How to Launch A Birthday Attack Against DES

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Abstract We present a birthday attack against DES. It is entirely based on the relationship $L_{i+1} = R_i$ and the simple key schedule in DES. It requires about 2^{16} ciphertexts of the same R_{16}, encrypted by the same key K. We conjecture it has a computational complexity of 2^{48}. Since the requirement for the birthday attack is more accessible than that for Differential cryptanalysis, Linear cryptanalysis or Davies' attack, it is of more practical significance.
Keywords. DES, Differential cryptanalysis, Linear cryptanalysis, Birthday attack.

1 Introduction

The Data Encryption Standard (DES) is a cipher selected as an official Federal Information Processing Standard for the United States in 1976 and which has subsequently enjoyed widespread use internationally [1, 3]. DES consequently came under intense academic scrutiny which motivated the modern understanding of block ciphers and their cryptanalysis. There are some analytical results which demonstrate theoretical weaknesses in the cipher, although they are infeasible to mount in practice. In recent years, the cipher has been superseded by the Advanced Encryption Standard (AES).

Although more information has been published on the cryptanalysis of DES than any other block cipher, the most practical attack to date is still a brute force approach. Various minor cryptanalytic properties are known, and three theoretical attacks are possible which, while having a theoretical complexity less than a brute force attack, require an unrealistic amount of known or chosen plaintext to carry out [4-16].

- Differential cryptanalysis was published in the late 1980s by E.Biham and A.Shamir. To break the full 16 rounds, differential cryptanalysis requires 2^{47} chosen plaintexts.
- Linear cryptanalysis was discovered by M.Matsui, and needs 2^{43} known plaintexts. Multiple linear cryptanalysis was suggested in 1994 (Kaliski and Robshaw), and was further refined by Biryukov et al (2004). It needs 2^{41} known plaintexts.

- Davies' attack is a specialised technique for DES, first suggested by D.Davies in the eighties, and improved by Biham and Biryukov (1997). The most powerful form of the attack requires 2^{50} known plaintexts, has a computational complexity of 2^{50}, and has a 51% success rate.

A birthday attack is a type of cryptographic attack [2]. Specifically, given a function f, the goal of the attack is to find two inputs x_1, x_2 such that $f(x_1) = f(x_2)$. Such a pair x_1, x_2 is called a collision. The method used to find a collision is to simply evaluate the function f for different input values that may be chosen randomly or pseudorandomly until the same result is found more than once. Because of the birthday paradox, this method can be rather efficient. Specifically, if a function $f(x)$ yields any of H different outputs with equal probability and H is *sufficiently large*, then we expect to obtain a pair of different arguments x_1 and x_2 with $f(x_1) = f(x_2)$ after evaluating the function for about $1.25\sqrt{H}$ different arguments on average.

Up to the present, nobody shows that how to apply a birthday attack to DES. In this paper, we present such an attack against DES. The attack is entirely based on the simple key schedule and the relationship $L_{i+1} = R_i$ in DES. It requires about 2^{16} ciphertexts of the same R_{16}, encrypted by the same key K. We conjecture it has a computational complexity of 2^{48}. Since the requirement for the birthday attack, namely enough special ciphertexts, is more accessible than that for Differential cryptanalysis, Linear cryptanalysis or Davies' attack, it is of more practical significance.

2 Description of DES

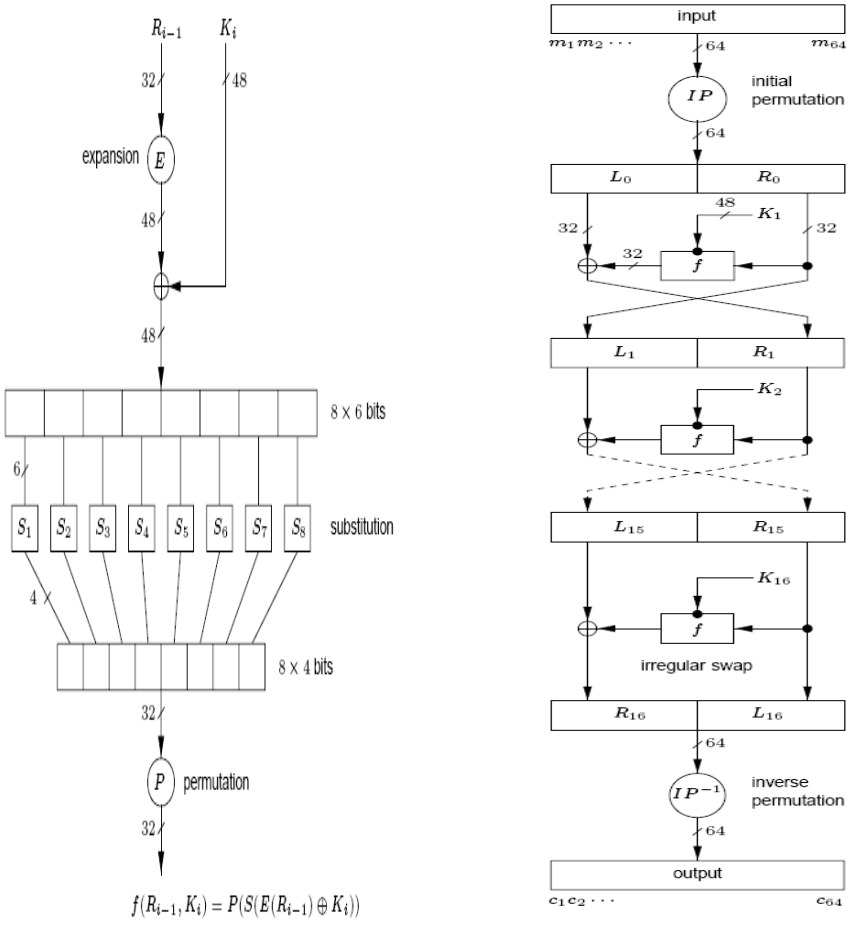
In this section we give a description of DES and some figures for the inner function f, computation path, S-box and key schedule.

DES processes plaintext blocks of $n = 64$ bits, producing 64-bit ciphertext blocks. The effective size of the secret key is $K = 56$ bits; more precisely, the input key K is specified as a 64-bit key, 8 bits of which (bits $8, 16, \cdots, 64$) may be used as parity bits. The 2^{56} keys implement (at most) 2^{56} of the $2^{64}!$ possible bijections on 64-bit blocks. Function f operates on two blocks of data: R_{n-1} and K_n. It produces 32-bit long block of data. Process of calculating f function consists of 4 steps:

1. E permutation
2. XOR with a subkey
3. S box transformation
4. P permutation

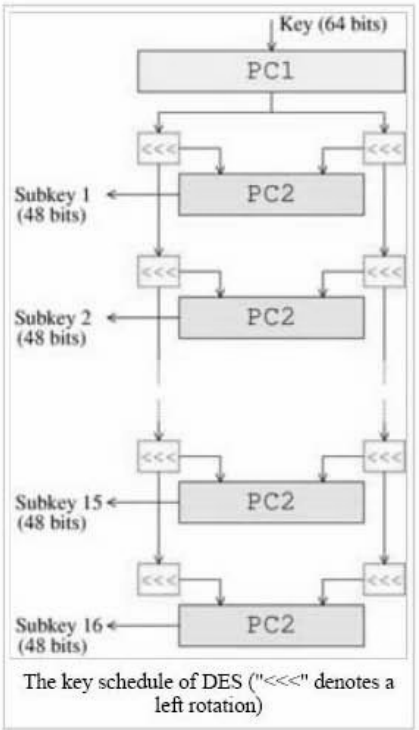
Each of the eight S-boxes replaces its six input bits with four output bits according to a non-linear transformation, provided in the form of a lookup table. The idea of transformation is straightforward: the first and the last bit of the first group of six bits form a binary number in the decimal range 0 to 3. This is the number of a row in the S1 table. The middle four bits of the group of six bits form a binary number in the decimal range 0 to 15. This is the number of a column in the S1 table. Those two coordinates indicates a decimal number, which as a 4-bit long binary number is the output. We repeat this operation for each of eight groups of six bits and as a result we obtain eight groups of 4 bits. The S-boxes provide the core of the security of DES. Without them, the cipher would be linear and trivially breakable.

DES inner function *f* and computation path



S[1]-box	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	E	4	D	1	2	F	B	8	3	A	6	C	5	9	0	7
1	0	F	7	4	E	2	D	1	A	6	C	B	9	5	3	8
2	4	1	E	8	D	6	2	B	F	C	9	7	3	A	5	0
3	F	C	8	2	4	9	1	7	5	B	3	E	A	0	6	D
⋮								...								
S[8]-box	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	D	2	8	4	6	F	B	1	A	9	3	14	5	0	C	7
1	1	F	D	8	A	3	7	4	C	5	6	B	0	E	9	2
2	7	B	4	1	9	C	E	2	0	6	A	D	F	3	5	8
3	2	1	E	7	4	A	8	D	F	C	9	0	3	5	6	B

S-box for DES



The key schedule of DES ("<<<" denotes a left rotation)

3 Basic idea

We first point out that it's easy to compute the R_{16} and L_{16} for a known ciphertext c (of 64-bit length). Refer to the above figure, which shows the inner function f and computation path of DES.

By the last round in DES, we have

$$R_{16} = L_{15} \oplus f(R_{15}, K_{16}), \quad L_{16} = R_{15}$$

Hence

$$f(L_{16}, K_{16}) = R_{16} \oplus L_{15}$$

Note that both L_{15} and K_{16} are **not** accessible.

Collision-assumption Suppose that there is a pair of ciphertexts (c, c') generated by the same key K_{16} and satisfying

$$R'_{16} = R_{16},\ L'_{16} \neq L_{16},\ L'_{15} = L_{15}$$

By the collision-assumption, we have

$$f(L'_{16}, K_{16}) = f(L_{16}, K_{16}) \tag{1}$$

Denote $E(L_{16})$ by EL_{16} where E is the expansion transformation in function f. Express EL_{16}, K_{16} as

$$\begin{aligned} EL_{16} &= EL_{16}[1] \,||\, EL_{16}[2] \,||\, EL_{16}[3] \,||\, EL_{16}[4] \,||\, EL_{16}[5] \,||\, EL_{16}[6] \,||\, EL_{16}[7] \,||\, EL_{16}[8] \\ K_{16} &= K_{16}[1] \,||\, K_{16}[2] \,||\, K_{16}[3] \,||\, K_{16}[4] \,||\, K_{16}[5] \,||\, K_{16}[6] \,||\, K_{16}[7] \,||\, K_{16}[8] \end{aligned}$$

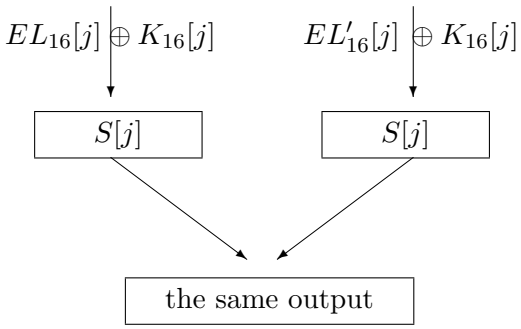
where each $EL_{16}[j], K_{16}[j], j = 1, \cdots, 8$, is of length 6-bit, $\alpha||\beta$ denotes the concatenation of the two strings α, β. Thus for each S-box $S[j], j = 1, \cdots, 8$, the input of $S[j]$ is

$$EL_{16}[j] \oplus K_{16}[j]$$

By the structure of f and Eq.(1), we have

$$S[j](EL_{16}[j] \oplus K_{16}[j]) = S[j](EL'_{16}[j] \oplus K_{16}[j]) \tag{2}$$

This can be depicted as follows.



Collision for S[j]-box

Claim-1 There are about 2^2 possible values for $K_{16}[j]$ satisfying Eq.(2) if $EL_{16}[j] \neq EL'_{16}[j]$, and 2^6 values for $K_{16}[j]$ if $EL_{16}[j] = EL'_{16}[j]$.

In fact, the pair $(EL_{16}[j] \oplus K_{16}[j], EL'_{16}[j] \oplus K_{16}[j])$ is just a collision of the nonlinear function $S[j]$. Roughly speaking, $S[j]$ can be viewed as a random or pseudorandom function. To find a collision of it for the given $EL_{16}[j], EL'_{16}[j]$, about 2^4 different arguments should be evaluated. Thus, there are 2^2 possible values for $K_{16}[j]$ if $EL_{16}[j] \neq EL'_{16}[j]$. Yet, all 6-bit values for $K_{16}[j]$ do satisfy Eq.(2) if $EL_{16}[j] = EL'_{16}[j]$.

For each box $S[j], j = 1, \cdots, 8$, integrate each string a of 6-bit with $EL_{16}[j], EL'_{16}[j]$. Check Eq.(2) to determine all candidates for $K_{16}[j]$. Thus the corresponding candidates for K_{16} are achieved.

Remark 1 This claim depends on the randomness of each S-box. We here give a rough estimate. It can be further refined by experiments on each S-box.

4 Description of the birthday attack against DES

Now we give a full description of the birthday attack against DES.

1 *Collecting proper ciphertexts.* Choose ciphertexts (64-bit) generated by the same key K. For each ciphertext c, compute its corresponding L_{16}, R_{16}. Collect the ciphertexts with the same R_{16} and denote the set by $\mathbb{C}_{R_{16},K}$. Denote $E(L_{16})$ by EL_{16}, where E is the expansion transformation in function f. Express EL_{16} as

$$EL_{16} = EL_{16}[1] \,||\, EL_{16}[2] \,||\, EL_{16}[3] \,||\, EL_{16}[4] \,||\, EL_{16}[5] \,||\, EL_{16}[6] \,||\, EL_{16}[7] \,||\, EL_{16}[8]$$

2 *Computing the candidates for each* $K_{16}[j], j = 1, \cdots, 8$. Randomly pick two ciphertexts $c, c' \in \mathbb{C}_{R_{16},K}$. Integrate each string a of 6-bit with $EL_{16}[j], EL'_{16}[j]$. Determine the candidates for $K_{16}[j]$ by checking

$$S[j](EL_{16}[j] \oplus a) \stackrel{?}{=} S[j](EL'_{16}[j] \oplus a)$$

3 *Local checking.* If there does not exist any candidate for some $K_{16}[i], i \in \{1, \cdots, 8\}$, then goto step 2.

4 *Determining the candidates for* K_{16}. Derive the candidates for K_{16} from the candidates for $K_{16}[1], \cdots, K_{16}[8]$.

5 *Determining the candidates for* K. Derive the candidates for K from K_{16} by the key schedule of DES.

6 *Distinguishing* K *from the candidates.* Given a plaintext and its corresponding ciphertext, the key (or its equivalent) can be distinguished from its candidates by evaluations.

7 *Outputting* K. If the key cannot be derived from the pair (c, c'), goto step 2. Otherwise, output the key.

Remark 2 In the above attack, we aim at finding a collision (L_{15}, L'_{15}), which is achieved by evaluating possible values for $K_{16}[j], j = 1, \cdots, 8$. This is the reason for calling it a *birthday attack*.

5 Complexity

5.1 On the complexity of evaluations

Clearly, the most complexity of evaluations results from deriving the candidates for $K_{16}[j], j = 1, \cdots, 8$. To derive the candidates for $K_{16}[j], j = 1, \cdots, 8$, we should evaluate all 6-bit values, which are integrated with $EL_{16}[j], EL'_{16}[j]$ separately. But all 8×2^6 evaluations can be run in parallel and be separately restricted in eight boxes. In this case, the time for one evaluation is less than that for an evaluation using one round in DES. Thus the complexity of evaluations is very small.

5.2 On the amount of rounds

The birthday attack against DES does not relate to the amount of rounds. It is entirely based on the inner function f and the key schedule in DES. This is a peculiar property of the birthday attack.

5.3 On the amount of ciphertexts

By $L_{15} = R_{16} \oplus f(L_{16}, K_{16})$ and the definition of $\mathbb{C}_{R_{16},K}$, we define

$$\mathcal{P}_{R_{16},K_{16}} : L_{16} \mapsto L_{15}$$

It's reasonable to assume that $\mathcal{P}_{R_{16},K_{16}}$ is random or pseudorandom. To find a collision for it, i.e.,

$$\mathcal{P}_{R_{16},K_{16}}(L_{16}) = L_{15} = L'_{15} = \mathcal{P}_{R_{16},K_{16}}(L'_{16})$$

about 2^{16} arguments should be evaluated. Practically speaking, it is not difficult to construct such a set $\mathbb{C}_{R_{16},K}$ satisfying $D \geq 2^{16}$ where D is the cardinal number of $\mathbb{C}_{R_{16},K}$, because each ciphertext is of only 64-bit.

5.4 On the amount of candidates for K in each iteration

Define the *block-distance* between $c, c' \in \mathbb{C}_{R_{16},K}$ as

$$d = \#\{\, \lambda \,:\, EL_{16}[\lambda] \neq EL'_{16}[\lambda] \,\}$$

By the claim-1 and the definition of block-distance, we know the amount of candidates for K_{16} mainly depends on the block-distance of the pair (EL_{16}, EL'_{16}). In the best case, i.e., the block-distance is the maximum, 8, the amount of candidates for K_{16} is about 2^{16}. In the worst case, i.e., the block-distance is 1, the amount is 2^{44}.

Obviously, we are concerned about the average amount of candidates for K in each iteration. On average, a K_{16} leads to $\frac{7}{6}$ candidates for K. We conjecture the amount of candidates for K in each iteration is 2^{18}. We refer to the previous figure of the key schedule in DES.

5.5 On the amount of iterations

In the worst case, the amount of iterations is $\frac{D(D-1)}{2}$, namely we should try all ciphertext pairs of $\mathbb{C}_{R_{16},K}$. We conjecture the average amount of iterations is 2^{30}. Hence, the birthday attack should evaluate 2^{48} candidates for K. Thus, the attack has a computational complexity of 2^{48}.

5.6 On the amount of plaintexts

In the proposed attack, we need a plaintext and the corresponding ciphertext to distinguish the key (or its equivalents) from its candidates. Note that the resulting amount of the key or its equivalents will be sharply decreased as the increase of plaintexts.

6 Conclusion

At present, we do not have any experiment to test the attack because of a lack of resources. But we believe the simple derivation of candidates for K from K_{16} and the relationship $L_{i+1} = R_i$ can be a serious problem in DES. Possibly, it is due to historical considerations instead of a contrived process.

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