

# Dynamic Group Key Agreement in Tree-Based Setting

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## Abstract

We present a provably secure tree based authenticated group key agreement protocol in dynamic scenario. Bilinear pairing and multi-signature are at the heart of our protocol. We prove that our protocol is provably secure in the standard security model of Bresson *et al.* An appropriate modification of Katz-Yung approach to tree based setting is adopted while proving its security against active adversaries. The protocol has an in-built hierarchical structure that makes it desirable for certain applications.

**Keywords:** bilinear pairing, multi-signature, group key agreement, dynamic operations, provable security <sup>1</sup>

## 1 Introduction

A group key agreement protocol allows a group of users to exchange information over public network to agree upon a common secret key from which a session key can be derived. This common session key can later be used to achieve desirable security goals, such as authentication, confidentiality and data integrity.

Tree based group key agreement protocols are typically essential while the users are grouped into a hierarchical structure. The leaves of the tree denote individual users and each internal node corresponds to a user that represents the set of users in the subtree rooted at that node. The representative users have more computational resources than other users in the subtree. In a tree based group key agreement protocol, the set of all users in each subtree agree upon a common secret key. Besides, making optimal use of precomputed values in the previous session, a group of users can save computation and communication in subsequent sessions in which users join or leave the group. Moreover, some subclass of users agree upon multiple common keys in a single session which facilitates a typical subclass of users of the group to securely communicate among themselves. These features make tree based key agreement protocols desirable for certain applications.

In this work, we present a provably secure tree based authenticated group key agreement in the dynamic scenario where a user can join or leave the group as his desire with updating sets of keys. Although designing constant round provably secure group key agreement schemes [13, 25, 26] get much attention in the current research, it does not eliminate the need of tree based key agreement. We can combine constant round protocols and tree based protocols to get hybrid group key agreement which are efficient in terms of both computation and communication. Consider the situation where there are collection of user sets, each having a common key agreed upon by executing an efficient constant round protocol among

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<sup>1</sup>This is the full version of the paper that will be presented at ACISP 2005.

the users in that user set. Now executing the constant round protocol among these user sets may not always be desirable and executing the protocol among all the users may be expensive from computation or communication point of view when number of users in each subgroup is large. For instance, if the number of groups is about 20 and each group is large, then a constant round key agreement using the protocol of [25] will involve a large number of computations as well as communications. In contrast, the tree based protocol (with the representatives of each group) will compute the common key in 3 rounds with lesser number of communications and verifications. Thus, a tree based scheme can be incorporated among these user sets to get an efficient multi-party key agreement protocol. There are quite a number of tree based key agreement protocols [6, 24]. A ternary tree based protocol was proposed by Barua *et al.* [5] that extends the basic Joux [21] protocol to multi-party setting. They have shown that the protocol is secure against passive adversaries. Dutta *et al.* [17] authenticate this unauthenticated protocol using multi-signature and provide a concrete security analysis against active adversaries in the standard model as formalized by Bresson *et al.* [14]. This security was achieved by modifying the Katz and Yung [22] technique to tree based setting. The present work further extends this static authenticated protocol [17] to dynamic authenticated protocol and provides a proof of security in the security model of Bresson *et al.* [14]. Our protocol is designed to ensure minimum modification to the computation already precomputed when a user leaves or joins the group. Besides, if the tree structure is not maintained after a join or leave operation, then the subsequent join or leave in the group can not be performed anymore. Thus retaining the tree structure is another important issue of our protocol while such dynamic operations (join or leave) are concerned.

## 2 Preliminaries

In this section, we put forward the required notations, definitions, models that we use in the discussions of the subsequent sections. We use the notation  $a \in_R S$  to denote that  $a$  is chosen randomly from the set  $S$ .

### 2.1 Cryptographic Bilinear Maps

Let  $G_1, G_2$  be two groups of the same prime order  $q$ . We view  $G_1$  as an additive group and  $G_2$  as a multiplicative group. A mapping  $e : G_1 \times G_1 \rightarrow G_2$  satisfying the following properties is called a cryptographic bilinear map:

- Bilinearity* :  $e(aP, bQ) = e(P, Q)^{ab}$  for all  $P, Q \in G_1$  and  $a, b \in Z_q^*$ .
- Non-degeneracy* : If  $P$  is a generator of  $G_1$ , then  $e(P, P)$  is a generator of  $G_2$ .
- Computability* : There exists an efficient algorithm to compute  $e(P, Q)$ .

Modified Weil Pairing [9] and Tate Pairing [4, 19] are examples of cryptographic bilinear maps.

### 2.2 Security Model

We assume that the reader is familiar with the model of Bresson *et al.* [14], which is the model in which we prove security of our dynamic key agreement protocol. For completeness, we review their definitions and refer the reader to [14] for more details.

Let  $\mathcal{P} = \{U_1, \dots, U_n\}$  be a set of  $n$  (fixed) users or participants. A user can execute the protocol for group key agreement several times with different partners, can join or leave the group at it's desire by executing the protocols for Insert or Delete. We assume that users do not deviate from the protocol and adversary never participates as a user in the protocol. This adversarial model allows concurrent execution of the protocol. The interaction between the adversary  $\mathcal{A}$  and the protocol participants occur only via

oracle queries, which model the adversary's capabilities in a real attack. These queries are as follows, where  $\Pi_U^i$  denotes the  $i$ -th instance of user  $U$  and  $\text{sk}_U^i$  denotes the session key after execution of the protocol by  $\Pi_U^i$ .

- $\text{Send}(U, i, m)$  : This query models an active attack, in which the adversary may intercept a message and then either modify it, create a new one or simply forward it to the intended participant. The output of the query is the reply (if any) generated by the instance  $\Pi_U^i$  upon receipt of message  $m$ . The adversary is allowed to prompt the unused instance  $\Pi_U^i$  to initiate the protocol with partners  $U_2, \dots, U_l, l \leq n$ , by invoking  $\text{Send}(U, i, \langle U_2, \dots, U_l \rangle)$ .
- $\text{Execute}(\{(V_1, i_1), \dots, (V_l, i_l)\})$  : Here  $\{V_1, \dots, V_l\}$  is a non empty subset of  $\mathcal{P}$ . This query models passive attacks in which the attacker eavesdrops on honest execution of group key agreement protocol among unused instances  $\Pi_{V_1}^{i_1}, \dots, \Pi_{V_l}^{i_l}$  and outputs the transcript of the execution. A transcript consists of the messages that were exchanged during the honest execution of the protocol.
- $\text{Join}(\{(V_1, i_1), \dots, (V_l, i_l)\}, (U, i))$  : This query models the insertion of a user instance  $\Pi_U^i$  in the group  $(V_1, \dots, V_l) \in \mathcal{P}$  for which  $\text{Execute}$  have already been queried. The output of this query is the transcript generated by the invocation of algorithm  $\text{Insert}$ . If  $\text{Execute}(\{(V_1, i_1), \dots, (V_l, i_l)\})$  has not taken place, then the adversary is given no output.
- $\text{Leave}(\{(V_1, i_1), \dots, (V_l, i_l)\}, (U, i))$  : This query models the removal of a user instance  $\Pi_U^i$  from the group  $(V_1, \dots, V_l) \in \mathcal{P}$ . If  $\text{Execute}(\{(V_1, i_1), \dots, (V_l, i_l)\})$  has not taken place, then the adversary is given no output. Otherwise, algorithm  $\text{Delete}$  is invoked. The adversary is given the transcript generated by the honest execution of procedure  $\text{Delete}$ .
- $\text{Reveal}(U, i)$  : This outputs session key  $\text{sk}_U^i$ . This query models the misuse of the session keys, *i.e.* known session key attack.
- $\text{Corrupt}(U)$  : This outputs the long-term secret key (if any) of player  $U$ . The adversarial model that we adopt is a weak-corruption model in the sense that only the long-term secret keys are compromised, but the ephemeral keys or the internal data of the protocol participants are not corrupted. This query models (perfect) forward secrecy.
- $\text{Test}(U, i)$  : This query is allowed only once, at any time during the adversary's execution. A bit  $b \in \{0, 1\}$  is chosen uniformly at random. The adversary is given  $\text{sk}_U^i$  if  $b = 1$ , and a random session key if  $b = 0$ . This oracle computes the adversary's ability to distinguish a real session key from a random one.

An adversary which has access to the  $\text{Execute}$ ,  $\text{Join}$ ,  $\text{Leave}$ ,  $\text{Reveal}$ ,  $\text{Corrupt}$  and  $\text{Test}$  oracles, is considered to be passive while an active adversary is given access to the  $\text{Send}$  oracle in addition. We also use notations

- $\text{sid}_U^i$  : session identity for instance  $\Pi_U^i$ . We set  $\text{sid}_U^i = S = \{(U_1, i_1), \dots, (U_k, i_k)\}$  such that  $(U, i) \in S$  and  $\Pi_{U_1}^{i_1}, \dots, \Pi_{U_k}^{i_k}$  wish to agree upon a common key.
- $\text{pid}_U^i$  : partner identity for instance  $\Pi_U^i$ , defined by  $\text{pid}_U^i = \{U_1, \dots, U_k\}$ , such that  $(U_j, i_j) \in \text{sid}_U^i$  for all  $1 \leq j \leq k$ .
- $\text{acc}_U^i$  : 0/1-valued variable which is set to be 1 by  $\Pi_U^i$  upon normal termination of the session and 0 otherwise.

The adversary can ask Send, Execute, Join, Leave, Reveal and Corrupt queries several times, but Test query is asked only once and on a fresh instance. We say that an instance  $\Pi_U^i$  is *fresh* unless either the adversary, at some point, queried  $\text{Reveal}(U, i)$  or  $\text{Reveal}(U', j)$  with  $U' \in \text{pid}_U^i$  or the adversary queried  $\text{Corrupt}(V)$  (with  $V \in \text{pid}_U^i$ ) before a query of the form  $\text{Send}(U, i, *)$  or  $\text{Send}(U', j, *)$  where  $U' \in \text{pid}_U^i$ . Finally adversary outputs a guess bit  $b'$ . Such an adversary is said to win the game if  $b = b'$  where  $b$  is the hidden bit used by the Test oracle. Let Succ denote the event that the adversary  $\mathcal{A}$  wins the game for a protocol XP. We define

$$\text{Adv}_{\mathcal{A}, \text{XP}} := |2 \text{Prob}[\text{Succ}] - 1|$$

to be the advantage of the adversary  $\mathcal{A}$  in attacking the protocol XP. The protocol XP is said to be a *secure unauthenticated group key agreement* (KA) protocol if there is no polynomial time *passive* adversary with non-negligible advantage. We say that protocol XP is a *secure authenticated group key agreement* (AKA) protocol if there is no polynomial time *active* adversary with non-negligible advantage. Next we define the advantage functions.

$$\begin{aligned} \text{Adv}_{\text{XP}}^{\text{KA}}(t, q_E) &:= \text{the maximum advantage of any passive adversary attacking protocol XP,} \\ &\quad \text{running in time } t \text{ and making } q_E \text{ calls to the Execute oracle.} \\ \text{Adv}_{\text{XP}}^{\text{AKA}}(t, q_E, q_J, q_L, q_S) &:= \text{the maximum advantage of any active adversary attacking protocol XP,} \\ &\quad \text{running in time } t \text{ and making } q_E \text{ calls to the Execute oracle, } q_J \text{ calls to} \\ &\quad \text{Join oracle, } q_L \text{ calls to the Leave oracle and } q_S \text{ calls to the Send oracle.} \end{aligned}$$

### 2.3 Decision Hash Bilinear Diffie-Hellman (DHBDH) Problem

Let  $(G_1, G_2, e)$  be as in Section 2.1. We define the following problem. Given an instance  $(P, aP, bP, cP, r)$  for some  $a, b, c, r \in_R Z_q^*$  and a one way hash function  $H : G_2 \rightarrow Z_q^*$ , to decide whether  $r = H(e(P, P)^{abc}) \bmod q$ . This problem is termed DHBDH problem as defined in [5] and is a combination of the bilinear Diffie-Hellman (BDH) problem and a variation of the hash Diffie-Hellman (HDH) problem. The DHBDH assumption is that there exists no probabilistic, polynomial time, 0/1-valued algorithm which can solve the DHBDH problem with non-negligible probability of success.

### 2.4 Multi-signatures

Multi-signatures allow a group of users to sign a message, such that a verifier can verify that all users indeed signed the message. We use the multi-signatures presented by Boldyreva in [12] which is based on the Boneh-Lynn-Shacham [10] (BLS) pairing based short signature. Formally, a multi-signature scheme consists of three algorithms  $\text{MSig} = (\mathcal{MK}, \mathcal{MS}, \mathcal{MV})$ , where  $\mathcal{MK}$  is the key generation algorithm;  $\mathcal{MS}$  is the signature generation algorithm and  $\mathcal{MV}$  is the signature verification algorithm. We denote by  $\text{Succ}_{\text{DSig}}(t)$  the maximum success probability of any adversary running in time  $t$  to forge signatures for a standard digital signature scheme  $\text{DSig} = (\mathcal{K}, \mathcal{S}, \mathcal{V})$ . Similarly, by  $\text{Succ}_{\text{MSig}}(t)$  the maximum success probability of any adversary running in time  $t$  to break the multi-signature scheme  $\text{MSig}$  based on  $\text{DSig}$ .

## 3 A Provably Secure Dynamic Group Key Agreement Protocol

Our protocol extends the authenticated tree-based multi-party group key agreement protocol of [17] to dynamic case where a user can leave or join the group. The protocol is designed to ensure minimum modification to the computations already precomputed when a user leaves or joins the group. By appropriately

modifying the proof in [17], the security of our protocol can be reduced to that of the unauthenticated protocol of [5] which is a provably secure scheme under DHBDH assumption.

Suppose a set of  $n$  users  $\mathcal{P} = \{U_1, U_2, \dots, U_n\}$  wish to agree upon a secret key. Let  $US$  be a subset of users. Quite often, we identify a user with its instance during the execution of a protocol. In case  $US$  is a singleton set, we will identify  $US$  with the instance it contains. Each user set  $US$  has a representative  $Rep(US)$  and for the sake of concreteness we take  $Rep(US) = U_j$  where  $j = \min\{k : \Pi_{U_k}^{d_k} \in US\}$ . We use the notation  $A[1, \dots, n]$  for an array of  $n$  elements  $A_1, \dots, A_n$  and write  $A[i]$  or  $A_i$  to denote the  $i$ th element of array  $A[\cdot]$ . Let  $G_1 = \langle P \rangle, G_2$  (groups of prime order  $q$ ) and  $e(\cdot, \cdot)$  be as described in Section 2.1. We choose a hash function  $H : G_2 \rightarrow Z_q^*$ . The public parameters are  $params = (G_1, G_2, e, q, P, H)$ . Each user  $U_i \in \mathcal{P}$  chooses  $s_i \in Z_q^*$  at random which it uses as its ephemeral key. These keys are session specific and determine the final common key for the users in a session.

### 3.1 Unauthenticated Key Agreement Protocol of [5]

This section presents an informal description of the unauthenticated protocol of [5]. Security of our dynamic key agreement protocol relies on the security of this scheme.

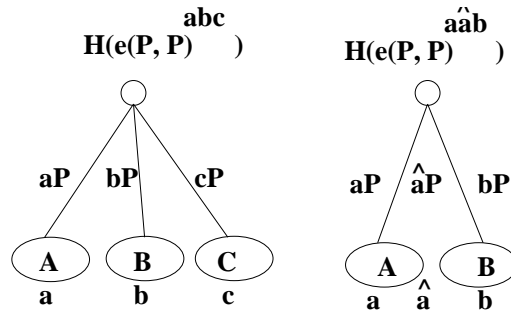


Figure 1: **procedure** CombineThree and **Procedure** CombineTwo

Let  $p = \lfloor \frac{n}{3} \rfloor$  and  $r = n \bmod 3$ . The set of users participating in a session is partitioned into three user sets  $US_1, US_2, US_3$  with respective cardinalities being  $p, p, p$  if  $r = 0$ ;  $p, p, p + 1$  if  $r = 1$ ; and  $p, p + 1, p + 1$  if  $r = 2$ . This top down recursive procedure **KeyAgreement** is invoked for further partitioning to obtain a ternary tree structure (cf. Figure 2). The lowest level 0 consists of singleton users having a secret key. We invoke **CombineTwo**, a key agreement protocol for two user sets and **CombineThree**, a key agreement protocol for three user sets in the key tree thus obtained. We demonstrate these two procedure in Figure 1. For more details, we refer [5].

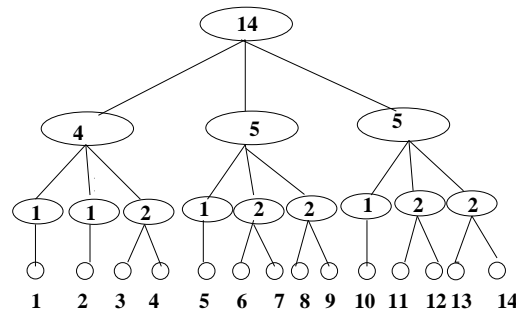


Figure 2: **procedure** KeyAgreement for  $n = 14$

All communications are done by representatives and users in each user set have a common agreed key. In `CombineThree`,  $a, b, c$  respectively are the common agreed key of user sets  $A, B, C$ . Representative of user set  $A$  sends  $aP$  to both the user sets  $B, C$ . Similarly, representative of  $B$  sends  $bP$  to both  $A, C$  and representative of  $C$  sends  $cP$  to both  $A, B$ . After these communications, each user can compute the common agreed key  $H(e(P, P)^{abc})$ . In `CombineTwo`, users in user set  $A$  has common agreed key  $a$ , users in user set  $B$  has common agreed key  $b$ . Representative of  $A$  sends  $aP$  to user set  $B$  and representative of  $B$  sends  $bP$  to user set  $A$ . Besides representative of user set  $A$  generates a random key  $\hat{a} \in Z_q^*$  and sends  $\hat{a}P$  to all the users in both  $A, B$ . After these communications, each user can compute the common agreed key  $H(e(P, P)^{a\hat{a}b})$ .

### 3.2 Authenticated Key Agreement Protocol of [17]

This protocol incorporates secure signature based authentication mechanism into the unauthenticated protocol of Barua, Dutta, Sarkar [17]. Each user  $U_i$  chooses a signing and a verification key  $sk_i$  (or  $sk_{U_i}$ ) and  $pk_i$  (or  $pk_{U_i}$ ) respectively as part of the basic signature scheme `DSig` and multi-signature scheme `MSig`.

Besides, this authentication mechanism uses a variable, partial session-identity  $\text{psid}_{U_{i_j}}^{d_j}$  for each instance  $\Pi_{U_{i_j}}^{d_j}$  which is initially set to be  $\{(U_{i_j}, d_j)\}$  and after completion of the session,  $\text{psid}_{U_{i_j}}^{d_j} = \text{sid}_{U_{i_j}}^{d_j} = \{(U_{i_1}, d_1), \dots, (U_{i_k}, d_k)\}$  where instances  $\Pi_{U_{i_1}}^{d_1}, \dots, \Pi_{U_{i_k}}^{d_k}$  are involved in this session. The session-identity  $\text{sid}_{U_{i_j}}^{d_j}$  uniquely identifies the session and is same for all instances participating in this session. The authenticated protocol consists of an algorithm for two-party authenticated key agreement `AuthCombTwo`, a three-party authenticated key agreement `AuthCombThree-A` and an authenticated key agreement `AuthCombThree-B` among three user sets. These procedures are invoked instead of `CombineTwo` and `CombineThree` in the key tree obtained by the procedure `KeyAgreement` in the unauthenticated protocol described above. We discuss `AuthCombTwo`, `AuthCombThree-A`, `AuthCombThree-B` informally and refer the reader to [17] for details. (See Figure 3.)

In `AuthCombTwo`, user instance  $\Pi_{U_1}^{d_1}$  has a secret key  $s_1$  with partial session-identity  $\text{psid}_{U_1}^{d_1} = \{(U_1, d_1)\}$ . Besides it generates a random key  $s \in Z_q^*$ , computes a signature  $\sigma_1$  using basic signature scheme `DSig` on  $m_1 = (s_1P, sP)$  and sends  $U_1|1|m_1|\sigma_1$  to user instance  $\Pi_{U_2}^{d_2}$ . Similarly, user instance  $\Pi_{U_2}^{d_2}$  with secret key  $s_2$  and partial session-identity  $\text{psid}_{U_2}^{d_2} = \{(U_2, d_2)\}$  computes a basic signature  $\sigma_2$  on  $m_2 = s_2P$  and sends  $U_2|1|m_2|\sigma_2$  to user instance  $\Pi_{U_1}^{d_1}$ . On receiving the message  $U_2|1|m_2|\sigma_2$ , user  $U_1$  verifies  $\sigma_2$  on  $U_2|1|m_2$  according to the verification algorithm  $\mathcal{V}$  of `DSig`. If verification fails, it sets  $\text{acc}_{U_1}^{d_1} = 0$ ,  $\text{sk}_{U_1}^{d_1} = \text{NULL}$  and aborts. Otherwise it sets  $\text{psid}_{U_1}^{d_1} = \text{psid}_{U_1}^{d_1} \cup \{(U_2, d_2)\}$  and computes the key  $H(e(P, P)^{s_1s_2})$ . In a similar way, user  $U_2$  performs verification  $\sigma_1$  on  $U_1|1|m_1$  and computes the key only if verification succeeds. Note that at the end, user instances  $\Pi_{U_1}^{d_1}, \Pi_{U_2}^{d_2}$  have the same partial session-identity :  $\text{psid}_{U_1}^{d_1} = \text{psid}_{U_1}^{d_1} \cup \text{psid}_{U_2}^{d_2} = \text{psid}_{U_2}^{d_2}$ . An analogous description holds for `AuthCombThree-A`. The algorithm `AuthCombThree-B` performs key agreement among three user sets  $US_1, US_2, US_3$  as follows: Suppose  $\Pi_{U_1}^{d_1}$  be the representative of user set  $US_1$  and users in this set have a common agreed key  $s_1$ . Similarly,  $\Pi_{U_2}^{d_2}, s_2$  are those for user set  $US_2$  and  $\Pi_{U_3}^{d_3}, s_3$  for user set  $US_3$ . Let  $m_1 = \text{psid}_{U_1}^{d_1}|t_1|s_1P$ , where  $t_1$  is the next expected message number to be sent by  $\Pi_{U_1}^{d_1}$ . For each user  $V$  in user set  $US_1$ ,  $\text{psid}_V^{d_1}|t_1|s_1P$  is same as  $m_1$ . Each user  $V \in US_1$  computes a basic signature  $\sigma_V$  on  $m_1$  using the scheme `DSig` and sends  $(V, \sigma_V)$  to  $\Pi_{U_1}^{d_1}$ . After accumulating these basic signatures, the representative  $\Pi_{U_1}^{d_1}$  constructs the multi-signature  $\text{msig}_1$  on message  $m_1$  using the scheme `MSig` and sends  $m_1|\text{msig}_1$  to  $US_2 \cup US_3$ . Similarly, representative  $\Pi_{U_2}^{d_2}$  of user set  $US_2$  sends  $m_2|\text{msig}_2$  to

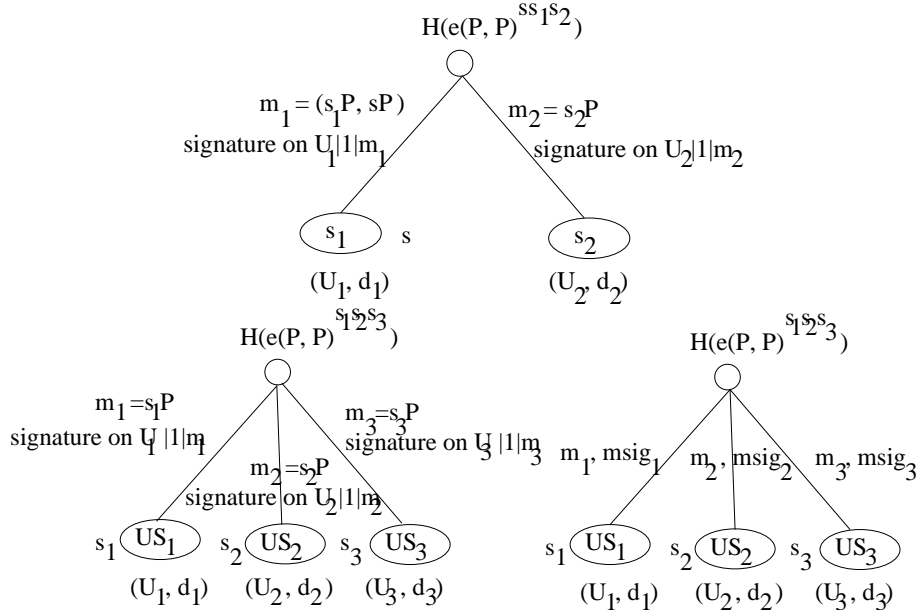


Figure 3: **procedure** AuthCombTwo, AuthCombThree-A and AuthCombThree-B

$US_1 \cup US_3$  and representative  $\Pi_{U_3}^{d_3}$  of user set  $US_3$  sends  $m_3|msig_3$  to  $US_1 \cup US_2$  where  $m_2 = \text{psid}_{U_2}^{d_2}|t_2|s_2P$  and  $m_3 = \text{psid}_{U_3}^{d_3}|t_3|s_3P$ ,  $t_2, t_3$  being the next expected message number to be sent by  $\Pi_{U_2}^{d_2}, \Pi_{U_3}^{d_3}$  respectively.

We define a variable  $\text{First}(\text{psid}_U^d)$  for the ease of discussion. If  $\text{psid}_U^d = \{(U_{i_1}, d_{i_1}), \dots, (U_{i_k}, d_{i_k})\}$ , then  $\text{First}(\text{psid}_U^d) = \{U_{i_1}, \dots, U_{i_k}\}$ . Now on receipt of messages  $m_2|msig_2$  and  $m_3|msig_3$ , each user instance  $\Pi_V^{d_V} \in US_1$  (1) checks  $\text{First}(\text{psid}_{U_2}^{d_2}) \subseteq \text{pid}_V^{d_V}$  and  $\text{First}(\text{psid}_{U_3}^{d_3}) \subseteq \text{pid}_V^{d_V}$ ; (2) verifies  $t_2, t_3$  are the next expected message number to be sent by  $\Pi_{U_2}^{d_2}, \Pi_{U_3}^{d_3}$  respectively; (3) verifies  $msig_2, msig_3$  are multi-signatures on  $m_2, m_3$  respectively. If any of these verification fails,  $\Pi_V^{d_V}$  sets  $\text{acc}_V^{d_V} = 0$  and  $\text{sk}_V^{d_V} = \text{NULL}$  and aborts. Otherwise computes the common key  $H(e(P, P)^{s_1s_2s_3})$  and sets  $\text{psid}_V^{d_V} = \text{psid}_V^{d_V} \cup \text{psid}_{U_2}^{d_2} \cup \text{psid}_{U_3}^{d_3}$ .

Similar verifications are done by each user in user sets  $US_2$  and  $US_3$ . Common key  $H(e(P, P)^{s_1s_2s_3})$  is computed only if verification holds. Note that at the end of an honest execution of this protocol, each user in the group  $US_1 \cup US_2 \cup US_3$  has a common partial session-identity.

### 3.3 Proposed Authenticated Dynamic Key Agreement Protocol

Dynamic key agreement consists of a key agreement protocol together with two additional algorithms, Insert and Delete. The procedure Insert enables a user to join a group. A user can leave a group by invoking the procedure Delete. In this subsection, we describe protocols for insertion and deletion for the above static tree-based authenticated protocol. Our protocol design makes an optimal use of the data precomputed in the procedure KeyAgreement. When a user joins or leaves a group, the structure of the key tree is disturbed and requires to be updated for any subsequent join or leave operation. Maintaining the tree structure of the key agreement protocol is a crucial part of our scheme. We refer this as the preservation of the structure of the procedure KeyAgreement.

Suppose we have a keytree  $T$  of  $n$  users  $\{1, 2, \dots, n\}$  according to the key agreement of [5] with  $k = R(n)$  rounds. For the sake of easy description, we take the user set  $\mathcal{P} = \{1, \dots, n\}$  instead of the set  $\{U_1, \dots, U_n\}$  and introduce some more notations. For  $1 \leq l \leq k$ , let  $U_i^{(l)}$  be the  $i$ -th user set at level  $l$  and  $s_i^{(l)}$  be the

common agreed key of users in the user set  $U_i^{(l)}$  at level  $l$ . Initially,  $U_i^{(0)} = \{i\}$ ,  $s_i^{(0)}$  is the private key randomly chosen by user  $i$  from  $Z_q^*$ .

### 3.3.1 Insertion

Let a new user  $\{u\}$  with private key  $x$  requests for join in the group  $\{1, \dots, n\}$ . He joins the tree in such a way that the structure of KeyAgreement is still preserved and updation of key path is optimal in the sense that minimum updation or recomputation is required. For instance, consider a group key agreement with  $n = 10$  members. In this case, the root node will have 3 subtrees, with the left and middle subtrees having 3 leaves each and the right subtree having 4 leaves. Now suppose a new user wants to join this group. He cannot join the first subgroup (of subusers) since this is contrary to the way we partition the user sets. So the entire group of users will have to be repartitioned. Similarly, he cannot join the third subgroup (of 4 users) without causing repartitioning. But if he joins the second subtree, then there is no need of repartitioning and so key updation is minimal and is only along a single path. This is illustrated in Figure 4. The following Lemma determines uniquely such a path that we call the optimal path of joining of the new user.

**Lemma 3.1** *For  $1 \leq l \leq k$ ,  $k = R(n)$ , define the following:*

- $i_l :=$  the index of the node at level  $l$  whose subtree will contain the new user  $\{u\}$  as a leaf,
- $\eta_l :=$  the number of leaf nodes in the subtree at  $i_l$ ,
- $N_l :=$  number of leaf nodes to the left of node  $i_l$  and
- $r_l = \eta_l \bmod 3$ .

*Clearly  $i_k = 1; \eta_k = n; r_k = n \bmod 3; N_k = 0$ . Then, for  $2 \leq l \leq k$ , we have  $i_{l-1} = 3i_l - r_l; \eta_{l-1} = \lfloor \frac{2\eta_l}{3} \rfloor$ ; and  $N_{l-1} = N_l + (2 - r_l)\eta_{l-1}$ . Thus user  $u$  joins the subgroup at  $i_1$  and  $i_k, i_{k-1}, \dots, i_1$  determine the optimal path along which the subgroup keys needs to be updated or recomputed.*

*Proof :* By induction. The case  $l = k$  is straightforward. Suppose the results hold for  $l < k$ . The node  $i_{l-1}$  is the  $(3 - r_l)$ -th child of the subtree  $T$  at  $i_l$  and there are  $3(i_l - 1)$  nodes at level  $(l - 1)$  to the left of subtree  $T$  in the original tree. Hence  $i_{l-1} = 3(i_l - 1) + (3 - r_l) = 3i_l - r_l$ .

The second one is easy to check. For the third one, we argue as follows: There are  $\eta_l$  leaf nodes in the subtree  $T$  at  $i_l$  and  $i_l$  has exactly three offsprings with subtrees  $T_L$  (left),  $T_M$  (middle) and  $T_R$  (right). If  $r_l = 0$ , i.e. if  $|T_L| = |T_M| = |T_R|$ , then  $i_{l-1}$  is the third child; if  $r_l = 1$ , then  $i_{l-1}$  is the second child and if  $r_l = 2$ , then  $i_{l-1}$  is the first child. In any case, the number of leaf nodes in the subtree at  $i_{l-1}$  is  $\eta_{l-1}$ .

In case 1 above, there are  $2\eta_{l-1}$  leaf nodes to the left of node  $i_{l-1}$  in the subtree  $T$  at  $i_l$ . In case 2, there are  $\eta_{l-1}$  leaf nodes to the left and in case 3, there are zero leaf nodes to the left. In each case, there are  $(2 - r_l)\eta_{l-1}$  leaf nodes to the left of  $i_{l-1}$  in  $T$ . Since there are  $N_l$  leaf nodes to the left of  $i_l$  in the original tree, the total number of leaf nodes to the left of  $i_{l-1}$  is  $N_l + (2 - r_l)\eta_{l-1} = N_{l-1}$ , by definition of  $N_{l-1}$ . This completes the proof. ■

We now describe our protocol Insert. This algorithm invokes a procedure FindKeyPath to find the optimal key path path of joining of a new member and updates path according to the algorithm UpdateKeyPath. The new user who has permission to join the group for a group key agreement computes it's optimal key path of joining  $i_k, \dots, i_1; N_k, \dots, N_1$  by using algorithm FindKeyPath, communicate this to all other members of the group and invokes algorithm UpdateKeyPath to update this path. The formal description of the procedures FindKeyPath and UpdateKeyPath are given below.

**procedure** FindKeyPath( $n$ )



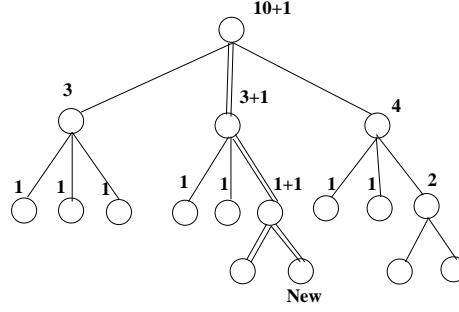


Figure 4: **procedure Insert**

1.  $k = R(n); i_k = 1; N_k = 0; \eta_k = n; r_k = n \bmod 3;$
  2.  $l = k$  downto 2 **do**
  3.  $i_{l-1} = 3i_l - r_l; \eta_{l-1} = \lfloor \frac{\eta_l}{3} \rfloor; r_{l-1} = \eta_{l-1} \bmod 3; N_{l-1} = N_l + (2 - r_l)\eta_{l-1};$
  4. **end do**
  5. **return**  $(i_k, \dots, i_1; \eta_k, \dots, \eta_1; N_k, \dots, N_1);$
- end FindKeyPath**

**procedure UpdateKeyPath(path)**

1. parse  $\text{path} = (i_k, \dots, i_1; \eta_k, \dots, \eta_1; N_k, \dots, N_1);$
2. The node  $i_1$  at level 1 has  $\eta_1$  children which are clearly leaf nodes. Note that  $\eta_1 \leq 3$ .
3. **if**  $\eta_1 = 1$  **then**
4. user  $N_{i_1} + 1$  chooses a new private key  $s_{N_{i_1}+1}^{(0)} \in_R \mathbb{Z}_q^*;$
5.  $\hat{U}_1 = U_{N_{i_1}+1}^{(0)}; \hat{s}_1 = s_{N_{i_1}+1}^{(0)}; \hat{U}_2 = \{u\}; \hat{s}_2 = x;$
6. **call**  $\text{AuthCombTwo}(\hat{U}[1, 2], \hat{s}[1, 2]);$
7.  $s_{i_1}^{(1)}$  is assigned the agreed key between user sets  $\hat{U}_1, \hat{U}_2$  and  $\hat{U}_{i_1}^{(1)} = \hat{U}[1, 2];$
8. **end if**
9. **if**  $\eta_1 = 2$  **then**
10. users  $N_{i_1} + 1, N_{i_1} + 2$  choose new private keys  $s_{N_{i_1}+1}^{(0)}, s_{N_{i_1}+2}^{(0)} \in_R \mathbb{Z}_q^*$  respectively;
11.  $\hat{U}_1 = U_{N_{i_1}+1}^{(0)}; \hat{s}_1 = s_{N_{i_1}+1}^{(0)}; \hat{U}_2 = U_{N_{i_1}+2}^{(0)}; \hat{s}_2 = s_{N_{i_1}+2}^{(0)}; \hat{U}_3 = \{u\}; \hat{s}_3 = x;$
12. **call**  $\text{AuthCombThree} - \text{A}(\hat{U}[1, 2, 3], \hat{s}[1, 2, 3]);$
13.  $s_{i_1}^{(1)}$  is assigned the agreed key between user sets  $\hat{U}_1, \hat{U}_2, \hat{U}_3$  and  $\hat{U}_{i_1}^{(1)} = \hat{U}[1, 2, 3];$
14. **end if**
15. **if**  $\eta_1 = 3$  **then**
16. users  $N_{i_1} + 1, N_{i_1} + 2, N_{i_1} + 3$  choose new private keys  $s_{N_{i_1}+1}^{(0)}, s_{N_{i_1}+2}^{(0)}, s_{N_{i_1}+3}^{(0)}$  respectively;
17.  $\hat{U}_1 = U_{N_{i_1}+1}^{(0)}; \hat{s}_1 = s_{N_{i_1}+1}^{(0)}; \hat{U}_2 = \{u\}; \hat{s}_2 = x;$
18. **call**  $\text{AuthCombTwo}(\hat{U}[1, 2], \hat{s}[1, 2]);$
19.  $s_{N_{i_1}+3}^{(0)}$  is assigned the agreed key between user sets  $\hat{U}_1, \hat{U}_2$
20.  $\hat{U}_3 = \hat{U}[1, 2]; \hat{s}_3 = s_{N_{i_1}+3}^{(0)}; \hat{U}_2 = U_{N_{i_1}+2}^{(0)}; \hat{s}_2 = s_{N_{i_1}+2}^{(0)}; \hat{U}_1 = U_{N_{i_1}+1}^{(0)}; \hat{s}_1 = s_{N_{i_1}+1}^{(0)};$
21. **call**  $\text{AuthCombThree} - \text{B}(\hat{U}[1, 2, 3], \hat{s}[1, 2, 3]);$
22.  $s_{i_1}^{(1)}$  is assigned the agreed key between user sets  $\hat{U}_1, \hat{U}_2, \hat{U}_3$  and  $\hat{U}_{i_1}^{(1)} = \hat{U}[1, 2, 3];$

```

23. end if
24.  $l = 1$  to  $k - 1$  do
25.    $j = i_l$ ;
26.   if  $i_l = 3i_{l+1} - 2$  i.e.  $r_{l+1} = 2$  then
27.     call AuthCombThree – B( $U^{(l)}[j, j + 1, j + 2], s^{(l)}[j, j + 1, j + 2]$ );
28.      $s_{i_{l+1}}^{(l+1)}$  is assigned the agreed key among user sets  $\hat{U}_j^{(l)}, \hat{U}_{j+1}^{(l)}, \hat{U}_{j+2}^{(l)}$ ;  $U_{i_{l+1}}^{(l+1)} = U^{(l)}[j, j + 1, j + 2]$ ;
29.   end if
30.   if  $i_l = 3i_{l+1} - 1$  i.e.  $r_{l+1} = 1$  then
31.     call AuthCombThree – B( $U^{(l)}[j - 1, j, j + 1], s^{(l)}[j - 1, j, j + 1]$ );
32.      $s_{i_{l+1}}^{(l+1)}$  is assigned the agreed key among user sets  $\hat{U}_{j-1}^{(l)}, \hat{U}_j^{(l)}, \hat{U}_{j+1}^{(l)}$ ;  $U_{i_{l+1}}^{(l+1)} = U^{(l)}[j - 1, j, j + 1]$ ;
33.   end if
34.   if  $i_l = 3i_{l+1}$  i.e.  $r_{l+1} = 0$  then
35.     call AuthCombThree – B( $U^{(l)}[j - 2, j - 1, j], s^{(l)}[j - 2, j - 1, j]$ );
36.      $s_{i_{l+1}}^{(l+1)}$  is assigned the agreed key among user sets  $\hat{U}_{j-2}^{(l)}, \hat{U}_{j-1}^{(l)}, \hat{U}_j^{(l)}$ ;  $U_{i_{l+1}}^{(l+1)} = U^{(l)}[j - 2, j - 1, j]$ ;
37.   end if
38. end do
end UpdateKeyPath

```

The algorithm UpdateKeyPath works as follows to update keys in level 1 on joining of the new user. In the key tree  $T$  with  $n$  users, the number of children of the node  $i_1$  (node at level 1) in the optimal key path is either 1 or 2 or 3. If  $i_1$  has 1 leaf node, then the user corresponding to this leaf node chooses a new private key, agree upon a common key with the new user by invoking algorithm AuthCombTwo and the corresponding user set for level 1 is modified to a set that includes these two users. In case  $i_1$  has 2 leaf nodes, the users corresponding to these leaves choose new private keys, a new user set for level 1 is constructed that contains these two users and the new user and algorithm AuthCombThree-A is invoked to agree upon a common key among them. If  $i_1$  has 3 leaves, then the users corresponding to these leaves choose new private keys. The rightmost user agree upon a common key with the new user by invoking algorithm AuthCombTwo and constructs a new user set that consists of the new user and itself. Then AuthCombThree-B is invoked for this new user set and the the other two leaves of  $i_1$  to agree upon a common key. Finally the corresponding user set for level 1 is modified to a set that includes these users.

The subsequent user sets are accordingly changed by algorithm UpdateKeyPath and key updates in level  $l + 1$  ( $1 \leq l \leq k - 1$ ) are done by invoking algorithm AuthCombThree-B among the three user sets which are subtrees of node  $i_{l+1}$ . The modified user set corresponding to the node  $i_l$  invokes AuthCombThree-B to agree upon a common key with the user sets corresponding to the other two subtrees (siblings of  $i_l$ ) of node  $i_{l+1}$  and a new user set for level  $l + 1$  is constructed that is the union of these three user sets. We proceed in this way and finally a common key is agreed among all the  $n + 1$  users. At the end, we newly index the members (leaves) as  $\{1, 2, \dots, n + 1\}$ .

**Remark 3.2** *The insertion algorithm presented above is an authenticated algorithm. If we invoke CombineTwo in lines 6, 18 instead of AuthCombTwo and CombineThree in lines 12, 21, 27, 31, 35 instead of AuthCombThree-A and AuthCombThree-B, we get an unauthenticated version of the insertion algorithm. We require  $k = \lfloor \log_3 n \rfloor + 1$  key updates in the execution of Insert. After insertion of the new member,  $R(n + 1) = k + 1$  if  $n = 3^k$ , and  $k$  otherwise.*

### 3.3.2 Deletion

Suppose key tree  $T$  with  $n$  leaf nodes  $\{1, 2, \dots, n\}$  has the tree structure used in the procedure `KeyAgreement` and suppose a member  $j_0$ ,  $1 \leq j_0 \leq n$ , wants to leave the group. For this we first introduce a procedure `Extract` which outputs the identity or index of a leaf node in  $T$  such that the structure of `KeyAgreement` is preserved in the tree after removal of this node. We take a designated user, called group controller (GC) to initiate the operation `Delete`. To be specific, we take one sibling of the node leaving the group as the GC which is trusted only for this purpose.

The procedure `Extract` works as follows. If all the three subtrees  $T_L, T_M, T_R$  of the tree  $T$  have equal number of leaf nodes, then removal of a leaf node from the leftmost subtree  $T_L$  will not disturb the tree structure and so we can extract a leaf node from  $T_L$ . Similarly, if the number of leaves in both  $T_L, T_M$  are same, say  $p$ , and that of right subtree  $T_R$  is  $p + 1$ , then we can extract a leaf from  $T_R$  without disturbing the tree structure. If the number of leaves in  $T_M, T_R$  are same, say,  $p + 1$  and that of  $T_L$  is  $p$ , then we can extract a leaf from  $T_M$  retaining the tree structure. We recursively apply this procedure on  $T_L, T_R$  or  $T_M$  chosen in this manner and finally reach to a leaf node. The index of the user corresponding to this leaf node is outputted to the GC.

**procedure** `Extract` ( $T, n$ )

1. **if**  $n = 1$  **then return** the index of the leaf node to GC;
  2. **if**  $n = 2$  **then return** the index of the left leaf node to GC;
  3. Let  $T_L, T_M, T_R$  be respectively the left, middle and right subtree of  $T$ ;
  4.  $p = \lfloor \frac{n}{3} \rfloor$ ;  $r = n \bmod 3$ ;
  5. **while** ( $p \geq 1$ ) **do**
  6.     **if**  $r = 0$  *i.e.*  $|T_L| = |T_M| = |T_R| = p$  **then**
  7.         **call** `Extract` ( $T_L, p$ );
  8.     **end if**
  9.     **if**  $r = 1$  *i.e.*  $|T_L| = |T_M| = p, |T_R| = p + 1$  **then**
  10.         **call** `Extract` ( $T_R, p + 1$ );
  11.     **end if**
  12.     **if**  $r = 2$  *i.e.*  $|T_L| = p, |T_R| = |T_M| = p + 1$  **then**
  13.         **call** `Extract` ( $T_M, p + 1$ );
  14.     **end if**
  15. **end do**
- end** `Extract`

Now a user corresponding to leaf  $j_0$  leaves the group, the tree structure is disturbed. We first find the highest level, say  $i$ , for which the subtree rooted at the internal node at, say  $j_i$ , lacks the tree structure. Consequently,  $j_i$  is the root of the highest level subtree with disturbed tree structure on removal of  $j_0$ . To retain the the tree structure, we may need to extract suitably a leaf node  $l_0$  from the tree  $T$  in such a way that the key updates required are minimal. We do this by using an algorithm `FindExtractNode` that uses the procedure `Extract` as a subroutine and removes the leaving member  $j_0$ , still preserving the structure of `KeyAgreement` in the resulting key tree. This algorithm outputs the index of the extracted leaf node  $l_0$  to GC and also the index of the internal node  $j_i$ . The procedure `FindExtractNode` is formally described below.

**procedure** `FindExtractNode` ( $T, n, j_0$ )

1.  $k = R(n)$ ;

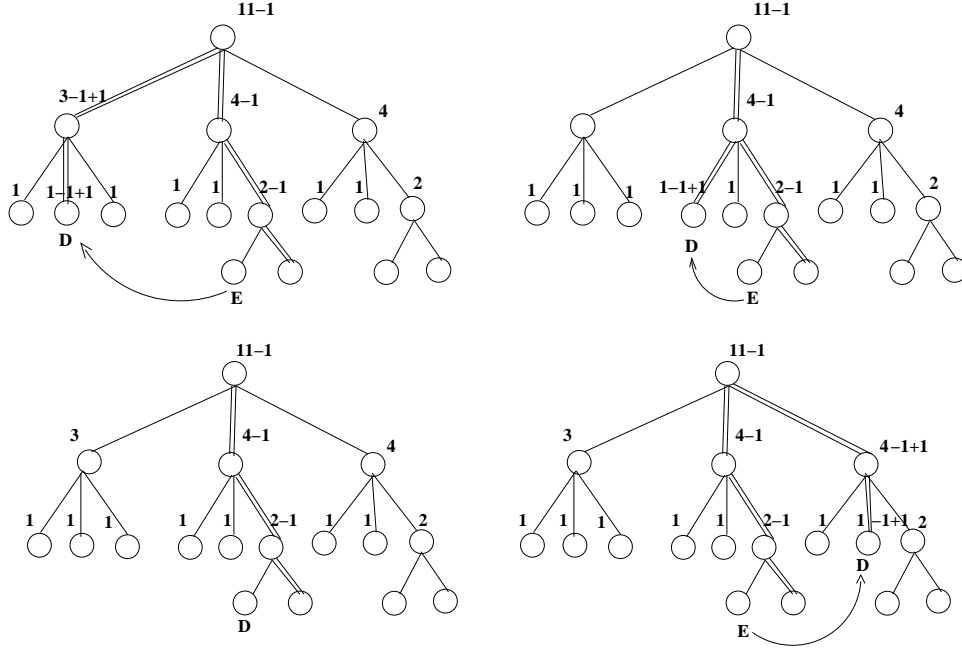


Figure 5: Different cases of **procedure** Delete with  $n = 11$  (D denotes the node to be deleted and E denote the node to be extracted to maintain the key structure)

2. After removing node  $j_0$ , let  $i (\leq k)$  be the highest level for which the subtree rooted at  $j_i$  lacks the structure of **KeyAgreement**. (In case  $i < k$ , this structure is retained in the other two siblings of  $j_i$ );
  3. Let the left, middle and right subtrees of offsprings of the node  $j_i$  be  $T_L, T_M, T_R$  respectively;
  4. **Case 1** : Before removing  $j_0$ , let  $|T_L| = |T_M| = |T_R| = p$ ; remove node  $j_0$ ;
  5.   **if** ( $j_0$  is leaf node of  $T_M$  or  $T_R$ ) **then**
  6.     **call** Extract ( $T_L, p$ );
  7.   **end if**
  8. **Case 2** : Before removing  $j_0$ , let  $|T_L| = |T_M| = p$ ;  $|T_R| = p + 1$ ; remove node  $j_0$ ;
  9.   **if** ( $j_0$  is leaf node of  $T_L$  or  $T_M$ ) **then**
  10.    **call** Extract ( $T_R, p + 1$ );
  11.   **end if**
  12. **Case 3** : Before removing  $j_0$ , let  $|T_L| = p$ ;  $|T_M| = |T_R| = p + 1$ ; remove node  $j_0$ ;
  13.   **if** ( $j_0$  is leaf node of  $T_L$  or  $T_R$ ) **then**
  14.    **call** Extract ( $T_M, p + 1$ );
  15.   **end if**
  16. Let  $l_0$  be the extracted node to be inserted as the  $j_0$ -th leaf in the tree  $T$ ;
  17. **return** ( $l_0, j_i$ );
- end** FindExtractNode

Next we describe the algorithm Delete below. Our algorithm Delete invokes FindExtractNode to obtain the index  $l_0$  of the node to be extracted (if required), finds from the tree  $T$  the path from root to the parent of the leaving leaf node  $j_0$  and also the path from root to the parent of the extracted leaf node  $l_0$ . Two new user sets are constructed in level 1: one user set includes the user corresponding to  $l_0$  together with the users corresponding to brothers of  $j_0$  and another user set includes only the users corresponding to

brothers of  $l_0$ . All the users in these two new user sets choose new private keys. The subsequent higher level user sets are modified accordingly and appropriate algorithms AuthCombTwo, AuthCombThree-A or AuthCombThree-B are invoked for successive key agreements in the key tree. We invoke AuthCombTwo for two party authenticated key agreement, AuthCombThree-A for three party authenticated key agreement and AuthCombThree-B for authenticated key agreement among three user sets. At the end of the procedure Delete, we newly index the users (leaves) as  $\{1, 2, \dots, n - 1\}$ . Some particular cases are shown in Figure 5 (See Appendix.)

**procedure** Delete ( $T, n, j_0$ )

1. Let  $k = R(n)$ .
2. **call** FindExtractNode( $T, n, j_0$ );
3. Let  $(l_0, j_i)$  be it's output.
4. **if**  $l_0 = j_0$  **then**
5.   let  $\text{path}_D = (j_k, \dots, j_1)$  be the path from root to parent of  $j_0$ . The siblings of user  $j_0$  choose random new keys and form a new user set  $U_{j_1}^{(1)}$ . The subsequent user sets  $U_{j_t}^{(t)}$  are accordingly modified replacing old  $U_{j_1}^{(1)}$  by the new one and the keys updated as in UpdateKeyPath.
6. **end if**
7. **else**
8.   let  $\text{path}_D = (j_k, \dots, j_i, j_{i-1}, \dots, j_1)$  and  $\text{path}_E = (j_k, \dots, j_i, l_{i-1}, \dots, l_1)$  be the paths from root to the parent of the leaving leaf node  $j_0$  and extracted leaf node  $l_0$  respectively, where  $j_t$  denotes the index of the node at level  $t$  whose subtree contains the leaving node  $j_0$  and  $l_t$  denotes the index of the node at level  $t$  whose subtree contains the extracted node  $l_0$ .
9.   User  $l_0$  and its brothers choose random new keys; also brothers of user  $j_0$  choose random new keys;
10.   User  $l_0$  and brothers of  $j_0$  construct new user set  $U_{j_1}^{(1)}$ ; brothers of user  $l_0$  constructs new set  $U_{l_1}^{(1)}$ .
11.   The subsequent user sets  $U_{j_t}^{(t)}, U_{l_t}^{(t)}$ , for  $2 \leq t \leq k$ , are modified accordingly replacing the old  $U_{j_1}^{(1)}$  and  $U_{l_1}^{(1)}$  values by their new values. Note that  $|U_{l_1}^{(1)}| \neq 0$ .
12.   **do in parallel**  $t = 1$  to  $i - 1$
13.     • user sets of level  $t - 1$  in  $U_{j_t}^{(t)}$  agree upon a common key by invoking appropriate subroutine for authenticated key agreement depending upon the cardinality of  $U_{j_t}^{(t)}$ .
14.     • user sets of level  $t - 1$  in  $U_{l_t}^{(t)}$  agree upon a common key by invoking appropriate subroutine for authenticated key agreement depending upon the cardinality of  $U_{l_t}^{(t)}$ .
15.   **end do**;
16.   **do**  $t = i$  to  $k$
17.     user sets of level  $t - 1$  in  $U_{j_t}^{(t)}$  agree upon a common key by invoking appropriate subroutine for authenticated key agreement depending upon the cardinality of  $U_{j_t}^{(t)}$ .
18.   **end do**;
19. **end else**
- end** Delete

**Remark 3.3** *The invocation of AuthCombTwo, AuthCombThree-A and AuthCombThree-B in the above procedure make the protocol Delete authenticated. If instead, we use unauthenticated CombineTwo and CombineThree, an unauthenticated version of the deletion algorithm is obtained. At most  $2\lfloor \log_3(n + 1) \rfloor$  key updates are required in the execution of Delete. After deletion of a member,  $R(n - 1) = k + 1$  if  $n = 3^k + 1$ , and  $k$  otherwise.*

## 4 Security Analysis

We will show that our dynamic authenticated key agreement protocol DAP is secure in the model as described in Subsection 2.2. In fact, we can convert any active adversary attacking the protocol DAP into a passive adversary attacking the unauthenticated protocol UP assuming that both DSig and MSig are secure and DHBDH problem is hard. No Corrupt query appears since long term secret keys are not used. So our protocol trivially achieves forward secrecy. We state the security results of the unauthenticated protocol [5] UP and authenticated protocol [17] in Theorem 4.1 and Theorem 4.2. For the proofs of these results, see [5, 17]. We prove our main result in Theorem 4.3 that states the security of our dynamic authenticated key agreement protocol DAP.

**Theorem 4.1** [5] *The group key agreement protocol UP described in Section 3.1 is secure against passive adversaries under the assumption that DHBDH problem is hard.*

**Theorem 4.2** [17] *The group key agreement protocol AP described in Section 3.2 satisfies the following:*

$$\text{Adv}_{\text{AP}}^{\text{AKA}}(t, q_E, q_S) \leq \text{Adv}_{\text{UP}}^{\text{KA}}(t', q_E + q_S/2) + |\mathcal{P}| \text{Succ}_{\text{DSig}}(t') + |\mathcal{P}| \text{Succ}_{\text{MSig}}(t')$$

where  $t' \leq t + (|\mathcal{P}|q_E + q_S)t_{\text{AP}}$ , where  $t_{\text{AP}}$  is the time required for execution of AP by any one of the users.

**Theorem 4.3** *The dynamic group key agreement protocol DAP described in Section 3.3 satisfies the following:*

$$\text{Adv}_{\text{DAP}}^{\text{AKA}}(t, q_E, q_J, q_L, q_S) \leq \text{Adv}_{\text{UP}}^{\text{KA}}(t', q_E + (q_J + q_L + q_S)/2) + |\mathcal{P}| \text{Succ}_{\text{DSig}}(t') + |\mathcal{P}| \text{Succ}_{\text{MSig}}(t')$$

where  $t' \leq t + (|\mathcal{P}|q_E + q_J + q_L + q_S)t_{\text{DAP}}$ , where  $t_{\text{DAP}}$  is the time required for execution of DAP by any one of the users,  $q_E, q_J, q_L$  and  $q_S$  are respectively the maximum number of execute, join, leave and send queries that an adversary can make.

*Proof (Sketch)* : Let  $\mathcal{A}'$  be an adversary which attacks the dynamic authenticated protocol DAP. Using this we construct an adversary  $\mathcal{A}$  which attacks the unauthenticated protocol UP. As in [17], we have the following claim.

**Claim** : Let Forge be the event that a signature (either of DSig or of MSig) is forged by  $\mathcal{A}'$ . Then  $\text{Prob}[\text{Forge}] \leq |\mathcal{P}| \text{Succ}_{\text{MSig}}(t') + |\mathcal{P}| \text{Succ}_{\text{DSig}}(t')$ .

Adversary  $\mathcal{A}$  maintains a list Tlist to store pairs of session IDs and transcripts. It also uses two lists Jlist and Llist to be specified later. Adversary  $\mathcal{A}$  generates the verification/signing keys  $pk_U, sk_U$  for each user  $U \in \mathcal{P}$  and gives the verification keys to  $\mathcal{A}'$ . If ever the event Forge occurs, adversary  $\mathcal{A}$  aborts and outputs a random bit. Otherwise,  $\mathcal{A}$  outputs whatever bit is eventually output by  $\mathcal{A}'$ . Note that since the signing and verification keys are generated by  $\mathcal{A}$ , it can detect occurrence of the event Forge.  $\mathcal{A}$  simulates the oracle queries of  $\mathcal{A}'$  using its own queries to the Execute oracle. We provide details below.

**Execute and Send queries:** These queries are simulated as in [17]. Apart from the usual send queries, there are two special type of send queries, Send<sub>J</sub> and Send<sub>L</sub>. If an unused instance  $\Pi_U^d$  wants to join the group  $\Pi_{U_{i_1}}^{d_1}, \dots, \Pi_{U_{i_k}}^{d_k}$ , then  $\mathcal{A}'$  will make Send<sub>J</sub>( $U, d, \langle U_{i_1}, \dots, U_{i_k} \rangle$ ) query. This query initiates Join( $\{(U_{i_1}, d_1), \dots, (U_{i_k}, d_k)\}, (U, d)$ ) query.  $\mathcal{A}$  first finds a unique entry of the form  $(S, T)$  in Tlist with  $S = \{(U_{i_1}, d_1), \dots, (U_{i_k}, d_k)\}$ . If no such entry,  $\mathcal{A}$  makes an execute query to its own execute oracle on  $S$  and gets a transcript  $T$ .  $\mathcal{A}$  then stores  $(S, U|d, T)$  in Jlist. Similarly, when Send<sub>L</sub>( $U, d, \langle U_{i_1}, \dots, U_{i_k} \rangle$ ) query is made,  $\mathcal{A}$  stores  $(S, U|d, T)$  in Llist.

**Join queries :** Suppose  $\mathcal{A}'$  makes a query  $\text{Join}(\{(U_{i_1}, d_1), \dots, (U_{i_k}, d_k)\}, (U, d))$ .  $\mathcal{A}$  finds an entry of the form  $(S, U|d, T)$  in  $\text{Jlist}$  where  $S = \{(U_{i_1}, d_1), \dots, (U_{i_k}, d_k)\}$ . If no such entry, then the adversary  $\mathcal{A}'$  is given no output. Otherwise,  $\mathcal{A}$  modifies  $T$  as follows:  $\mathcal{A}$  can find the path of joining of  $U$  in the key tree with leafs  $U_{i_1}, \dots, U_{i_k}$  and detect the positions in  $T$  where the new messages are to be injected or where the old messages are to be replaced by new messages.  $\mathcal{A}$  does these modifications in  $T$  according to the unauthenticated version (see Remark 3.2) of the algorithm `Insert` described in Section 3.3.1 and gets a modified transcript  $T_M$ . It then patches appropriate basic signatures and multi-signatures with each message in  $T_M$  according to the modifications described in Section 3.2. Thus  $\mathcal{A}$  expands the transcript  $T_M$  into a transcript  $T'$  for DAP. It returns  $T'$  to  $\mathcal{A}'$ .

**Leave queries :** These queries are simulated as Join queries with modified transcript  $T_M$  obtained from unauthenticated transcript  $T$  according to the algorithm `Delete` in Section 3.3.2.

**Reveal/Test queries :** Suppose  $\mathcal{A}'$  makes the query  $\text{Reveal}(U, i)$  or  $\text{Test}(U, i)$  for an instance  $\Pi_U^i$  for which  $\text{acc}_U^i = 1$ . At this point the transcript  $T'$  in which  $\Pi_U^i$  participates has already been defined. If  $T'$  corresponds to the transcript of the authenticated protocol, then  $\mathcal{A}$  finds the unique pair  $(S, T)$  in  $\text{Tlist}$  such that  $(U, i) \in S$ . Assuming that the event `Forge` does not occur,  $T$  is the unique unauthenticated transcript which corresponds to the transcript  $T'$ . Then  $\mathcal{A}$  makes the appropriate `Reveal` or `Test` query to one of the instances involved in  $T$  and returns the result to  $\mathcal{A}'$ . Otherwise,  $T'$  is the transcript for `Join` or `Leave`, as the case may be. Since  $T'$  has been simulated by  $\mathcal{A}$ ,  $\mathcal{A}$  is able to compute the modified session key and hence send an appropriate reply to  $\mathcal{A}'$ .

As long as `Forge` does not occur, the above simulation for  $\mathcal{A}'$  is perfect. Whenever `Forge` occurs, adversary  $\mathcal{A}$  aborts and outputs a random bit. So  $\text{Prob}_{\mathcal{A}', \text{AP}}[\text{Succ} | \text{Forge}] = \frac{1}{2}$ . Using this, one can show

$$\text{Adv}_{\mathcal{A}, \text{UP}} \geq \text{Adv}_{\mathcal{A}', \text{DAP}} - \text{Prob}[\text{Forge}]$$

The adversary  $\mathcal{A}$  makes an `Execute` query for each `Execute` query of  $\mathcal{A}'$ .  $\mathcal{A}'$  makes  $q_J$  `Join` queries and  $q_L$  `Leave` queries. These queries are initialized respectively by `SendJ` and `SendL` queries of  $\mathcal{A}'$ . Now each of `SendJ` and `SendL` query of  $\mathcal{A}'$  makes at most one `Execute` query of  $\mathcal{A}$ . Thus there are at most  $q_J + q_L$  `execute` query made by  $\mathcal{A}$  to respond all the `SendJ` and `SendL` queries of  $\mathcal{A}'$ . Also  $\mathcal{A}$  makes an `Execute` query for each session started by  $\mathcal{A}'$  using `Send` queries. Since a session involves at least two instances, such an `Execute` query is made after at least two `Send` queries of  $\mathcal{A}'$ . Thus there are  $(q_S - q_J - q_L)/2$  `execute` queries of  $\mathcal{A}$  to respond all other `Send` queries of  $\mathcal{A}'$ , where  $q_S$  is the number of `Send` queries made by  $\mathcal{A}'$ . Hence the total number of `Execute` queries made by  $\mathcal{A}$  is at most  $q_E + q_J + q_L + (q_S - q_J - q_L)/2 = q_E + (q_J + q_L + q_S)/2$ , where  $q_E$  is the number of `Execute` queries made by  $\mathcal{A}'$ . Also since  $\text{Adv}_{\mathcal{A}, \text{UP}}(t, q_E, q_J, q_L, q_S) \leq \text{Adv}_{\text{UP}}^{\text{KA}}(t', q_E + q_J/2 + q_L/2 + q_S/2)$  by assumption, we obtain:

$$\text{Adv}_{\text{DAP}}^{\text{AKA}} \leq \text{Adv}_{\text{UP}}^{\text{KA}}(t', q_E + (q_J + q_L + q_S)/2) + \text{Prob}[\text{Forge}].$$

This yields the statement of the theorem. ■

## 5 Conclusion

We describe a dynamic group key agreement in tree-based setting and prove that the scheme is provably secure under the assumption that DHBDH problem is hard. Our protocol is an extension of the protocols of Barua *et al.* [5] and Dutta *et al.* [17]. The bilinear pairing based Joux [21] protocol and multi-signature by Boldyreva [12] are used. The protocol is proven to be secure in the standard formalized security model of Bresson *et al.* [14] by appropriately modifying the Katz-Yung [22] technique to tree-based setting. One can be benefited in both communication and computation power by combining our protocol with an efficient

constant round protocol. Designing such hybrid group key agreement protocols may be desirable for certain applications, in particular when large number of user groups are concerned.

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