}^{*(\ell)} = \overrightarrow{s}^{\prime(\ell)} | (\ell = \ell^*) \wedge \neg E1 \wedge \neg D \wedge F] \cdot \Pr[\ell = \ell^* | \neg E1 \wedge \neg D \wedge F] \\ &> \tfrac{1}{k \lambda Q}.\end{aligned}$$

Next, we show that the probability

$$\Pr[\neg E1 \wedge \neg E2 \wedge \neg D \wedge F] = \epsilon - (\tfrac{q^{m+1}}{(m+1)! Q^{km}} + \tfrac{q}{p} + (\tfrac{q^{m+1}}{(m+1)! Q^m})^\lambda).$$

By Lemma 2, we see that

$$\Pr[E1] = \Pr[|S_k| \geq 1] < \frac{q^{m+1}}{(m+1)!Q^{km}}$$

and

$$\Pr[D] = \Pr[|S_1| \geq \lambda] < (\frac{q^{m+1}}{(m+1)!Q^m})^\lambda.$$

The $E2$ is the event such that for some $j \in [1,q] \setminus J$, $\sum_{i=1}^{\ell}(s_{ji} - s'_i)z_i = 0$. For each $j \in [1,q] \setminus J$, there always exists a s_{ji} such that $(s_{ji} - s'_i) \neq 0$ for some i (by the definition of J); and hence, for each $j \in [1,q] \setminus J$ we obtain

$$\Pr[\sum_{i=1}^{\ell}(s_{ji} - s'_i)z_i = 0] = \frac{1}{p},$$

where the probability goes over the choice of $z_1, \ldots, z_\ell$. By the union bound, we obtain

$$\Pr[E2] < \frac{q}{p}.$$

We can easily verify the following.

$$\Pr[\neg E1 \wedge \neg E2 \wedge \neg D \wedge F] \;=\; \Pr[F] - \Pr[(E1 \vee E2 \vee D) \wedge F] \;>\; \Pr[F] - (\Pr[E1] + \Pr[E2] + \Pr[D]).$$

Therefore, we obtain the desired result

$$\Pr[\neg E1 \wedge \neg E2 \wedge \neg D \wedge F] \;=\; \epsilon - (\tfrac{q^{m+1}}{(m+1)!Q^{km}} + \tfrac{q}{p} + (\tfrac{q^{m+1}}{(m+1)!Q^m})^\lambda).$$

Putting it all together, we obtain the lower bound of the success probability of $\mathcal{B}$

$$\Pr[S] > \frac{1}{k\lambda Q}(\epsilon - \frac{q^{m+1}}{(m+1)!Q^{km}} - \frac{q}{p} - (\frac{q^{m+1}}{(m+1)!Q^m})^\lambda).$$

4.3 Parameter Selection for Short Public Key

The description of our construction did not explain how to choose m, k, and Q. In this subsection, we show how to minimize public key size. From Theorem 1, we obtained three conditions for polynomial-time reduction to the DH problem. First, $k\lambda Q$ should be polynomially bounded in λ. Second, $\frac{q^{m+1}}{(m+1)!Q^{km}}$ and $(\frac{q^{m+1}}{(m+1)!Q^m})^\lambda$ should be a negligible function in λ. For simple analysis, we assume that $Q = Cq$ for small constant $C > 1$ and compute conditions for m and k when $\frac{q^{m+1}}{(m+1)!Q^{km}}$ and $(\frac{q^{m+1}}{(m+1)!Q^m})^\lambda$ are smaller than $\frac{1}{2^\lambda}$. We compute asymptotically minimal values of m and k for short public key size, and then provide practical parameters with reasonable reduction loss, which is comparable to that of Waters signature in [38].

Condition 1. $k\lambda q$ is polynomially bounded in λ.
Condition 2. $\frac{q^{m+1}}{(m+1)!Q^{km}} < \frac{1}{2^\lambda}$.
Condition 3. $(\frac{q^{m+1}}{(m+1)!Q^m})^\lambda < \frac{1}{2^\lambda}$.

From the condition 2, at least the denominator should be larger than 2^λ. Since $Q = Cq$ and $(m+1)! \approx \sqrt{2\pi(m+1)}(\frac{m+1}{e})^{m+1}$ (by Stirling's approximation), where e is the Euler's number, $km = \Omega(\frac{\lambda}{\log\lambda})$ or $m = \Omega(\frac{\lambda}{\log\lambda})$. For minimizing public key size, we should minimize $m + k$ since the size of public key is $\Theta(m+k)$. Therefore, $m = \Theta(\sqrt{\frac{\lambda}{\log\lambda}})$ and $k = \Theta(\sqrt{\frac{\lambda}{\log\lambda}})$ are (asymptotically) minimal parameters for minimal public key size. In fact, if we set $m = \Theta(\sqrt{\frac{\lambda}{\log\lambda}})$ and $k = \Theta(\sqrt{\frac{\lambda}{\log\lambda}})$, then the condition 1 and 3 also hold.

Next, we provide practical parameters for $\lambda \in \{80, 128, 192, 256\}$ and $q \in \{2^{30}, 2^{40}\}$, where λ is the security parameter and q is the bound for adversarial signing queries. If the above condition 2 and condition 3 hold, our

$\mathsf{KeyGen}(\lambda)$: Run $\mathcal{G} \to (p, \mathbb{G}, \mathbb{G}_t, e)$.
Choose $v, u_1, \ldots, u_m, g_1, \ldots, g_k, h, g \xleftarrow{\$} \mathbb{G}$, $\alpha \xleftarrow{\$} \mathbb{Z}_p$, and pick a PRF key K at random.
Output $PK = \{v, u_1, \ldots, u_m, g_1, \ldots, g_k, h, g, g^\alpha, K\}$ and $SK = \{\alpha\}$,
where $PRF : \{0,1\}^* \to \{0,1\}^{k\lceil \log Q \rceil}$ be a pseudorandom function family.

$\mathsf{Sign}(PK, M, SK)$: Uniformly choose $t \xleftarrow{\$} \mathbb{Z}_p$.
Compute $PRF_K(M) = (s_1, \ldots, s_k)$, $\sigma_1 = (v \prod_{i=1}^m u_i^{M^i})^\alpha (h \prod_{i=1}^k g_i^{s_i})^t$, and $\sigma_2 = g^{-t}$.
Output $\sigma = (\sigma_1, \sigma_2)$.

$\mathsf{Verify}(PK, M, \sigma)$: Parse σ to (σ_1, σ_2).
Compute $PRF_K(M) = (s_1, \ldots, s_k)$.
Check whether $e(\sigma_1, g) e(\sigma_2, h \prod_{i=1}^k g_i^{s_i}) \stackrel{?}{=} e(g^\alpha, v \prod_{i=1}^m u_i^{M^i})$.
Output 1 if the above equation holds; otherwise, 0.

Fig. 4. Signatures with tag compression

security proof loses $4k\lambda Q$ factor in the simulation (when we ignore negligible factors), which is asymptotically larger than Waters signature scheme's reduction loss. However, for practical choices of λ and q, k is a small constant (at most 3 in our example parameters) and Q is Cq with small constant $C > 1$ ($C = 2^3$ in our example parameters); and thus, the reduction loss in our example parameters is at most $96\lambda q$, which is comparable to that given in [38][8], where we assume that the message size is 2λ.[9] The example practical parameters are given in the table 1, where we set $Q = 2^3 q$ and thus all reduction loss of signature scheme with parameters in the table 1 are less than or equal to $96\lambda q$. To get the table 1, we firstly set $Q = 2^3 q$ and $q \in \{2^{30}, 2^{40}\}$, and then find small m and k satisfying the above three conditions. The size of a tag vector is a $k\lceil \log Q \rceil$-bit string, which is asymptotically smaller than 2λ if $k = \Theta(\sqrt{\frac{\lambda}{\log \lambda}})$ and Q is a polynomial in λ; and thus, we can assume that a tag vector is a field element of $\mathbb{Z}_p$. In particular, when we apply practical parameters in the table 1, the size of a tag vector is still smaller than 2λ (e.g., a tag vector is 129-bit string when $k = 3$ and $Q = 2^{43}$).

5 Extensions

In this section, we provide two extensions of the main construction in Figure 3. We give a (EU-q-wCMA secure) variant of the main scheme that has shorter signatures (two group elements). Next, an instantiation using asymmetric pairings (type-2 pairings or type-3 pairings) instead of symmetric pairings (type-1 pairings) is considered.

5.1 Tag Compression using Pseudorandom Functions

In this subsection, we introduce a trick for tag compression using (non-adaptive) pseudorandom functions (PRF). Note that similar techniques are used in the RSA-based signatures [27, 28, 24, 39] to compress random prime numbers used in each signature. If we use this trick, we can reduce the signature size of EU-q-wCMA secure scheme to two group elements by augmenting signing/verification costs and adding constant factor in public key size (that is, public key size is still $\Theta(\sqrt{\frac{\lambda}{\log \lambda}})$ group elements); each signature has a tag vector that is uniformly chosen from its domain. Thus, a signer can use pseudorandom functions (PRF) mapping from messages to tag vectors, and publishes the PRF the signer used along with its key. Even though the signer publishes the PRF key, (in the weak security model) we can use the fact that the distribution of tag vectors is indistinguishable

[8] Hofheinz et al. proposed a variant of Waters signatures using a special encoding for optimal security reduction $\Theta(\frac{1}{q})$ [25]. In [25], however, they do not provide a concrete constant factor of Θ notation for practical security parameters, but only asymptotic analysis for optimal security reduction.

[9] For arbitrarily large message, both our scheme and Waters' scheme take a collision resistant hash function value, and then run signing algorithms. To prevent the birthday attack, the output of hash function should be larger than 2λ.

$\mathsf{KeyGen}(\lambda)$: Run $\mathcal{G} \to (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_t, e)$.
Choose $\mathsf{v}, \mathsf{u}_1, \ldots, \mathsf{u}_\mathsf{m}, \mathsf{g}_1, \ldots, \mathsf{g}_\mathsf{k}, \mathsf{h}, \alpha \xleftarrow{\$} \mathbb{Z}_p, g \xleftarrow{\$} \mathbb{G}_2, g' \xleftarrow{\$} \mathbb{G}_1$, and pick a PRF key K at random.
Compute $v = g^{\mathsf{v}}, u_i = g^{\mathsf{u}_i}, g_i = g^{\mathsf{g}_i}, h = g^{\mathsf{h}}$.
Output $PK = \{v, u_1, \ldots, u_m, g_1, \ldots, g_k, h, g \in \mathbb{G}_2, g'^{\alpha} \in \mathbb{G}_1, K\}$
and $SK = \{\mathsf{v}, \mathsf{u}_1, \ldots, \mathsf{u}_m, \mathsf{g}_1, \ldots, \mathsf{g}_k, \mathsf{h}, \alpha \in \mathbb{Z}_p, g' \in \mathbb{G}_1\}$,
where $PRF : \{0,1\}^* \to \{0,1\}^{k\lceil \log Q \rceil}$ be a pseudorandom function family.

$\mathsf{Sign}(PK, M, SK)$: Uniformly choose $t \xleftarrow{\$} \mathbb{Z}_p$
Compute $PRF_K(M) = (s_1, \ldots, s_k), \sigma_1 = g'^{(\mathsf{v}+\sum_{i=1}^{m} \mathsf{u}_i M^i)\alpha + (\mathsf{h}+\sum_{i=1}^{k} \mathsf{g}_i s_i)t}$ and $\sigma_2 = g'^{-t} \in \mathbb{G}_1$.
Output $\sigma = (\sigma_1, \sigma_2)$.

$\mathsf{Verify}(PK, M, \sigma)$: Parse σ to (σ_1, σ_2).
Compute $PRF_K(M) = (s_1, \ldots, s_k)$.
Check whether $e(\sigma_1, g)e(\sigma_2, h\prod_{i=1}^{k} g_i^{s_i}) \stackrel{?}{=} e(g'^{\alpha}, v\prod_{i=1}^{m} u_i^{M^i})$.
Output 1 if the above equation holds; otherwise, 0.

Fig. 5. Instantiation using asymmetric pairings

from the uniform distribution. In some application, short signatures are important even though public key size and signing/verifying costs increase. Then, the signature scheme with tag compression technique is appropriate in such applications.

Let $PRF : \{0,1\}^* \to \{0,1\}^{k\lceil \log Q \rceil}$ be a pseudorandom function family. The signer randomly chooses a PRF key K, uses $PRF_K(M)$ as the tag vector associated with the signature of M, and publishes the description of PRF and the key K of PRF as a part of PK. Then, we can remove tag vectors from signatures since everyone who knows a message and PK can compute the corresponding tag vector. Hence, the resulting scheme can have shorter signatures (two group elements). The signature scheme with tag compression technique is given in Figure 4.

This simple modified signature scheme is also EU-q-wCMA secure; in the security proof, we need the uniformity of all tag vectors used in signing queries, which is guaranteed by the proof of Lemma 2. More precisely, as we mentioned in Section 3.2, two properties $|S_1| < \lambda$ and $|S_k| < 1$ are essentially used in the security proof. If each tag vector is computed by $\overrightarrow{s}_i = R(M_i)$, where $R(\cdot)$ is a random function, then we obtain the above two properties since Lemma 2 with an appropriate parameter selection of m, k, and Q implies that $\Pr[|S_1| \geq \lambda]$ and $\Pr[|S_k| \geq 1]$ are negligible functions in λ. (In our choices of practical parameters, both probability are less than $\frac{1}{2^\lambda}$.) Therefore, if we change a random function R to a pseudorandom function PRF with randomly chosen key K, then we also obtain the same result such that $|S_1| < \lambda$ and $|S_k| < 1$. If not (that is, $|S_1| \geq \lambda$ or $|S_k| \geq 1$), we can construct a distinguisher, which distinguish the output of PRF_K from random strings, where K is randomly chosen. Note that for this case, it is no matter whether the key K is published in the public key of the signature scheme or not. By using these two properties $|S_1| < \lambda$ and $|S_k| < 1$, we can prove the security of a signature scheme with tag compression since other parts are essentially same to the proof of Theorem 1.

5.2 Instantiation using Asymmetric Pairings

Although we described our construction using type-1 pairings in Section 3, we can easily modify our construction to be instantiated using type-2 pairings or type-3 parings as in Figure 5. The scheme using type-1 parings and its security proof does not use pairing's symmetry property. Our main idea to achieve sublinear public key is to divide adversarial types according to tag vectors in the security proof, and this technique is independent of pairing's types. Therefore, we can prove the security of the instantiation using asymmetric pairings by following the same proof strategy used for the scheme with type-1 pairings. For type-2 pairings, the security can be reduced to the co-DHP: given $\phi : \mathbb{G}_2 \to \mathbb{G}_1$ and $g', g^b \in \mathbb{G}_1, g, g^a \in \mathbb{G}_2$ such that $g' = \phi(g)$, compute g'^{ab}. For type-3 pairings, the security of the proposed DH-based signature scheme can be reduced to the co-DHP*: given $g', g'^a, g'^b \in \mathbb{G}_1, g, g^a \in \mathbb{G}_2$, compute g'^{ab}. Note that the security of the Waters signature scheme using asymmetric pairings is also based on the same problems [1, 11].

Security Parameter λ		$q = 2^{30}$				$q = 2^{40}$			
		m	k	PK size	Sig. size	m	k	PK size	Sig. size
80	w/o tag comp.	7	2	$12\tau_{\mathbb{G}_2} + \tau_{\mathbb{G}_1} + 2\tau_{\mathbb{G}_{ch}}$	$2\tau_{\mathbb{G}_1} + 2\tau_{\mathbb{Z}_p}$	8	2	$13\tau_{\mathbb{G}_2} + \tau_{\mathbb{G}_1} + 2\tau_{\mathbb{G}_{ch}}$	$2\tau_{\mathbb{G}_1} + 2\tau_{\mathbb{Z}_p}$
	w/ tag comp.	7	2	$12\tau_{\mathbb{G}_2} + \tau_{\mathbb{G}_1} + 2\tau_{\mathbb{G}_{ch}} + \lvert K\rvert$	$2\tau_{\mathbb{G}_1} + \tau_{\mathbb{Z}_p}$	8	2	$13\tau_{\mathbb{G}_2} + \tau_{\mathbb{G}_1} + 2\tau_{\mathbb{G}_{ch}} + \lvert K\rvert$	$2\tau_{\mathbb{G}_1} + \tau_{\mathbb{Z}_p}$
128	w/o tag comp.	7	2	$12\tau_{\mathbb{G}_2} + \tau_{\mathbb{G}_1} + 2\tau_{\mathbb{G}_{ch}}$	$2\tau_{\mathbb{G}_1} + 2\tau_{\mathbb{Z}_p}$	8	2	$13\tau_{\mathbb{G}_2} + \tau_{\mathbb{G}_1} + 2\tau_{\mathbb{G}_{ch}}$	$2\tau_{\mathbb{G}_1} + 2\tau_{\mathbb{Z}_p}$
	w/ tag comp.	7	2	$12\tau_{\mathbb{G}_2} + \tau_{\mathbb{G}_1} + 2\tau_{\mathbb{G}_{ch}} + \lvert K\rvert$	$2\tau_{\mathbb{G}_1} + \tau_{\mathbb{Z}_p}$	8	2	$13\tau_{\mathbb{G}_2} + \tau_{\mathbb{G}_1} + 2\tau_{\mathbb{G}_{ch}} + \lvert K\rvert$	$2\tau_{\mathbb{G}_1} + \tau_{\mathbb{Z}_p}$
192	w/o tag comp.	7	2	$12\tau_{\mathbb{G}_2} + \tau_{\mathbb{G}_1} + 2\tau_{\mathbb{G}_{ch}}$	$2\tau_{\mathbb{G}_1} + 2\tau_{\mathbb{Z}_p}$	8	2	$13\tau_{\mathbb{G}_2} + \tau_{\mathbb{G}_1} + 2\tau_{\mathbb{G}_{ch}}$	$2\tau_{\mathbb{G}_1} + 2\tau_{\mathbb{Z}_p}$
	w/ tag comp.	7	2	$12\tau_{\mathbb{G}_2} + \tau_{\mathbb{G}_1} + 2\tau_{\mathbb{G}_{ch}} + \lvert K\rvert$	$2\tau_{\mathbb{G}_1} + \tau_{\mathbb{Z}_p}$	8	2	$13\tau_{\mathbb{G}_2} + \tau_{\mathbb{G}_1} + 2\tau_{\mathbb{G}_{ch}} + \lvert K\rvert$	$2\tau_{\mathbb{G}_1} + \tau_{\mathbb{Z}_p}$
256	w/o tag comp.	7	3	$13\tau_{\mathbb{G}_2} + \tau_{\mathbb{G}_1} + 2\tau_{\mathbb{G}_{ch}}$	$2\tau_{\mathbb{G}_1} + 2\tau_{\mathbb{Z}_p}$	8	2	$13\tau_{\mathbb{G}_2} + \tau_{\mathbb{G}_1} + 2\tau_{\mathbb{G}_{ch}}$	$2\tau_{\mathbb{G}_1} + 2\tau_{\mathbb{Z}_p}$
	w/ tag comp.	7	3	$13\tau_{\mathbb{G}_2} + \tau_{\mathbb{G}_1} + 2\tau_{\mathbb{G}_{ch}} + \lvert K\rvert$	$2\tau_{\mathbb{G}_1} + \tau_{\mathbb{Z}_p}$	8	2	$13\tau_{\mathbb{G}_2} + \tau_{\mathbb{G}_1} + 2\tau_{\mathbb{G}_{ch}} + \lvert K\rvert$	$2\tau_{\mathbb{G}_1} + \tau_{\mathbb{Z}_p}$

Table 1. Practical parameters for EU-q-CMA secure signature scheme: m and k and are parameters used in the description of the main construction for signatures scheme, and q is the maximum of the number of signatures that an adversary can get. In this table, the security reduction loss is at most $96\lambda q$, which is comparable to that of Waters signature scheme in [38]. 'w/o tag comp.' means the signature scheme without applying tag compression technique in the figure 3 and 'w/ tag comp.' means the signature scheme with tag compression technique in the figure 4 and figure 5. '$\tau_{\mathbb{G}_1}$' and '$\tau_{\mathbb{G}_2}$' are the bit-lengths to represent elements in $\mathbb{G}_1$ and $\mathbb{G}_2$, respectively. For type-1 pairings, $\tau_{\mathbb{G}_1} = \tau_{\mathbb{G}_2}$. '$\tau_{\mathbb{Z}_p}$' is the size of prime p that is order of cyclic groups $\mathbb{G}_1$, $\mathbb{G}_2$, and $\mathbb{G}_t$. '$\tau_{\mathbb{G}_{ch}}$' is the bit length to represent an element in the group $\mathbb{G}_{ch}$ (of order $p' \leq p$), over which the chameleon hashes defined. '$|K|$' is a size of a PRF key.

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