# Weakness of Shim's New ID-based Tripartite Multiple-key Agreement Protocol 

Jue-Sam Chou*, Chu-Hsing Lin** and Chia-Hung Chiu**<br>jschou@mail.nhu.edu.tw, chlin@thu.edu.tw, hdilwy@islab.csie.thu.edu.tw<br>*Department of Information management, Nanhua university<br>Chaiyi Taiwan<br>**Department of Computer Science and Information Engineering, Tunghai University<br>Taichung Taiwan


#### Abstract

In this article we show that Shim's new ID-based tripartite multiple-key agreement protocol still suffers from the impersonation attack, a malicious user can launch an impersonation attack on their protocol.


Keyword: ID-based, Weil-paring, impersonation attack, tripartite authenticated key agreement, unknown key share attack.

## 1. Introduction

The first one-round tripartite Diffiee-Hellman key agreement protocol [1] was proposed by Joux in 2000. However, Joux's protocol does not authenticate the three communicating entities, and is vulnerable to the man-in-the-middle attack. Recently Liu et al. proposed an ID-based one round authenticated tripartite key agreement protocol with pairing[2,4-12] (LZC protocol) which results in eight session keys in the agreement. However, their scheme could not prevent the "unknown key share" attack proposed by Shim et al. in 2005[3]. In [3], they suggest a method to resist the unknown key share attack. This article will show that their protocol is still vulnerable to the impersonation attack.

## 2. The Background

In this section, we will first briefly review the basic concept and some properties of bilinear pairing
then review the Shim's protocol.

### 2.1. Bilinear pairing

Let $\mathbb{G}_{1}$ be a cyclic group generated by $P$, whose order is a prime $q$ and $\mathbb{G}_{2}$ be a cyclic multiplicative group of the same order $q$. We assume that the discrete logarithm problem (DLP) in both $\mathbb{G}_{1}$ and $\mathbb{G}_{1}$ are hard. Let $e: \mathbb{G}_{1} \times \mathbb{G}_{1} \rightarrow \mathbb{G}_{2}$ be a pairing which satisfies the following conditions:

1. Bilinear: $e(a P, b Q)=e(P, Q)^{a b}$, for any $a, b \in \mathbb{Z}$ and $P, Q \in \mathbb{G}_{1}$.
2. Non-degenerate: there exists $P \in \mathbb{G}_{1}$ and $Q \in \mathbb{G}_{1}$ such that $e(P, Q) \neq 1$.
3. Computability: there is an efficient algorithm to compute $e(P, Q)$ for all $P, Q \in \mathbb{G}_{1}$

### 2.2. Shim's protocol

- Setup: Key generation center (KGC) chooses a random $s \in \mathbb{Z}_{q}^{*}$ and set $P_{p u b}=s P$. The KGC publishes the system parameters $\left\langle\mathbb{G}_{1}, \mathbb{G}_{2}, q, e, P, P_{p u b}, H, H_{1}\right\rangle$ and keep $s$ as a secret master key, which is known only by itself.
- Private key extraction: A user submits his identity information ID to KGC. KGC computes the user's public key as $Q_{I D}=H_{1}(I D)$ and returns $S_{I D}=s Q_{I D}$ to the user as his private key.

Assume that there are three entities $A, B, C$. Each chooses two random numbers then computers their corresponding parameters. For examples, $A$ chooses random numbers $a$ and $a^{\prime}$, and computes $P_{A}=a P, P_{A}^{\prime}=a^{\prime} P, T_{A}=S_{A}+a^{2} P+a^{\prime} P_{p u b} . B$ chooses random numbers $b$ and $b^{\prime}$, and computes $P_{B}=b P, P_{B}^{\prime}=b^{\prime} P, T_{B}=S_{B}+b^{2} P+b^{\prime} P_{p u b} . C$ chooses random numbers $C$ and $c^{\prime}$, and computes $P_{C}=c P, P_{B}^{\prime}=c^{\prime} P, T_{C}=S_{C}+c^{2} P+c^{\prime} P_{p u b}$. After the computing, they broadcast their values $\left(P_{A}, P_{A}^{\prime}, T_{A}\right),\left(P_{B}, P_{B}^{\prime}, T_{B}\right)$ and $\left(P_{C}, P_{C}^{\prime}, T_{C}\right)$ to all the other parties.

When receiving the other party's communicational parameters, each party performs his/her own verifying equation. For example, $A$ checks whether the following equation holds.

$$
\begin{aligned}
e\left(T_{B}+T_{C}, P\right) & =e\left(S_{B}+b^{2} P+b^{\prime} P_{p u b}+S_{C}+c^{2} P+c^{\prime} P_{p u b}, P\right) \\
& =e\left(s P_{B}+b^{\prime} s P+s P_{C}+c^{\prime} s P, P\right) e\left(b^{2}, P\right) e\left(c^{2}, P\right) \\
& \stackrel{?}{=}\left(Q_{B}+Q_{C}+P_{B}^{\prime}+P_{C}^{\prime}, P_{p u b}\right) e\left(P_{B}, P_{B}\right) e\left(P_{C}, P_{C}\right)
\end{aligned}
$$

$B$ and $C$ also do their corresponding verification to check if the equations hold.
If each equation holds, then $A, B$ and $C$ compute the eight session keys respectively, as in the LZC protocol, as follows.

$$
\text { A computes: } \begin{aligned}
& K_{A}^{(1)}=e\left(P_{B}, P_{C}\right)^{a}, K_{A}^{(2)}=e\left(P_{B}, P_{C}^{\prime}\right)^{a}, K_{A}^{(3)}=e\left(P_{B}^{\prime}, P_{C}\right)^{a}, K_{A}^{(4)}=e\left(P_{B}^{\prime}, P_{C}^{\prime}\right)^{a} \\
& \\
& K_{A}^{(5)}=e\left(P_{B}, P_{C}\right)^{a^{\prime}}, K_{A}^{(6)}=e\left(P_{B}, P_{C}^{\prime}\right)^{a^{\prime}}, K_{A}^{(7)}=e\left(P_{B}^{\prime}, P_{C}\right)^{a^{\prime}}, K_{A}^{(8)}=e\left(P_{B}^{\prime}, P_{C}^{\prime}\right)^{a^{\prime}}
\end{aligned}
$$

$B$ computes:

$$
K_{B}^{(1)}=e\left(P_{A}, P_{C}\right)^{b}, K_{B}^{(2)}=e\left(P_{A}, P_{C}^{\prime}\right)^{b}, K_{B}^{(3)}=e\left(P_{A}, P_{C}\right)^{b^{\prime}}, K_{B}^{(4)}=e\left(P_{A}, P_{C}^{\prime}\right)^{b^{\prime}}
$$

$$
\begin{aligned}
& K_{B}^{(5)}=e\left(P_{A}^{\prime}, P_{C}\right)^{b}, K_{B}^{(6)}=e\left(P_{A}^{\prime}, P_{C}^{\prime}\right)^{b}, K_{B}^{(7)}=e\left(P_{A}^{\prime}, P_{C}\right)^{b^{\prime}}, K_{B}^{(8)}=e\left(P_{A}^{\prime}, P_{C}^{\prime}\right)^{b^{\prime}} \\
& K_{C}^{(1)}=e\left(P_{A}, P_{B}\right)^{c}, K_{C}^{(2)}=e\left(P_{A}, P_{B}\right)^{c^{\prime}}, K_{C}^{(3)}=e\left(P_{A}, P_{B}^{\prime}\right)^{c}, K_{C}^{(4)}=e\left(P_{A}, P_{B}^{\prime}\right)^{c^{\prime}}
\end{aligned}
$$

C computers:

$$
K_{C}^{(5)}=e\left(P_{A}^{\prime}, P_{B}\right)^{c}, K_{C}^{(6)}=e\left(P_{A}^{\prime}, P_{B}\right)^{c^{\prime}}, K_{C}^{(7)}=e\left(P_{A}^{\prime}, P_{B}^{\prime}\right)^{c}, K_{C}^{(8)}=e\left(P_{A}^{\prime}, P_{B}^{\prime}\right)^{c^{\prime}}
$$

We can find that $K_{A}^{(1)}=K_{B}^{(1)}=K_{C}^{(1)}=e(P, P)^{a b c}=K^{(1)}$. Similarly, we also have $K_{A}^{(i)}=K_{B}^{(i)}=$ $K_{C}^{(i)}=K^{(i)}$, for $i=2,3, \ldots, 8$. Each entity then takes the eight computed values $K^{(i)}(i=1,2, \ldots, 8)$ as the final session keys, where

$$
\begin{aligned}
& K^{(1)}=e(P, P)^{a b c}, K^{(2)}=e(P, P)^{a b c^{\prime}}, K^{(3)}=e(P, P)^{a b^{\prime} c}, K^{(4)}=e(P, P)^{a b^{\prime} c^{\prime}} \\
& K^{(5)}=e(P, P)^{a^{\prime} b c}, K^{(6)}=e(P, P)^{a^{\prime} b c^{\prime}}, K^{(7)}=e(P, P)^{a^{b^{\prime} b^{\prime} c}}, K^{(8)}=e(P, P)^{a^{\prime} b^{\prime} c^{\prime}}
\end{aligned}
$$

## 3. Our Attack

In this section, we show that how the Shim's protocol is insecure against the impersonation attack. Assume that there is an adversary $X$, who wants to impersonate $B$ to communicate with $A$ and $C$. He will first compute $P_{X}=x P, P_{X}^{\prime}=x^{\prime} P-Q_{B}, T_{X}=x^{\prime} P_{p u b}+x^{2} P$ and broadcast them to $A$ and $C$. After receiving the broadcast parameters sent by $X$ and $C$, A can pass his/her verification step as follows.

$$
\begin{aligned}
e\left(T_{X}+T_{C}, P\right) & =e\left(x^{\prime} P_{p u b}+x^{2} P+S_{C}+c^{2} P+c^{\prime} P_{p u b}, P\right) \\
& =e\left(x^{\prime} P+Q_{C}+c^{\prime} P, P_{p u b}\right) e\left(x^{2} P+c^{2} P, P\right) \\
& =e\left(x^{\prime} P-Q_{B}+Q_{B}+Q_{C}+c^{\prime} P, P_{p u b}\right) e(x P, x P) e(c P, c P) \\
& =e\left(P_{X}^{\prime}+Q_{B}+Q_{C}+c^{\prime} P, P_{p u b}\right) e(x P, x P) e(c P, c P) \\
& =e\left(Q_{B}+Q_{C}+P_{X}^{\prime}+P_{C}^{\prime}, P_{p u b}\right) e\left(P_{X}, P_{X}\right) e\left(P_{C}, P_{C}\right)
\end{aligned}
$$

$C$ can obtain his parameters sent from other parties and also pass his/her verification by the equation $e\left(T_{A}+T_{X}, P\right)=e\left(Q_{A}+Q_{B}+P_{X}^{\prime}+P_{A}^{\prime}\right) e\left(P_{A}, P_{A}\right) e\left(P_{X}, P_{X}\right)$.
After that, $A$ can compute the session keys as follows.

$$
\begin{aligned}
& K_{A}^{(1)}=e\left(P_{X}, P_{C}\right)^{a}, K_{A}^{(2)}=e\left(P_{X}, P_{C}^{\prime}\right)^{a}, K_{A}^{(3)}=e\left(P_{X}^{\prime}, P_{C}\right)^{a}, K_{A}^{(4)}=e\left(P_{X}^{\prime}, P_{C}^{\prime}\right)^{a} \\
& K_{A}^{(5)}=e\left(P_{X}, P_{C}\right)^{a^{\prime}}, K_{A}^{(6)}=e\left(P_{X}, P_{C}^{\prime}\right)^{a^{\prime}}, K_{A}^{(7)}=e\left(P_{X}^{\prime}, P_{C}\right)^{a^{\prime}}, K_{A}^{(8)}=e\left(P_{X}^{\prime}, P_{C}^{\prime}\right)^{a^{\prime}}
\end{aligned}
$$

And $C$ can compute the session keys as follows:

$$
\begin{aligned}
& K_{C}^{(1)}=e\left(P_{A}, P_{X}\right)^{c}, K_{C}^{(2)}=e\left(P_{A}, P_{X}\right)^{c^{\prime}}, K_{C}^{(3)}=e\left(P_{A}, P_{X}^{\prime}\right)^{c}, K_{C}^{(4)}=e\left(P_{A}, P_{X}^{\prime}\right)^{c^{\prime}} \\
& K_{C}^{(5)}=e\left(P_{A}^{\prime}, P_{X}\right)^{c}, K_{C}^{(6)}=e\left(P_{A}^{\prime}, P_{X}\right)^{c^{\prime}}, K_{C}^{(7)}=e\left(P_{A}^{\prime}, P_{X}^{\prime}\right)^{c}, K_{C}^{(8)}=e\left(P_{A}^{\prime}, P_{X}^{\prime}\right)^{c^{\prime}}
\end{aligned}
$$

Each entity, $A$ and $C$, then takes the following eight computed values $K^{(i)}=(i=1, \ldots, 8)$ as their final session keys

$$
\begin{aligned}
& K^{(1)}=e(P, P)^{a c c}, K^{(2)}=e(P, P)^{a \alpha^{\prime}}, K^{(3)}=e(P, P)^{\alpha \alpha^{\prime} c} e\left(Q_{B}, P\right)^{-a c}, K^{(4)}=e(P, P)^{\alpha \alpha^{\prime} c} e\left(Q_{B}, P\right)^{-a c^{\prime}} \\
& K^{(5)}=e(P, P)^{\alpha x c}, K^{(6)}=e(P, P)^{\alpha x^{\prime}}, K^{(7)}=e(P, P)^{\alpha \alpha c} e\left(Q_{B}, P\right)^{-a c}, K^{(8)}=e(P, P)^{\alpha \alpha c^{\prime}} e\left(Q_{B}, P\right)^{-a^{\prime} c^{\prime}}
\end{aligned}
$$

The adversary $X$ can also get the same session keys $K^{(1)}, K^{(2)}, K^{(5)}$ and $K^{(6)}$ as $A$ and $C$ by computing:

$$
\begin{aligned}
& K_{X}^{(1)}=e\left(P_{A}, P_{C}\right)^{x}=e(P, P)^{a x c} \equiv K^{(1)}, K_{X}^{(2)}=e\left(P_{A}, P_{C}^{\prime}\right)^{x}=e(P, P)^{a x c} \equiv K^{(2)} \\
& K_{X}^{(5)}=e\left(P_{A}^{\prime}, P_{C}\right)^{x}=e(P, P)^{a^{\prime} x c} \equiv K^{(5)}, K_{X}^{(6)}=e\left(P_{A}^{\prime}, P_{C}^{\prime}\right)^{x}=e(P, P)^{a^{\prime} x c^{\prime}} \equiv K^{(6)}
\end{aligned}
$$

As a result, $X$ can share these four keys $K^{(1)}, K^{(2)}, K^{(5)}, K^{(6)}$ in the eight session keys. Under this situation, $A$ and $C$ think these four session keys are shared with $B$, but indeed, they are shared with $X$. Besides, both $A$ and $C$ come to share the same eight session keys. Thus, the impersonation attack on four of the eight session keys can be successfully mounted. More precisely, the attacker $X$ can use these four session keys to communicate with $A$ and $C$, and he can have one half of the probability to realize what the communication contents are between $A$ and $C$.

## 4. Conclusion

In this article, we show that Shim et al.'s new ID-based tripartite multiple-key agreement protocol in [3] can not resist an impersonation attack. How to design a secure and efficient ID-based authenticated tripartite multiple-key agreement scheme to prevent all kinds of attacks remains an open problem.

## Reference:

[1] A. Joux, "A one-round protocol for tripartite Diffie-Hellman", Proceedings of the $4^{\text {th }}$ International Algorithmic Number Theory Symposium (ANTS-IV), LNCS 1838, July, 2000, pp.385-394.
[2] S. Liu, F. Zhang, K. Chen, "ID-based tripartite key agreement protocol with pairing", 2003 IEEE International Symposium on Information Theory, 2003, pp. 136-143, or available at Cryptology ePrint Archive, Report 2002/122.
[3] Kyungah Shim and Sungsik Woo, "Weakness in ID-based one round authenticated tripartite multiple-key agreement protocol with pairings", Applied Mathematics and Computation, Volume: 166, Issue: 3, July 26, 2005, pp. 523-530.
[4] R. Barua, R.Dutta, P. Sarkar, "Extending Joux Protocol to Multi Party Key Agreement", Indocrypt 2003, Also available at http://eprint.iacr.org/2003/062.
[5] P. S. L. M. Berreto, H. Y. Kim and M.Scott, "Efficient algorithms for pairing-based cryptosystems", Advances in Cryptology - Crypto ‘2002, LNCS 2442, Springer-Verlag (2002), pp. 354-368.
[6] A. Boldyreva, "Efficient Threshold Signature, Multisignature and Blind Signature Schemes Based on the Gap-Diffie-Hell,am-Group Signature Scheme", PKC2003, LNCS 2139, Springer-Verlag, 2003, pp. 31-46.
[7] D. Boneh, X.Boyen, "Short Signature without Random Oracles", In Christian Cachin and Jan Camenisch, editors. Proceedings of Eurocrypt 2004, LNCS, Springer-Verlag, 2004.
[8] D. Boneh, M. Franklin, "Identity Based Encryption from the Weil Pairing", SIAM J. of Computing, Vol. 32, No. 3, pp. 586-615, 2003, Extended Abstract in Crypto 2001.
[9] D. Boneh, B. Lynn, H. Shacham, "Short signature from the Weil paring". In Proceedings of Asiacrypt 2001.
[10] D. Boneh, I. Mironov, V. Shoup, "A secure Signature Scheme from Bilinear Maps", CT-RSA-2003, pp. 98-110.
[11] D. Boneh, A. Silverberg, "Applications of Multilinear forms Cryptography", Report 2002/080, http://eprint.iacr.org, 2002.
[12] E. Fujisaki, T. Okamoto, "Secure Integration of Asymmetric and Symmetric Encryption Schemes, in Advances in Cryptology - Crypto'99, LNCS 1666, Springer-Verlag, 1999.

